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Quasi-Three-Dimensional Nonreflecting Boundary Conditions for Euler Equations Calculations

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This article presents a theory for the construction of steady-state quasi-three-dimensional nonreflecting boundary conditions for the Euler equations. These allow calculations to be performed on truncated domains without the generation of spurious nonphysical reflections at the far-field boundaries. The theory is based upon Fourier analysis and eigenvectors applied to the linearized Euler equations. It is presented within the context of transonic axial flow turbomachinery computations. The effectiveness of the new boundary conditions is demonstrated by comparing results obtained using this new formulation and calculations performed with the standard one-dimensional approach.

Nomenclature

c = speed of sound
h_t = stagnation enthalpy
k, l = wave numbers in the x and y directions, respectively
P = pitch
p = static pressure
p_t = stagnation pressure
R = radius, $\sqrt{y^2 + z^2}$
s = entropy
t = time
U = vector of primitive variables
u, v, w = Cartesian velocity components in x, y, z directions, respectively
u_x, u_θ, u_R = cylindrical velocity components in x, θ, R directions, respectively
u^R, v^L = right and left eigenvectors
α_θ, α_R = tangential and radial flow angles, respectively
β = pressure wave parameter
γ = ratio of specific heats
ρ = static density
σ = under-relaxation factor
Φ = vector of linearized characteristic variables
Ω = angular speed

Subscripts
F = flux-averaged quantity
inl = inlet
out = outlet

Superscripts
n = time index
$\bar{}$ = circumferential arithmetic average
$\hat{}$ = Fourier transformed quantity
$\tilde{}$ = linearized perturbation

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I. Introduction

ONE typical difficulty occurring in a numerical simulation of a turbomachine flowfield is the handling of the boundary conditions (b.c.). This is because in an internal flow environment the computation has to be performed on truncated domains, whose far-field boundaries do not represent an undisturbed known flowfield as in external aerodynamics. Typically, most of the codes available today are not capable of preventing spurious, nonphysical reflections at inflow and outflow boundaries. This leads to erroneous performance predictions, since the calculated flowfield is dependent on the position of the far-field boundary condition. Also for secondary flow calculations accurate boundary conditions are needed since using the standard one-dimensional approach corrupts the solution locally, as shown later in the results section.

The theoretical foundations of nonreflecting boundary conditions for model initial boundary value problems have been established by mathematicians specializing in the analysis of partial differential equations, see for instance Refs. 1 and 2. Some applications involving the Euler equations of fluid dynamics have been done. For two-dimensional steady-state flow, exact nonreflecting boundary conditions for the solution of the linearized Euler equations can be derived using Fourier expansion in the direction along the inlet and the exit boundaries. This has been done by Ferm and Gustafsson for an airfoil and a channel flow.[3,4] Hirsch and Verhoff[5] used a similar approach for cascade flows, though expanding the characteristic variables instead of the primitive ones used by Ferm and Gustafsson. In Ref. 6 Giles presented a unified theory on the formulation of nonreflecting boundary conditions and the application to the Euler equations. In particular, he derived different types of boundary conditions. These include exact one-dimensional and two-dimensional as well as approximate two-dimensional boundary conditions to be used for steady and unsteady flows. A different approach has been proposed by Bayliss and Turkel.[7] They used the asymptotic behavior of the wave equation to derive a boundary condition formulation for external flows.

The purpose of this article is to present a quasi-three-dimensional nonreflecting boundary condition formulation that can be used in a numerical simulation of steady-state inviscid flowfields. The objective in formulating the nonreflecting boundary conditions is to prevent nonphysical reflections at inflow and outflow boundaries as well as at stator/rotor interfaces. The method is an adaptation of the exact two-dimensional steady nonreflecting boundary conditions of Giles[6] to three dimensions. The theoretical approach, based upon Fourier analysis and eigenvectors is presented here, as well