ANALYSIS OF THE EFFECT OF MISTUNING ON TURBOMACHINERY AEROELASTICITY

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Abstract. This paper looks at the effect of alternate and random mistuning on flutter and forced response in turbomachinery. Two levels of asymptotic analysis are used, and their accuracy is assessed by comparison with the exact solution obtained by direct numerical computation. Monte Carlo simulations are used to assess the effects of random mistuning. The results demonstrate the effectiveness of mistuning in improving flutter stability, and the dependence of the maximum amplitude of forced response on the mistuning pattern, the ratio of mistuning to coupling, and the mode number of the excitation.

1. Introduction

Blade flutter and forced response may lead to dangerous mechanical failures if not properly accounted for in the design phase of an engine. The aeroelastic analysis of bladed rotors is dramatically simplified by the assumption of cyclic symmetry, which allows one to investigate this problem by considering a single blade with a suitable periodic boundary condition, rather than the whole bladed disk. However, probabilistic factors like manufacturing tolerances make questionable the validity of tuned analyses. The structurally tuned and mistuned assemblies can behave in a remarkably different fashion. There is evidence that (a) mistuning improves the flutter boundary [6, 8], (b) mistuning can either increase or reduce the blade forced response [6, 3]. The use of perturbation techniques for turbomachinery aeroelasticity [1] has proved that both effects are influenced by the ratio between the level of mistuning and the inter-blade coupling, which can be aerodynamic [8], mechanical [10, 11] or both [7]. The particular mistuning pattern also plays a significant role [2]. One is interested in mistuning for (a) accounting for the effects of stochastic factors like manufacturing tolerances on the flutter boundaries [9], (b) assessing the applicability of selected mistuning patterns as a measure of passive flutter control and (c) understanding its side-effects on the blade forced response.

In this paper, the mechanisms through which alternate and random mistuning affect the free and forced response are enlightened by means of asymptotic expansions, matrix perturbation theory, exact numerical solution of the aeroelastic equations and Monte Carlo simulations.

2. Model problem

To simplify the analysis, we consider a model problem with N blades each with a single degree-of-freedom $u_j(t), j = 1, 2, ..., N$. After a suitable non-dimensionalisation, the equations of motion in the absence of any external forcing are assumed to be of the form

$$\begin{aligned} \ddot{u}_j + (1+\epsilon\,\sigma\,m_j)\; u_j \;&=\; \epsilon\,(\; a_{-1}u_{j-1} + a_0 u_j + a_1 u_{j+1} \\ &\qquad + b_{-1}\dot{u}_{j-1} + b_0\dot{u}_j + b_1\dot{u}_{j+1}\;)\,. \end{aligned} \tag{1}$$

with the blade indices being modulus N so that $u_0 \equiv u_N$ and $u_1 \equiv u_{N+1}$. The left-hand side of the equation has the structural inertial and stiffness terms, with $\epsilon\sigma m_j$ being the structural mistuning which is assumed to have zero mean and r.m.s. variation $\epsilon\sigma$. The right-hand side has the forces due to aerodynamic coupling, with it being assumed that a blade only experiences forces due to its motion and its two neighbours, and the unsteadiness is of a low frequency so the motion is well represented by the displacement and velocity of each blade.

Looking for eigenmodes with $u_j(t)$ being the j^{th} element of the vector $\exp(st)\,\boldsymbol{u}$ gives the equation

$$\Big((s^2{+}1)I + \epsilon\,(\sigma M - A - sB)\Big)\,\boldsymbol{u} = 0, \tag{2}$$

in which M is a diagonal matrix, and A and B are tridiagonal circulant matrices. By defining $\boldsymbol{u}_0 = \boldsymbol{u},\ \boldsymbol{u}_1 = s\boldsymbol{u}$, this can also be written as

$$s\begin{pmatrix} \boldsymbol{u}_0 \\ \boldsymbol{u}_1 \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I - \epsilon(\sigma M - A) & \epsilon B \end{pmatrix}\begin{pmatrix} \boldsymbol{u}_0 \\ \boldsymbol{u}_1 \end{pmatrix}.$$

In this form, one can use standard mathematical software such as MATLAB to obtain the $2N$ eigenvalues. When ϵ is small, these come in complex conjugate pairs, with N eigenvalues near i and N counterparts near $-i$. Of particular interest is the pair of eigenvalues with the largest real component $\mathcal{R}(s)$ since these give the component of the general solution which grows fastest in time (if $\mathcal{R}(s) > 0$) or decays to zero most slowly (if $\mathcal{R}(s) < 0$).

3. First level asymptotic analysis

A key feature of aeroelasticity in turbomachinery is that ϵ, a constant representing the order of magnitude of the structural mistuning and aerodynamic effects, is very small. A representative value of 0.01 will be used for all numerical results in this paper. As a result, it is appropriate to use asymptotic analysis with the N eigenvalues near i having an expansion of the form

$$s = i + \epsilon\, s^{(1)} + O(\epsilon^2).$$

Substituting this into Eqn. (2) and neglecting higher order terms yields

$$\left(s^{(1)} I - \tfrac{1}{2}\, i \,(\sigma M - A - iB)\right) \boldsymbol{u} = 0. \tag{3}$$

What this equation shows is that $s^{(1)}$, which determines the stability of the aeroelastic system, depends crucially on the value of σ which represents the relative level of structural mistuning compared to aeroelastic coupling.

For any matrix, the average of its eigenvalues is equal to the average of its diagonal terms. Hence, the average value of the real part of the eigenvalues $\mathcal{R}(s^{(1)})$ is equal to $b_0/2$, which must be negative for stability. The best that can be achieved through mistuning is that all of the eigenvalues have this same negative real part.

Another simple observation is that in the absence of any aerodynamic coupling each blade vibrates at its own natural frequency, so the eigenvalues are $s_j^{(1)} = \frac{1}{2}\, i\, \sigma m_j$, and the j^{th} element is the only non-zero in the corresponding eigenvector. It is the off-diagonal terms in A and B which introduce coupling between the blades and cause more than one blade to vibrate in each eigenmode.

4. Travelling wave representation

When analysing periodic systems, it is common to use a Fourier series representation in which the displacement of each blade and the structural mistuning are both expressed as a sum of circumferential Fourier modes,

$$u_j = \sum_{k=0}^{N-1} e^{ij\beta_k}\, \widehat{u}_k, \qquad m_j = \sum_{k=0}^{N-1} e^{ij\beta_k}\, \widehat{m}_k, \qquad \beta_k = \frac{2\pi k}{N}.$$

The blade equations can be expressed collectively as $\boldsymbol{u} = F \hat{\boldsymbol{u}}$. Inserting this into Eqn. (3) and pre-multiplying by F^{-1} gives the transformed equation

$$\left(s^{(1)} I - \tfrac{1}{2}\, i \left(\sigma \widehat{M} - \widehat{A} - i \widehat{B}\right)\right) \hat{\boldsymbol{u}} = 0, \tag{4}$$

in which it can be shown [4] that the matrix $\widehat{M}=F^{-1}MF$ is a circulant matrix with $\widehat{M}_{kl}=\widehat{m}_{k-l}$, while $\widehat{A}=F^{-1}AF$ and $\widehat{B}=F^{-1}BF$ are both diagonal matrices, with their diagonal elements being

$$\widehat{A}_{kk}=e^{-i\beta_k}a_{-1}+a_0+e^{i\beta_k}a_1 \qquad \widehat{B}_{kk}=e^{-i\beta_k}b_{-1}+b_0+e^{i\beta_k}b_1.$$

Because $\widehat{A}$ and $\widehat{B}$ are diagonal matrices, the tuned eigenvalues in the absence of any structural mistuning are $s_k^{(1)}=\lambda_k$ where

$$\begin{aligned}\lambda_k \;=\; \tfrac{1}{2}\,(\widehat{B}_{kk}-i\widehat{A}_{kk}) \;&=\; \tfrac{1}{2}\Big(b_0+(b_1+b_{-1})\cos\beta_k+(a_1-a_{-1})\sin\beta_k\Big)\\ &\quad-\tfrac{1}{2}\,i\,\Big(a_0+(a_1+a_{-1})\cos\beta_k-(b_1-b_{-1})\sin\beta_k\Big),\end{aligned}$$

and the k^{th} element of the corresponding eigenvector is the only non-zero element, so the eigenmode is a travelling wave in which all blades vibrate with an equal amplitude and inter-blade phase angle β_k. It is the off-diagonal terms in $\widehat{M}$ which introduce coupling between the Fourier modes.

5. Alternate mistuning

In alternate mistuning with an even number of blades, every second blade is identical. $\widehat{m}_{N/2}$ is the only non-zero term in the Fourier representation for the mistuning, and so the off-diagonal term $\widehat{M}_{k\;k+N/2}$ causes coupling between the Fourier modes k and $k+N/2$. Isolating the eigenvalue/eigenvector equations for these two modes alone, we have

$$\begin{pmatrix} s^{(1)}-\lambda_k & -\frac{1}{2}\,i\,\sigma\widehat{m} \\ -\frac{1}{2}\,i\,\sigma\widehat{m} & s^{(1)}-\lambda_{k'} \end{pmatrix}\begin{pmatrix}\hat{v}_k\\ \hat{v}_{k'}\end{pmatrix}=0, \tag{5}$$

where for simplicity in notation we have omitted the subscript $N/2$ in $\widehat{m}_{N/2}$, and have defined $k'\equiv k+N/2$. Equating the determinant to zero yields

$$s^{(1)}=\tfrac{1}{2}\left(\lambda_k+\lambda_{k'}\pm\sqrt{(\lambda_k-\lambda_{k'})^2-\sigma^2\widehat{m}^2}\right). \tag{6}$$

When σ is very small, asymptotic approximation of the square root term yields the approximate roots

$$s_k^{(1)}\approx\lambda_k-\frac{\sigma^2\widehat{m}^2}{4(\lambda_k-\lambda_{k'})}, \qquad s_{k'}^{(1)}\approx\lambda_{k'}+\frac{\sigma^2\widehat{m}^2}{4(\lambda_k-\lambda_{k'})}. \tag{7}$$

If $\mathcal{R}(\lambda_k)>\mathcal{R}(\lambda_{k'})$, so mode k is less stable than mode k', then these equations show that the effect of the mistuning is to improve the stability of

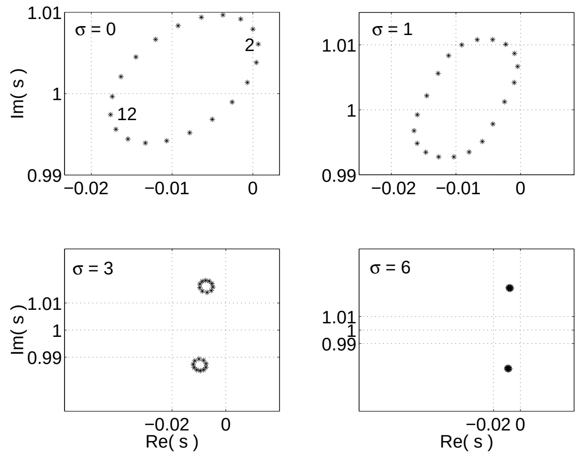

Figure 1. Effect of alternate mistuning.

mode k, while at the same time decreasing the stability of mode k' by an equal amount.

When σ is very large, asymptotic analysis yields

$$s^{(1)} \approx \pm\tfrac{1}{2}\, i\, \sigma \widehat{m} + \tfrac{1}{2}\,(\lambda_k + \lambda_{k'}) \pm \frac{(\lambda_k - \lambda_{k'})^2}{4\, i\, \sigma \widehat{m}}. \tag{8}$$

To leading order, the two roots have the same real component, which is equal to $\frac{1}{2}\mathcal{R}\,(\lambda_k + \lambda_{k'}) = \frac{1}{2} b_0$.

Figure 1 shows the eigenvalues for increasing levels of mistuning. The values of the aerodynamic constants are $N = 20$, $\epsilon = 0.01$, $a_{-1} = -0.4443$, $a_0 = -0.3587$, $a_1 = 0.5296$, $b_{-1} = -0.0054$, $b_0 = -1.7000$, $b_1 = 1.5688$, corresponding to the first bending mode of an LP turbine. The first two plots demonstrate the stabilising effect achieved through the coupling of modes k and k'. As σ increases further, the eigenvalues split into two groups, clustered around the frequencies of the weaker and stiffer blades, with nearly constant aerodynamic damping for all modes, as predicted. The stability parameter $\delta = \max \mathcal{R}(s)/\epsilon = \max \mathcal{R}(s^{(1)})$ determined from the exact aeroelastic eigenvalue problem, Eqn. (2), the first level approximation, Eqn. (6), and the second level approximations, Eqns. (7) and (8), is plotted versus σ in Fig. 2. The exact curve shows that the system becomes stable when the coupling and mistuning are of the same order ($\sigma \approx 1$). For $\sigma \approx 6.0$ the system has nearly achieved its maximum theoretical stability ($\frac{1}{2} b_0$) and further increases in mistuning have little effect on stability. The exact results are well predicted by the first level asymptotic analysis, whereas the two second level approximations are in good agreement for $\sigma \ll 1$ and $\sigma \gg 1$, as appropriate.

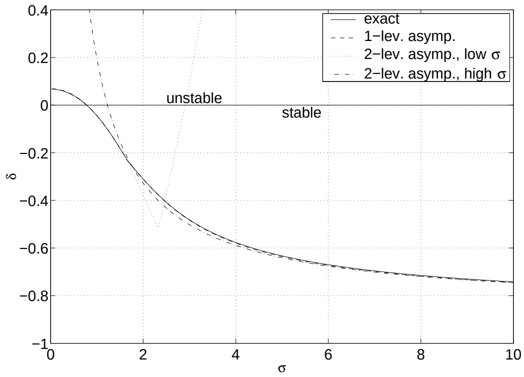

Figure 2. Aeroelastic stability with alternate mistuning. Exact and asymptotic analyses

6. Random mistuning

With random mistuning, we can also perform asymptotic analyses when $\sigma \ll 1$ or $\sigma \gg 1$. This is a second level of asymptotic analysis since we are applying it to Eqn. (3) (or Eqn. (4), its Fourier transform counterpart) which itself comes from an asymptotic approximation to Eqn. (2) for $\epsilon \ll 1$.

When $\sigma \ll 1$ we use Eqn. (4) with $\sigma\widehat{M}$ being regarded as a small perturbation to the tuned system with eigenvalues λ_k. Using second order perturbation theory [4], one obtains

$$s_k^{(1)} \approx \lambda_k - \frac{\sigma^2}{4} \sum_{l \neq k} \frac{\widehat{M}_{kl}\widehat{M}_{lk}}{\lambda_k - \lambda_l}. \tag{9}$$

Now, $\widehat{M}_{kl}\widehat{M}_{lk} = |\widehat{m}_{k-l}|^2$, and hence,

$$\mathcal{R}(s_k^{(1)}) \approx \mathcal{R}(\lambda_k) - \sigma^2 \sum_{l \neq k} \frac{|\widehat{m}_{k-l}|^2}{4} \; \frac{\mathcal{R}(\lambda_k) - \mathcal{R}(\lambda_l)}{|\lambda_k - \lambda_l|^2}.$$

Considering the index k for which $\mathcal{R}(\lambda_k)$ is greatest, this result shows that the effect of mistuning is always stabilising for the least stable mode.

An interesting situation arises if the tuned eigenvalues λ_l form a circle in the complex plane and so can be written as $\lambda_l = \lambda_0 + re^{i\theta_l}$ with $\theta = 0$ corresponding to the least stable mode. In this case,

$$\frac{1}{\lambda_k - \lambda_l} = \frac{1}{r} \; \frac{1 - \cos\theta_l + i\sin\theta_l}{(1-\cos\theta_l)^2 + \sin^2\theta_l} = \frac{1}{2r}\left(1 + i\cot\frac{\theta_l}{2}\right).$$

and hence,

$$\mathcal{R}(s_k^{(1)}) \approx \mathcal{R}(\lambda_k) - \frac{\sigma^2}{8r} \sum_{l \neq k} |\widehat{m}_{k-l}|^2.$$

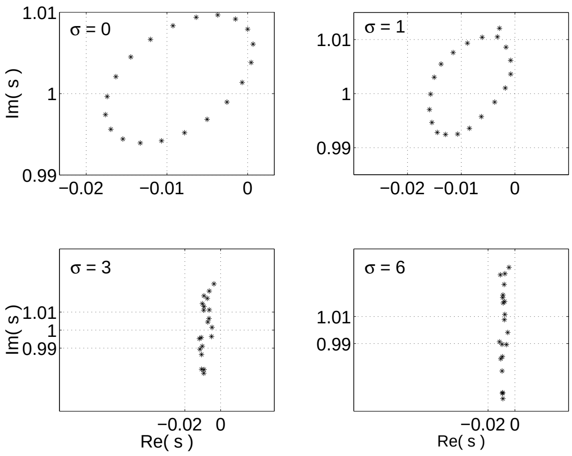

Figure 3. Effect of random mistuning.

Since the average level of mistuning is assumed to be zero, Parseval's theorem [4] states that

$$\sum_{l\neq k} |\widehat{m}_{k-l}|^2 = \frac{1}{N}\sum_j m_j^2.$$

Thus, the amount by which the mistuning stabilises the least stable mode is independent of the pattern of mistuning in the particular case when the eigenvalues of the perfectly tuned system form a circle in the complex plane.

When $\sigma \gg 1$ we use Eqn. (3) with $A+iB$ being regarded as a small perturbation to σM. The unperturbed eigenvalues are $\frac{1}{2}\, i\,\sigma m_j$, and applying second order perturbation theory gives

$$s_j^{(1)} \approx \tfrac{1}{2}\, i\,\sigma m_j + \tfrac{1}{2}\,(b_0 - i a_0) + \frac{i}{2\sigma}\sum_{k=j\pm 1} \frac{(a_{-1}+ib_{-1})(a_1+ib_1)}{m_j - m_k}. \qquad (10)$$

Considering only the real part of this, one obtains

$$\mathcal{R}(s_j^{(1)}) \approx \tfrac{1}{2}\, b_0 - \frac{1}{2\sigma}\sum_{k=j\pm 1} \frac{a_{-1}b_1 + a_1 b_{-1}}{m_j - m_k}$$

This shows that at very high levels of mistuning the system is stable because of the dominance of the b_0 term which is negative in real applications. The physical interpretation of this is that at high levels of mistuning each blade vibrates on its own at its own natural frequency. The forces it experiences are due solely to its own motion, and these self-induced forces are always stabilising. As the level of mistuning decreases, or equivalently

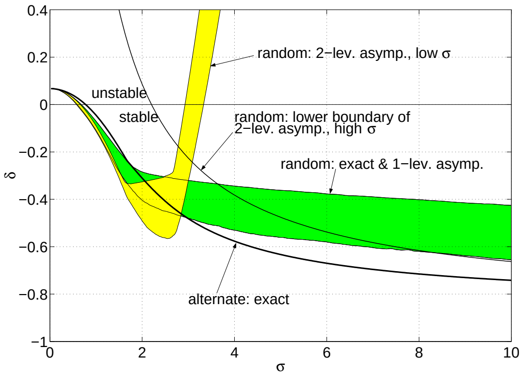

Figure 4. Aeroelastic stability with random mistuning; exact and asymptotic analyses with Monte Carlo simulations

the aerodynamic terms increase in strength, the aerodynamic forces cause the neighbouring blades to vibrate as well. The additional forces that this generates on the central blade may be stabilising or destabilising.

Using the same aerodynamic coefficients as before, Figure 3 shows the change in the eigenvalues for a particular random pattern of mistuning. Again the mistuning stabilises the unstable eigenvalues, but there is now a loss of regularity in the eigenvalue clouds during the transition from travelling wave to individual blade eigenmodes. To make the results independent of the particular choice of mistuning pattern, a Monte Carlo simulation has been carried out with 1000 different random patterns. Figure 4 has stability bands for the middle 80%, omitting the results for the best and worst 10%. We make the following observations: (a) the effect of random mistuning is always stabilising, (b) the agreement between exact and first level asymptotic stability is very good (the differences cannot be distinguished in the plot), (c) the second level asymptotic analysis for $\sigma \ll 1$ is accurate in predicting the stabilising effect of low levels of mistuning, (d) the second level asymptotic analysis for $\sigma \gg 1$ gives poor results unless the level of mistuning is unrealistically high.

7. Forced response

In forced response, equation (1) is modified through the addition of a prescribed aerodynamic forcing term

$$\begin{aligned} \ddot{u}_j + (1+\epsilon\,\sigma\, m_j)\; u_j \;&=\; \epsilon\,(\; a_{-1}u_{j-1} + a_0 u_j + a_1 u_{j+1} \\ &\qquad + b_{-1}\dot{u}_{j-1} + b_0 \dot{u}_j + b_1 \dot{u}_{j+1}\;)\;+\;\epsilon\, f_j(t). \qquad (11) \end{aligned}$$

The forcing term $f_j(t)$ can be decomposed into a sum of components each of which has a particular frequency ω and inter-blade phase angle β. Because of linearity, the effect of each of these can be superimposed, so from here onwards we consider a single such component.

Writing the forcing terms collectively as $\epsilon\, e^{i\omega t}\boldsymbol{f}$, the response of the blades is $e^{i\omega t}\boldsymbol{u}$, where $\boldsymbol{u}$ is given by

$$\Big((-\omega^2\!+\!1)I + \epsilon\,(\sigma M - A - i\omega B)\Big)\,\boldsymbol{u} = \epsilon\,\boldsymbol{f}. \tag{12}$$

A Fourier series transformation of this equation yields

$$\Big((-\omega^2\!+\!1)I + \epsilon\,(\sigma \widehat{M} - \widehat{A} - i\omega \widehat{B})\Big)\,\hat{\boldsymbol{u}} = \epsilon\,\hat{\boldsymbol{f}}, \tag{13}$$

in which only a single component of $\hat{\boldsymbol{f}}$ is non-zero, corresponding to the prescribed inter-blade phase angle.

If $|1\!-\!\omega^2| \gg \epsilon$, the effect of the $O(\epsilon)$ terms is negligible, and the approximate solution is

$$\boldsymbol{u} \approx \frac{\epsilon}{1\!-\!\omega^2}\,\boldsymbol{f}.$$

On the other hand, if the forcing frequency is close to the natural frequency of the blades, so $1-\omega^2 = O(\epsilon)$, then the other $O(\epsilon)$ terms become significant. In this case it is appropriate to make the substitution $\omega = 1 + \epsilon\omega^{(1)}$, and then ignoring terms which are $O(\epsilon^2)$ yields

$$\Big(-2\omega^{(1)}I + \sigma M - A - iB\Big)\,\boldsymbol{u} = \boldsymbol{f}. \tag{14}$$

and

$$\Big(-2\omega^{(1)}I + \sigma \widehat{M} - \widehat{A} - i\widehat{B}\Big)\,\hat{\boldsymbol{u}} = \hat{\boldsymbol{f}}. \tag{15}$$

If there is no mistuning then equation (15) can be solved to obtain

$$\widehat{u}_k = \frac{\hat{f}_k}{-2\omega^{(1)} - \widehat{A}_{kk} - i\widehat{B}_{kk}} = \frac{\hat{f}_k}{-2\,(\omega^{(1)} + i\lambda_k)}.$$

If $\omega^{(1)}$ is treated as a variable, the peak response is

$$|\widehat{u}_k| = \frac{|\hat{\boldsymbol{f}}|}{-2\,\mathcal{R}(\lambda_k)} = \frac{|\hat{\boldsymbol{f}}|}{-(b_0 + (b_1\!+\!b_{-1})\cos\beta_k + (a_1\!-\!a_{-1})\sin\beta_k)}, \tag{16}$$

when $\omega^{(1)} = \mathcal{I}(\lambda_k) = -\frac{1}{2}\Big(a_0 + (a_1\!+\!a_{-1})\cos\beta_k - (b_1\!-\!b_{-1})\sin\beta_k\Big)$.

At the opposite extreme, if the aerodynamic forces are weak, then the off-diagonal terms in matrices A and B can be ignored, to leading order, and so the approximate solution to Eqn. (14) is

$$u_j = \frac{f_j}{-2\omega^{(1)} + \sigma m_j - a_0 - ib_0},$$

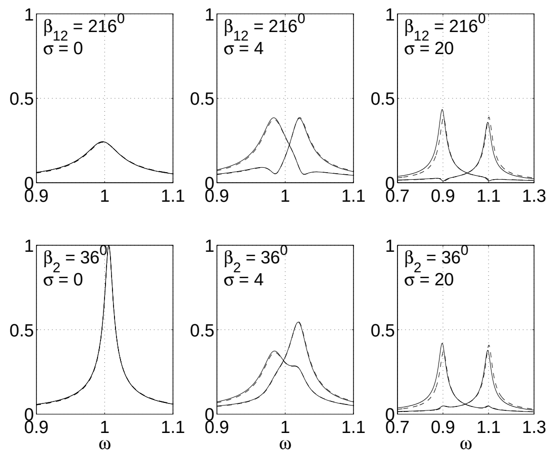

Figure 5. Blade response of alternately mistuned assembly for two engine orders and for different levels of mistuning. (—: exact, – –: asymptotic)

and hence the peak response of blade j, when $\omega^{(1)} = \frac{1}{2}\left(\sigma m_j - a_0\right)$, is

$$|u_j| = \frac{|\hat{\boldsymbol{f}}|}{-b_0}. \tag{17}$$

Figure 5 shows the exact and asymptotic blade response of the tuned and alternately mistuned assembly versus the exciting frequency ω for two excitations with inter-blade phase angle $\beta_{12} = 216^o$ and $\beta_2 = 36^o$, corresponding to the most and least damped modes of the tuned rotor, respectively. All ordinates are normalised by the maximum peak response of the tuned assembly. We make the following observations: (a) the agreement between exact and asymptotic analysis is excellent; (b) the maximum peak response of the tuned assembly occurs when the least damped harmonic is excited; (c) with alternate mistuning there are two peaks and a wider frequency range over which there is a significant response; (d) mistuning increases the maximum peak response when the most damped travelling wave is excited, but decreases the peak response when the least damped mode is excited; (e) the response becomes increasingly independent of the inter-blade phase angle of the forcing at high $\sigma' s$.

Effect (d) is due to the coupling of travelling waves k and k'. If the excited harmonic k has high damping, the mistuning transfers energy to the lightly damped harmonic k' producing a significant response. On the other hand, if the excited harmonic k has low damping, then the mistuning reduces the response by transferring energy to the more heavily damped harmonic k'. Effect (e) is in agreement with Eqn. (17).

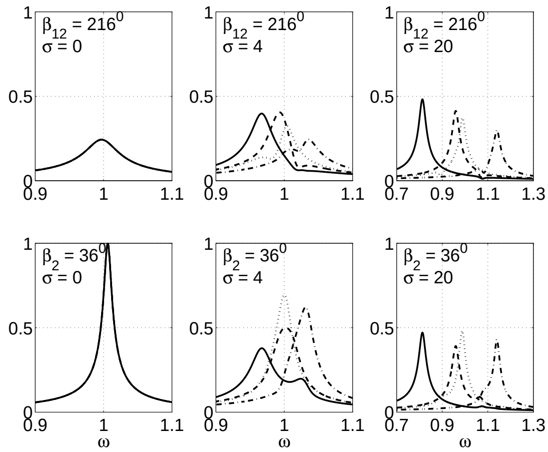

Figure 6. Blade response of randomly mistuned assembly for two engine orders and for different levels of mistuning. (—: blade 7, – –: blade 19, · · ·: blade 13, -.-.-: blade 10)

Observations $(b) \rightarrow (e)$ remain true for the randomly mistuned assembly, for which results are presented in Fig. 6 for a particular random mistuning pattern. The leftmost plots are the same as those in Fig. (5). The others show the response of four selected blades. Each peak corresponds to the resonance of a particular blade, each of which has a different natural frequency. Note that for $\sigma = 4$ the maximum peak response for both engine orders is not that of the weakest blade (blade 7).

8. Conclusions

The theoretical and numerical analyses presented in this paper confirm the stabilising effect of mistuning on blade flutter, and show that it depends on both the tuned eigenvalues and the ratio of mistuning to aerodynamic coupling. When the structural mistuning is lower than the aerodynamic coupling, each mode is best viewed as a combination of travelling waves and mistuning enhances the stability of the least stable mode by transferring energy to the more stable harmonics in which it is dissipated. When the structural mistuning is large, each mode is highly localised, corresponding primarily to the oscillation of a single blade, and to a lesser extent its neighbours, with the aerodynamic forces providing damping.

Alternate mistuning is particularly effective in improving flutter stability and this suggests its use as a measure for passive flutter control. Random mistuning is also stabilising, but its effectiveness depends on the particular mistuning pattern. Monte Carlo simulation, considering multiple random perturbations, is effective in assessing this.

With regards to forced response, structural mistuning leads to multiple resonant peaks and a widening of the frequency range over which resonance may occur. The maximum peak response can be either increased or decreased compared to the tuned case, depending whether the forcing inter-blade phase angle corresponds to one of the least damped modes or one of the most damped.

The computational costs of the analyses in this paper are minimal, a few seconds for each eigenvalue computation, making the approach very suitable for design optimisation, or the assessment of random mistuning during the design process. In particular, the single asymptotic models and the Monte Carlo simulations can be straightforwardly extended to 3D aeroelastic analyses and introduced into everyday design practice, as shown in [5]. This is made possible by the fact that turbomachinery blades are usually designed to keep the structural mode frequencies well separated. Consequently, aeroelasticity analyses can consider a single degree-of-freedom per blade, corresponding to the structural mode under investigation.

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