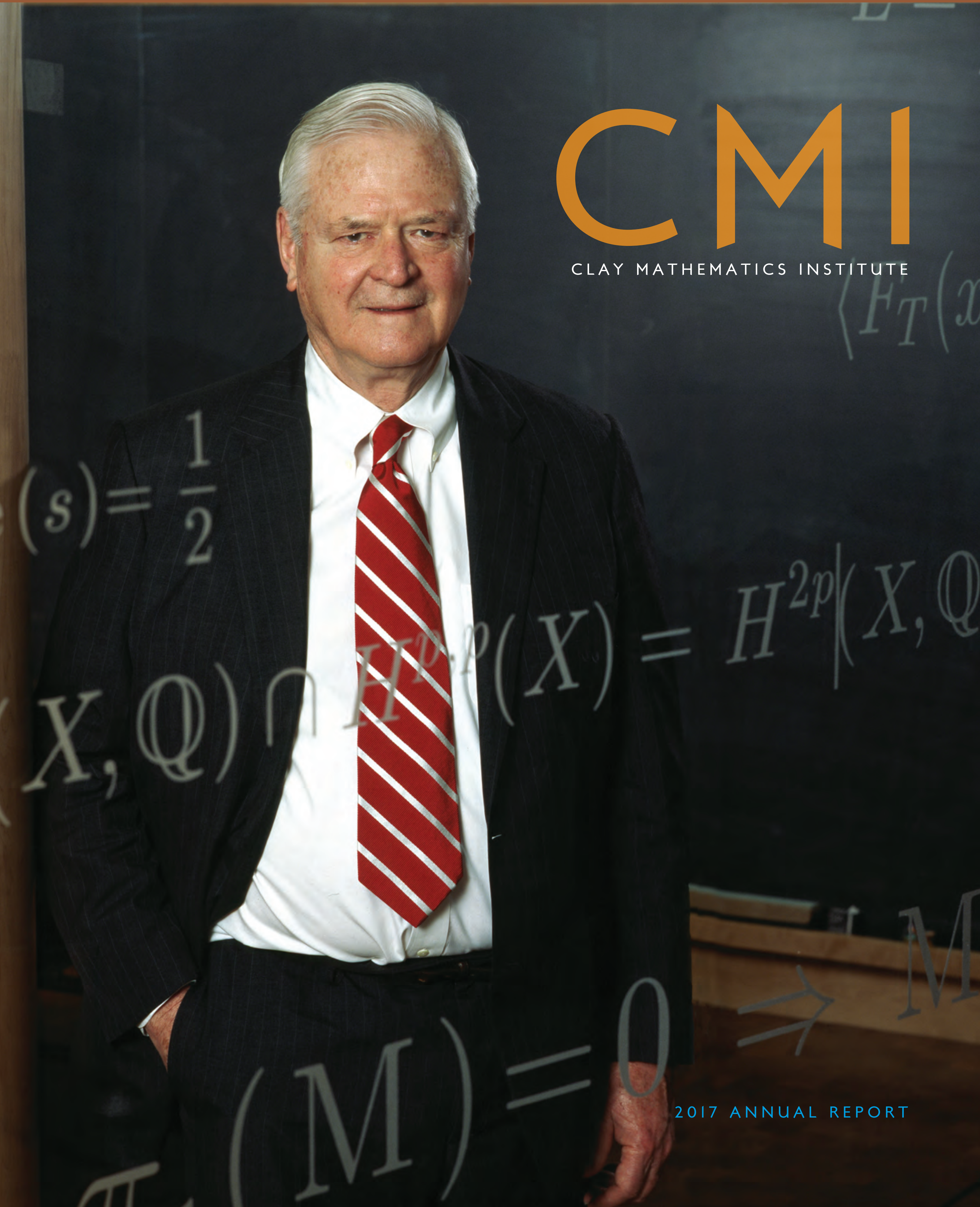

CMI

CLAY MATHEMATICS INSTITUTE

2017 ANNUAL REPORT

CLAY MATHEMATICS INSTITUTE

MISSION

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.

The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

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LETTER FROM THE PRESIDENT

At the end of July, we heard the sad news of the passing of Landon T Clay, whose inspiration and generosity led to his creation of the Clay Mathematics Institute, and whose wisdom and passion for mathematics has driven much of its success.

The 2017 Annual Report leads with an obituary and tributes to Mr Clay. They recall how his lifelong engagement with mathematics began with the correspondence course he took while stationed on Tinian with the United States Army Air Forces at the end of the Second World War and how his highly successful business career put him in a position to realise his philanthropic ambitions over a wide range of scientific and cultural interests. Amongst all these, he took greatest pride in the Clay Mathematics Institute. It will continue to thrive as a lasting memorial to his vision and generosity.

Earlier in the year, in April, CMI had announced three new Clay Research Awards: one to Maryna Viazovska, and two joint Awards, to Aleksandr Logunov and Eugenia Malinnikova, and to Jason Miller and Scott Sheffield. The first two Awards were presented by Richard Clay at the Research Conference in September. There is an account of the conference in this Report, as well an interview with Maryna Viazovska. The third award will be presented at CMI's 20th Anniversary Conference in September 2018.

It is a measure of the standing of the Awards that all three contributions were also recognised in the list of invited speakers at the 2018 International Congress, which was released in June: Viazovska, Logunov, Malinnikova, and Miller were all included. Amongst the plenary speakers there are to be two members of CMI's Scientific Advisory Board, Simon Donaldson and Andrei Okounkov, a former Clay Research Fellow, Peter Scholze, and two other recent recipients of Clay Research Awards, Rahul Pandharipande and Geordie Williamson. In all, there are ten current and former Research Fellows amongst the invitees.

CMI continues to pursue its mission across many countries and age groups. A few of the 2017 events are reported in detail here, but there were many others in which CMI enhanced mathematical activities through partnerships with other organisations. Most were traditional conferences, workshops, research programs, and summer schools, but others were intended to help shape the mathematical landscape in less direct ways, for example by supporting the growing number of events for women in mathematics or by supporting the STEM for Britain competition run by the UK's Parliamentary and Scientific Committee. CMI's involvement in the latter not only serves to reinforce in one corner of the world the message that mathematics has a central place in the global scientific enterprise but also to draw the attention of political leaders to the remarkable impact of even very 'pure' areas of mathematical research.

All this is part of Landon Clay's legacy: a foundation that is not merely a generous source of funding but also a significant influence in developing and promoting the discipline for the benefit of all.

N. M. J. Woodhouse

Landon Clay

Founder of the Clay Mathematics Institute

With the passing of Landon T. Clay on July 29, 2017, the mathematical world has lost one of its most generous benefactors and the Clay Mathematics Institute has lost its founder and inspirational guide.

Landon Clay was not a mathematician by education—he studied English at Harvard. Nor was his interest that of an amateur problem solver; rather he was driven by a deep appreciation of the beauty and importance of mathematical ideas and by his desire to share his understanding of the contribution they have made and continue to make to human development.

Landon Clay's generous philanthropy and wide-ranging intellectual curiosity led him to support a broad diversity of scientific and educational projects, but above others the Clay Mathematics Institute carries the stamp of his ingenuity and imagination. It grew out of his own study of mathematics as a serviceman in the final months of the Second World War and his realisation that the contributions of mathematicians were constantly undervalued by those telling the story of our civilisation.

His creation of the Millennium Prizes was an inspired step towards a solution. There are many prizes in mathematics and they are hugely valuable in recognising the achievements of individuals and in encouraging others with the message that their long, patient, and private efforts need not go unrewarded. But often they have little impact beyond the apparently closed and introspective world of mathematical research, not least because the work they celebrate is so far from familiar territory. When the questions are incomprehensible, little attention will be paid to the answers nor to those who find them, however brilliant and original.

Landon Clay appreciated, in a way that those who distribute public funds cannot permit themselves, the importance of giving the brightest young mathematicians the space and freedom to develop their ideas free of financial concerns and of the constant distractions of assessment and proposal writing.

The subtlety of the vision behind the Millennium Prizes is that they focus attention not on the prizes themselves (although the offer of a million dollars does grab attention), but on the problems—the Millennium Prize Problems. They provide reference points for the vast *terra incognita* of mathematics, whether in popular culture or in formal education. They allow others to glimpse that the frontiers of mathematics are alive and that they are places where each generation can do great things. The inclusion of the Riemann Hypothesis in the list helped to inspire many brilliant and thoughtful books that not only set out the question in simple terms and explain why it is important to answer it, but also spread understanding of the nature of the mathematical enterprise and of the absolutely central importance of proof. It is not enough that the predicted location of the zeros of the zeta function has been verified in trillions of cases—that is evidence, but not the answer to the question. When the answer is finally found, the reaction should not be one of incomprehension and indifference, but rather of understanding that something of huge significance has been achieved.

Landon Clay appreciated, in a way that those who distribute public funds cannot permit themselves, the importance of giving the brightest young mathematicians the space and freedom to develop their ideas free of financial concerns and of the constant distractions of assessment and proposal writing. The Clay Research Fellowships give precisely that—five years of freedom. The achievements of those who have benefited have been spectacular, illustrated not least by the award of three of the four Fields Medals at the last International Congress in 2014 to former Clay Fellows. One of those was to Maryam Mirzakhani, whose own recent untimely death is also mourned by the whole mathematics community.

The achievements of the Clay Research Fellows and the Clay Mathematics Institute itself will stand as lasting memorials to Landon Clay's vision and generosity.

—Nick Woodhouse

Addenda

For nearly twenty years Landon Clay has passionately supported mathematics through his founding of the Clay Mathematics Institute. Landon always felt that mathematicians were not sufficiently appreciated or supported and wanted to help redress what he felt was an injustice. In creating his institute he has succeeded far beyond any of our initial hopes in bringing the excitement, depth, beauty and value of mathematics to the general public. At the same time he has created a means for the support of mathematicians that is a model for other foundations in its willingness to help throughout the world without any precondition except that it should further mathematics at the highest levels. It has been a great personal privilege to have known Landon and to have served on the scientific advisory board of the institute during this time. Landon's guidance and support have been pivotal throughout the institute's history. We will greatly miss one of mathematics' most generous and visionary supporters.

—Andrew Wiles

Twenty years ago, Landon Clay created something novel—a private foundation devoted entirely to promoting and supporting mathematics. His efforts have really borne fruit, as the Clay Mathematics Institute has had a wide-ranging impact in advancing public awareness of mathematics and in supporting mathematics around the world. All of us working in mathematics and mathematical physics are deeply appreciative of Landon Clay's creation.

—Edward Witten

Obituary

Landon T. Clay 1926 – 2017

Landon Thomas Clay was born in New York City on March 12, 1926, and died on July 29, 2017, at his home in Peterborough, NH.

He spent his early childhood in Augusta, Georgia, where his family had an interest in the John P. King textile mill, and was educated at the Middlesex School in Concord, Massachusetts. On graduating from high school in 1944 he joined the United States Army Air Forces and was posted to Tinian in the Marianas, where the military airfield had grown to become the world's largest airport. His work there as a Specialist in B-29 armaments fuelled his technical and scientific interests. It also left him with great admiration for the Boeing Corporation, a successful target for his investment activities in the 1950s.

His service on Tinian was extended well beyond the close of hostilities by the War Department's points system, which imposed long periods of idleness on latecomers while they waited to be returned to the USA. The tropical ocean and the occasional magical glimpse of Guam, silhouetted "like a resting lion on the distant horizon", did not entirely mitigate the feeling that he might be losing out from this hiatus in his education. So Landon enrolled in a correspondence course with the University of Wisconsin and taught himself calculus—the beginning of a lifelong engagement with mathematics.

In the fall of 1947, he was finally able to take up a place at Harvard. Despite his efforts on Tinian, he was disconcerted to find that his fellow students in the advanced calculus class knew their calculus much better than he did, having studied it at school. So he chose instead to focus on English literature, graduating cum laude in 1950, a year ahead of schedule. He later regretted passing up the opportunity to extend his studies in a fourth year at Harvard.

After leaving Harvard, he embarked on a career in investment. He had been told that he would do better to stay in Boston, where he could be a big fish in a small pond, rather than move to New York, where he might be a small fish in a big pond. But his aim was to be a big fish in a big pond, so he ignored the advice and started out in New York. Later he moved back to Boston, first to the Massachusetts Investors Trust and then to Vance, Sanders & Co, where he was hired to establish an independent research department and in 1971 became CEO. In 1979, he organised the merger with Eaton & Howard to form the Eaton Vance Corporation, of which he became Chairman and CEO.

Early in his career, he had become convinced of the benefit of long-term investment in early stage companies. His love of science and mathematics gave him a particular interest in encouraging start-up companies working in those areas, companies that might not only prove to be financially successful but which also had the potential to bring substantial benefits to humankind. As a young newcomer at MIT, he volunteered to cover industries that established staff found of little interest, such as technology. He was the first buy-side analyst to follow American Research and Development, calling on George Doriot, one of the first American venture capitalists. He unearthed financial reports in the Massachusetts statehouse indicating that one of ARD's portfolio companies, Digital Equipment Corporation, was clearly worth many times the value of ARD itself, so he invested heavily in ARD before DEC's minicomputer success became widely appreciated. He was a founding investor in ADE Corporation, which struggled for years to sell its non-contact gauging technology to the tire industry until the semiconductor industry developed, whereupon ADE became a major supplier of metrology equipment. For many years, over 90% of the silicon wafers used in semiconductor manufacturing passed through an ADE machine to measure their flatness. He was an early investor in Apple Inc in the period of the first personal computers. Eaton Vance itself rode the development of the mutual fund industry to become one of the best-performing companies in the US stock market between its formation in 1979 and Landon Clay's retirement in 1997.

Landon Clay had a close relationship with his brother Harris, which was reflected in a lifetime of co-investment. Trusting in one another's expertise, Harris invested in Landon's financial services, technology, and mining companies, while Landon followed Harris's lead to take bold positions in oil and gas prospects across Canada. At certain times the two brothers controlled well over a million acres of Canadian mineral rights, and they watched the Apollo 11 moon landing from a motel north of the Arctic Circle, where they were inspecting properties.

Boldness and independence were characteristic of all of Landon Clay's projects, from his interest in collecting art of the early Americas before that field was fashionable, to his raising premium limousin and later wagyu cattle on his family's farm in Kentucky, to building rose and foliage farms in Guatemala while that country was racked by civil war, to making a bet on the development of real estate in Central Square in Cambridge, MA (equidistant between two premier universities) that proved a few decades too early.

In accord with his longstanding passion for investing in risk-taking, high-potential companies, following retirement from Eaton Vance at the age of 71, Landon Clay founded East Hill Management. This was set up as a private investment firm specialising in life science and technology start-ups and in early-stage mineral properties, in which Landon had developed particular expertise. It gave him great pleasure to back the ground-breaking ideas of bright young scientists in risky but potentially transformative commercial ventures. East Hill's various investments in the commercialisation of intellectual property brought him into close contact with the academic world, where he liked nothing better than to spend time in conversation with people who were the leading experts in their fields.

It was through a friendship with the chemist Jeremy Knowles, and later through his investments in spin-outs from the University, that Landon became involved with Oxford, from where the Clay Mathematics Institute now runs its scientific activities. One of these companies was Oxitec, which releases genetically modified mosquitoes into the wild to control mosquito-borne diseases. It is currently involved in the battle against the Zika virus.

Landon Clay served on the boards of many charitable foundations and energetically pursued his own wide-ranging philanthropic interests. With his wife Lavinia D. Clay, he supported numerous scientific and cultural bodies including Harvard College and the Harvard-Smithsonian Center for Astrophysics, the Cold Spring Harbor Laboratory, the Bodleian Library and the Mathematics and Chemistry departments at the University of Oxford, the Sea Turtle Conservancy, the Boston Museum of Fine Arts, the Clay Center at Massachusetts General Hospital, the Archaeological Exploration of Sardis, the Clay Center for Science and Technology at Dexter and Southfield Schools, the Magellan Telescopes at Las Campanas Observatory in Chile, the Middlesex School, and the Whitehead Institute for Biomedical Research. But he is most widely known as the founder of the Clay Mathematics Institute, in which he rightly took enormous pride.

His philanthropy continued the proud traditions of his family, a family that had made distinguished contributions to the political, social, and economic history of the United States. He was a great grandson of Brutus Junius Clay, the son of an influential and wealthy political family in Kentucky. Brutus Junius served in the Kentucky House of Representatives and in 1863 was elected to the United States House of Representatives on the Union Democratic ticket. He was a cousin of Senator Henry Clay, the 'Great Compromiser', and older brother to Cassius Marcellus Clay, the prominent champion of emancipation. Two of Cassius Marcellus's daughters, Brutus Junius's nieces Laura and Mary Barr Clay, were leading members of the women's suffrage movement. In 1920, Laura Clay was a candidate for the presidential nomination at the Democratic National Convention, the first woman to mount a serious challenge for the Presidency.

On his mother's side, Landon Clay was a great grandson of Landon A. Thomas, a nephew of the famous businesswoman and philanthropist Emily Harvie Thomas Tubman, who had grown up under the guardianship of Henry Clay. Emily Tubman was also intolerant of slavery, freeing all her family's slaves when she inherited her husband's estate in 1836. Many settled in Liberia, where William Tubman, the grandson of two of her freed slaves, served as President from 1944 to 1971. Emily Tubman founded the textile mill in Augusta with which Landon Clay's family was associated and was responsible for many philanthropic projects in Augusta and elsewhere.

Landon T. Clay is survived by his wife, Lavinia D. Clay, and by his sons Thomas M. Clay, Richard T. Clay, Landon H. Clay, and Cassius M. C. Clay. Lavinia Clay was very much involved in the creation of the Clay Mathematics Institute, and served on its Board of Directors up until 2016. Landon and Lavinia Clay's four sons serve on the Board today.


2017
Clay Research Conference
and Workshops
24–29 September
Mathematical Institute, Andrew Wiles Building
University of Oxford, UK
Clay Research Conference
27 September 2017
Plenary Lecturers:
Larry Guth, Massachusetts Institute of Technology
Ovidiu Savin, Columbia University
Bertrand Toën, Université de Toulouse
Tamar Ziegler, Hebrew University of Jerusalem
Conference Workshops
24–28 September 2017
Ergodic Theory: Numbers, Fractals, and Geometry
Organisers:
Manfred Einsiedler (ETHZ)
Tom Ward (Leeds)
Tamar Ziegler (HUJI)
Harmonic Analysis and Related Areas
Organisers:
Larry Guth (MIT)
Nets Katz (Caltech)
25–29 September 2017
Modern Moduli Theory
Organisers:
Dominic Joyce (Oxford)
Kevin McGerty (Oxford)
Balázs Szendrői (Oxford)
Nonlocal PDEs
Organisers
Luis Caffarelli (Austin)
Ovidiu Savin (Columbia)
REGISTRATION
Registration for the Clay Research Conference is free but required. Participation in the workshops is by invitation; a limited number of additional places is available. To register for the Conference and to register interest in a workshop, email admin@claymath.org.
For full details, including the schedule, titles and abstracts when they become available, see www.claymath.org
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The 2017 Clay Research Conference

Nick Woodhouse

The 2017 Clay Research Conference was held in Oxford on September 27, with plenary talks by Larry Guth (MIT), Ovidiu Savin (Columbia), Bertrand Toën (Toulouse) and Tamar Ziegler (HUJ). They are summarised below and are also available online in CMI's video library at www.claymath.org/library/video-catalogue.

The day concluded with presentations of the 2017 Clay Research Awards. Laudations for the awardees were given by Carlos Kenig, who spoke about the work of Aleksandr Logunov and Eugenia Malinnikova, and by Henry Cohn, who spoke about the work of Maryna Viazovska. Richard Clay presented the Awards.

Larry Guth

Larry Guth

Larry Guth spoke on applications of Fourier analysis to the problem of counting solutions to Diophantine equations. His initial focus was on sums of cubes: how many solutions are there to the equation

$$a_1^3 + a_2^3 + a_3^3 = b_1^3 + b_2^3 + b_3^3 \qquad (1)$$

with $a_i, b_i \in \mathbb{Z}$ and $1 \le a_i, b_i \le N$? A heuristic argument suggests that the number of solutions, $S(N)$, satisfies

Conjecture 1. *For all* $\varepsilon > 0$, $S(N) \le C_\varepsilon N^{3+\varepsilon}$ *for some constant* C_ε.

The proof remains out of reach, but although the Fourier approach has not led to progress in the three-cubes problem, it does suggest a way to tackle other Diophantine problems of the same type.

In the three-cubes case, the starting point is the introduction of the trigonometric polynomial

$$f(X) = \sum_{a=1}^{N} \mathrm{e}^{2\pi \mathrm{i} a^3 X} .$$

By evaluating the integral on the right-hand side, we have the following.

Lemma 1. $S(N) = \int_0^1 |f(X)|^6 \, \mathrm{d}X$.

Starting from this formula for $S(N)$, the idea is to use estimates from Fourier analysis to obtain bounds on the number of solutions. For example, from the triangle inequality, $|f(X)| \le N$ while from orthogonality,

$$\int_0^1 |f|^2 = N .$$

By combining these, one has

Proposition 1. $S(N) \leq N^5$.

This is not much of an advance because the bound can be obtained directly without using Fourier analysis, but it does suggest that one might do better than the trivial bound by exploring the properties of f in more depth.

Suppose that we are given a function g with $|g(X)| \leq N \ \forall X$ and $\int_0^1 |g|^2 = N$. There are two extreme behaviours. If $|g(X)|$ is 'spread out', with values close to $\sqrt{N}$ throughout the unit interval, then

$$\int_0^1 |g|^6 \sim N^3 .$$

If f had this behaviour, then the conjecture would follow. On the other hand, if $|g(X)|$ is 'focused', taking low values except on a set of total length $1/N$ on which it takes values close to N, then

$$\int_0^1 |g|^6 \sim N^5 .$$

In the Fourier approach, progress towards establishing the conjecture would involve showing that f exhibits behaviour closer to the first case than to the second.

This is hard, but there is another case in which the approach has recently proved fruitful, namely that of Vinogradov's conjecture concerning the number $J_{s,k}(N)$ of solutions to the system of Diophantine equations

$$\begin{aligned} a_1 + \cdots + a_s &= b_1 \cdots + b_s \\ a_1^2 + \cdots + a_s^2 &= b_1^2 \cdots + b_s^2 \\ &\vdots \\ a_1^k + \cdots + a_s^k &= b_1^k \cdots + b_s^k \end{aligned} \tag{2}$$

with $a_i, b_i \in \mathbb{Z}$ and $1 \leq a_i, b_i \leq N$.

Conjecture 2. $J_{s,k}(N) \leq C(s, k, \varepsilon)N^{\varepsilon}(N^s + N^{2s-(1+\cdots+k)})$.

On the right, N^s counts the number of diagonal solutions ($a_i = b_i$) while the second term comes from a heuristic guess for the number of non-diagonal solutions.

Vinogradov proved the conjecture for $s \geq Ck^2 \log k$. Its truth generally was been established by Bourgain, Demeter, and Guth in a paper published in *Annals of Mathematics* in 2016. Trevor Wooley had earlier proved the conjecture for $k = 3$, and more recently has given a different proof for the general case, by using his method of efficient congruencing.

At first sight it is puzzling that the second conjecture should be more tractable than the first when if anything it looks more complex. The answer lies in the additional symmetry in the second case: both equations (1) and (2) are invariant under dilation of the variables; but the system (2) has additional symmetry under translation. This is exploited more or less explicitly in all the approaches to Vinogradov's conjecture.

Guth outlined the idea behind his proof (with Bourgain and Demeter), starting with the special case $k = 2$. Here there is an older and simpler proof using unique factorization in $\mathbb{Z}[\mathrm{i}]$, but it is instructive to take a different route that uses Fourier analysis because the method then generalizes to give a proof of the full conjecture.

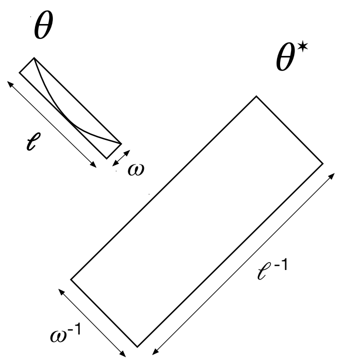
When $k = 2$, the key object is the two-variable function

$$f(X) = \sum_{a=1}^{N} e^{2\pi i \omega_a . X}$$

where the ‘frequencies’ ω_a all lie on the parabola $\{(a, a^2)\}$ in the plane. They are defined in such a way that

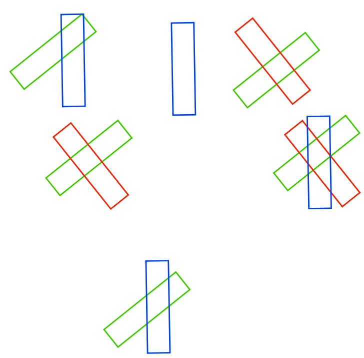
$$J_{s,k}(N) = \int_{[0,1]^2} |f(X)|^{2s} \, dX.$$

As with the first conjecture, the problem is to show that $|f(X)|$ is ‘spread-out’ rather than ‘focused’. This is achieved by a multiscale approach. First the parabola is partitioned into the union of small arcs θ_j. Correspondingly, the sum defining f decomposes into sums of the functions



$$f_{\theta_j} = \sum_{\omega_a \in \theta_j} e^{2\pi i \omega_a . X} .$$

Each arc determines a rectangle θ^* in the X-plane, ‘dual’ to the smallest rectangle θ containing the arc itself. The dual rectangle is orthogonal to θ, with inverse width and length. Its translates tile the X-plane. A key lemma is that $|f_\theta|$ is roughly constant on each tile.



The proof establishes the behaviour of f by considering that of the individual terms in the sum. The functions f_θ could be either spread out or they could be concentrated. If each is spread out, then f must be as well. On the other hand, suppose that each f_θ is focused in a sparse set of translates of θ. Because the tilings by the different θs (shown on the left in different colors) are slanted in different directions, one can argue from the lemma that the places where the different $|f_\theta|$s are large cannot coincide, and so f is forced to be more spread out than the individual f_θs. This idea goes back to the 1970s, under the heading *restriction theory*, but it was not clear that, on its own, it would lead to estimates sharp enough to prove the conjecture, not least because in higher dimensions one comes up against Kakeya-type questions about the extent to which tubes pointing in different directions can overlap. The new advance comes from combining restriction theory with a multi-scale approach, in which one considers nested sequences of partitions, with the small arcs θ_i grouped into larger arcs τ_i. This gives much more detailed constraints on the extent to which it is possible for f to be focused.

Guth concluded his lecture by working through one particular example that illustrated how the information at different scales combines to tie down the behaviour of f itself in enough detail to prove the conjecture.

Bertrand Toën

Bertrand Toën

Bertrand Toën began by posing a fundamental question in algebraic geometry: given p homogeneous polynomials F_i in $n+1$ complex variables, is it possible to read off the topology of the algebraic variety

$$X := \{(x_0, \ldots, x_n) \,|\, F_i(x) = 0\} \subset \mathbb{P}^n_{\mathbb{C}},$$

from the F_is? In low dimensions, there are two well known cases in which it is possible:

- when $n = p = 1$, X is a finite set with cardinality equal to $\deg(F_1)$ (counting according to multiplicity);
- when $n = 2$, $p = 1$ and X is smooth, X is a Riemann surface of genus $(d-1)(d-2)/2$, where $d = \deg(F_1)$.

More generally, in all dimensions, the Euler characteristic is given by

$$\chi(X) = \sum_{p,q} (-1)^{p+q} \dim H^p(X, \Omega^q_X), \tag{1}$$

where Ω^q_X is the sheaf of holomorphic q-forms. The right-hand side can be found purely in terms of the F_i. Eqn (1) is important because it relates a topological invariant on the left to an algebraic invariant on the right. The formula follows directly from the Hodge decomposition

$$H^i(X, \mathbb{Q}) \otimes \mathbb{C} \simeq \bigoplus_{p+q=i} H^p(X, \Omega^q_X)$$

but it also has an independent proof which works also in a more general setting, by combining the Gauss-Bonnet theorem with the Hirzebruch-Riemann-Roch formula. These also hold over other fields. So, for example, for a variety defined by homogenous polynomials over any algebraically closed field k, one has

$$\chi(X) := \sum_i (-1)^i \dim H^i_{\text{et}}(X, \mathbb{Q}_\ell) = \sum_{p,q} \dim H^p(X, \Omega^q_X), \tag{2}$$

where the $H^i_{\text{et}}(X, \mathbb{Q}_\ell)$ are Grothendieck's ℓ-adic cohomology groups and the sheaf cohomology groups on the right are defined by using the Zariski topology. This is a special case of the *trace formula*: if f is an algebraic endomorphism of X, then

$$\sum_i (-1)^i \text{Trace}\big(f : H^i_{\text{et}}(X, \mathbb{Q}_\ell)\big) = [\Gamma_f . \Delta_X],$$

where the right-hand side is the intersection number of the graph Γ_f with the diagonal $\Delta_X \subset X \times X$. The trace formula reduces to (2) when f is the identity.

Toën explained the extension of these ideas to families of algebraic varieties, which tend to degenerate to varieties with singularities. For example, compactified moduli spaces, where the compactification is achieved by adding singular varieties at infinity; or, in arithmetic geometry, in the case of varieties defined over number fields, a variety can have bad (i.e. singular) reductions modulo some primes in the ring of integers.

In a general setting, one studies such degenerations by considering a family of algebraic varieties defined by homogeneous polynomials with coefficients in the ring of functions on a parameter space S. For each $s \in S$, the polynomials determine an algebraic variety $X_s \subset \mathbb{P}^n_{k(s)}$, where $k(s)$ is the residue field of s. The particular examples of interest are where S is a formal "small" disk, such as the spectrum of $\mathbb{C}[[t]]$, $k[[t]]$ for an algebraically closed field k, or the p-adic integers $\mathbb{Z}_p$.

In all these cases, $S = \mathrm{Spec}\, A$ for some discrete valuation ring A. There are then just two points in S, the *special point* $o \in S$ where $k := k(s) = A/m$ is the residue field; and the *generic point* $\eta \in S$ where $K := k(\eta) = \mathrm{Frac}\,(A)$ is the fraction field. Corresponding to these, we have two algebraic varieties: the special fibre $X_{\overline{k}}$ and the generic fibre $X_{\overline{K}}$ (the bar denotes algebraic closure).

The *Variational problem* is to understand the change in topology from $X_{\overline{K}}$ to $X_{\overline{k}}$. In particular, to find an algebraic construction that yields the difference in Euler characteristics.

When $X \to S$ is a submersion, both varieties are smooth, the topology is constant, and the difference is zero. So the interesting question is what happens when $X_{\overline{K}}$ is smooth, but $X_{\overline{k}}$ may be singular. In the characteristic zero case ($A = \mathbb{C}[[t]]$), a formula of Milnor's give the difference in terms of the cohomology of the twisted de Rham complex. For higher characteristic, it was conjectured by Deligne and Bloch that

$$\chi(X_{\overline{k}}) - \chi(X_{\overline{K}}) = [\Delta_X \,.\, \Delta_X]_0 + \mathrm{Sw}(X_{\overline{K}})$$

(Bloch's formula). Toën's colour-coding uses blue for topological objects, red for algebraic objects, and brown for arithmetic objects. The first term on the right is Bloch's *localized intersection number*, which counts the singularities. The second is the *Swan conductor*. The formula is known to be true in a number of special cases, but is open in the mixed characteristic case (for example $A = \mathbb{Z}_p$).

With this background, Toën turned to his new approach to establishing the conjecture, by looking at it from the point of view of non-commutative geometry. In this language, the degenerate case $X_{\overline{\eta}} = 0$ is the commutative case. The general case involves a *quantum parameter*, a choice of a uniformizer $\pi \in A$ and thence a coordinate on S.

Toën's strategy is to seek a non-commutative variety X_π such that $\chi_\pi = \chi(X_{\overline{k}}) - \chi(X_{\overline{K}})$ and apply a non-commutative trace-formula for χ_π. Realizing this involves adopting a very broad definition under which a non-commutative variety over a commutative ring k is simply a k-linear category. There are great many examples, in particular, for any k-algebra, the differential graded category $\mathcal{D}(A)$ of complexes of A-modules. Indeed at first sight the definition seems too weak: there is no true geometry, no topology and no points. But there is a good notion of differential forms, defined by using the Hochschild complex; and, recently, a good notion ℓ-adic cohomology that allows a definition of Euler characteristic.

The non-commutative variety X_π is defined for the family $X \subset \mathbb{P}^n \times S$ by introducing the notion of matrix factorisation for a uniformiser π of A. This is a pair of vector bundles E_0, E_1 on X together with two morphisms

$$E_0 \xrightarrow{\partial} E_1 \xrightarrow{\partial} E_0$$

such that ∂^2 is multiplication by π. Then X_π is the (dg-)category of matrix factorisations. By assembling all these ingredients, Toën and Vezzosi have established a new case in which Bloch's formula is true. The general case, in which Swan's conductor appears, remains under investigation.

Ovidiu Savin

Ovidiu Savin's talk reviewed the connection between non-local equations and local equations in one higher dimension. Local PDEs are equations of the form

$$F(u, u_t.u_x, u_{tt}, u_{tx}, u_{xx}, \ldots) = 0$$

where in order to check the equation at a point, it is enough to know u in a neighbourhood of the point. By contrast, to check a nonlocal PDE at a point it is necessary to know the solution in the whole space.

The talk focused on a simple example, the fractional Laplacian. This has several definitions, but the preferred one is in terms of an extension property that allows the fractional Laplacian in $\mathbb{R}^n$ to be realised in terms of a local operator in $\mathbb{R}^{n+1}$.

Savin began the review by recollecting the definition of a fractional derivative in one dimension. For a 'nice functions' $u : \mathbb{R} \to \mathbb{R}$, D is said to be a *derivative* of u if

a) D is linear in u

b) D is translation invariant, and

c) D has *order* σ under dilation; that is

$$Du_\lambda(x) = \lambda^\sigma Du(\lambda x), \quad \text{where} \quad u_\lambda(x) := u(\lambda x)\,.$$

Up to a constant, there is only one derivative of order 1 that is also *isotropic*, that is, invariant under reflection in the origin. One way to see this is to use translation invariance to show that $(D\cos x) = a\cos x + b\sin x$. Isotropy implies that $b = 0$; and we can choose $a = 1$. It follows that $De^{i\xi x} = |\xi|e^{i\xi x}$, and hence that D is given in general by its action on the Fourier transform of a function by $\widehat{Du} = |\xi|\hat{u}$.

This derivative is 'half' a second-order derivative. It can be defined in other ways. First, as a weighted average of second-order increments: define Du for $\beta \in (-1, 1)$ by

$$Du(x) = \int_0^\infty \frac{u(x+h) + u(x-h) - 2u(x)}{h^{2+\beta}}\,dh\,. \tag{1}$$

Then D is a derivative of order $1 + \beta$, so we can get the desired operator by taking $\beta = 0$.

Second, one can construct Du by taking the y-derivative at $y = 0$ of the harmonic extension of u to $\mathbb{R}^2_+$. That is $Du(x) = U_y(x, 0)$ where

$$\Delta U = 0 \quad \text{and} \quad U(x, 0) = u(x)\,.$$

In all three approaches, it is clear that D is nonlocal since $Du(x)$ depends on values of u far from x.

The definitions via the Fourier transform and via the weighted average adapt in straightforward ways for other values of σ. The definition in terms of the harmonic extension also generalises, but in a less obvious way. The key is to replace the harmonic extension by the *Caffarelli-Silvestre extension*. Define

$$L_\sigma U : U = U_{xx} + U_{yy} + (1-\sigma)\frac{U_y}{y}\,.$$

and extend u by taking $L_\sigma u = 0$ and $U(x, 0) = u(x)$. We then get a derivative of order σ by putting $Du(x) = \partial_{y^\sigma} U(x, 0)$.

Formally L_σ can be viewed as the Laplace operator acting on the rotation of U around the x-axis in $2 - \sigma$ dimensions. The solutions of $L_\sigma U = 0$ minimize the *energy*

$$\int_{\mathbb{R}^2_+} |\nabla U|^2 y^{1-\sigma}\, \mathrm{d}x\mathrm{d}y\,.$$

This extension was already known in a probabilistic context.

In higher dimensions, one can extend the 'weighted average' construction in (1) to define fractional powers of the Laplacian on $\mathbb{R}^n$. If $u \in C^2$ near x and if $u(y)$ decays more slowly than $|y|^\sigma$ at infinity, then the (nonlocal) definition is

$$\Delta^{\sigma/2} u(x) = c_{n,\sigma}\, \mathrm{P.V.} \int_{\mathbb{R}^n} \frac{u(x+y) - u(x)}{|y|^{n+\sigma}}\, \mathrm{d}y.$$

This operator appears in the theory of stochastic processes as the generator of an isotropic σ-stable Lévy process X_t, with

$$\lim_{t \to 0^+} \frac{E\big(u(x + X_t)\big) - u(x)}{t} = \Delta^{\sigma/2} u(x)\,.$$

(In the case of Brownian motion $\sigma = 2$ and the operator on the right is the Laplacian.) The nonlocal character of the operator reflects the fact that the Lévy process can make discontinuous jumps. The corresponding *fractional heat equation*, $u_t = \Delta^{\sigma/2}$ models *anomalous diffusion*. It arises in financial mathematical and in a wide variety of physical and geometric problems.

The Dirichlet problem for the classical Laplacian has a probabilistic interpretation in terms of Brownian motion in the unit ball B_1. If g is a given function on the boundary ∂B_1, then the expected value $u(x)$ of g at the exit point when the process starts at x satisfies

$$\Delta u = 0 \text{ in } B_1, \quad u(x) = g(x) \text{ on } \partial B_1$$

The same is true for the fractional Laplacian and the Lévy process, except that boundary data must be given on the complement of B_1 rather than just on its boundary, to allow for the possibility of jumps from inside the ball to the outside. So the second equation above is replaced by $u(x) = g(x)$ on $\mathbb{R}^n \backslash B_1$.

So how does one address existence, uniqueness and regularity questions for such problems? In the case of the classical elliptic boundary value problem for the Laplacian Δ, the essential tools are the following.

1) Energy methods: the solution minimizes $\int_{B_1} |\nabla u|^2\, \mathrm{d}x$ subject to $u = g$ on ∂B_1.
2) The maximum principle: if $u \le v$ on ∂B_1 and $\Delta u \ge \Delta v$ then $u \le v$ in B_1
3) Holder estimates: $||u||_{C^\alpha(B_{1/2})} \le C||u||_{L^\infty(B_1)}$.

By iterating the last estimate one can successively improve statements about the smoothness of u. In the case of linear equations with rough coefficients, such a *Harnack inequality* serves the same purpose. For nonlinear equations with rough coefficients, such as the *Bellman equation* in control theory, the approach can be adapted by taking derivatives to obtain linear equations.

For the fractional Laplacian, the key is to pass to a local equation, Laplace's equation, in one extra variable. That is to consider the problem

Ovidiu Savin

$$\Delta U = 0 \quad \text{in } \mathbb{R}^{n+1}\backslash B_1 \times \{0\}, \qquad U = g \quad \text{on } \mathbb{R}^n \times \{0\}\backslash B_1 \,.$$

This, together with boundary regularity ($U \in C^{1/2} \implies u \in C^{1/2}$), allows conclusions about the regularity of u. Apart from the need to take care over the requirement to specify data outside the unit ball, the energy methods and the maximum principle extend in a straightforward way to the fractional Laplacian without the need to extend to $\mathbb{R}^{n+1}$.

More recent developments cover the case of nonlocal equations with rough coefficients and integro-differential equations with measurable kernels. There is a Harnack inequality in the latter case that reduces to the standard one in the limiting local case.

Finally, Savin turned to the *obstacle problem* for $\Delta^{1/2}$: for some given function φ, the problem is to find $u \geq \varphi$ such that

$$\Delta^{1/2} u \leq 0, \qquad \Delta^{1/2} u = 0 \quad \text{in } \{u > \varphi\} \,.$$

This can be reformulated in $\mathbb{R}^{n+1}$ in terms of the extension U and the standard Laplacian as an obstacle problem with a 'thin' obstacle:

$$\Delta U \leq 0 \quad \text{in } B_1, \qquad U(x, 0) \geq \varphi(x), \qquad \Delta U = 0 \ \text{ outside } \{U = \varphi\} \cap \{x_{n+1} = 0\} \,.$$

The regularity problems for U and the free boundary have been addressed in this case by using a monotonicity formula to explore the behaviour of U near the free boundary $\partial\{u > \phi\}$. However the formula is very closely tied to the specific geometric properties of the Laplacian and unit ball. A new approach by Caffarelli and others avoids the use of the monotonicity formula, and so may be used more generally.

Tamar Ziegler

Tamar Ziegler

Tamar Ziegler began by posing the following question.

> (Q1) We are given an infinite abelian group G, a vector space V over some field, and a map $\rho : G \to \mathrm{End}_k(V)$ which is an *almost homomorphism* in the sense that
>
> $$\operatorname{rank}\big(\rho(x + y) - \rho(x) - \rho(y)\big) < r \qquad \forall x, y \in G \,.$$
>
> Does there exist a homomorphism σ close to ρ in the sense that
>
> $$\operatorname{rank}\big(\rho(x) - \sigma(x)\big) < R \qquad \forall x \in G\,?$$

Here R is a constant depending on r, as well as on the choice of the group and the field. The question opens a window on recent developments in ergodic theory and additive combinatorics.

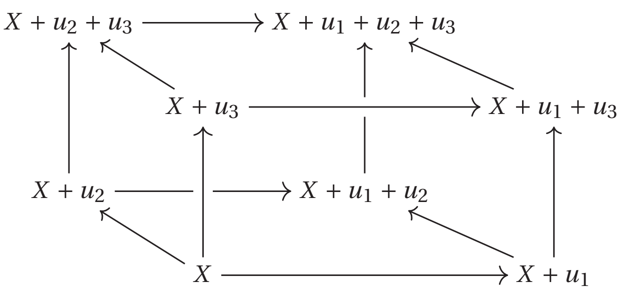
A good starting point for developing techniques to address such problems is Roth's 1952 theorem that a subset $E \subset \mathbb{Z}$ of positive density necessarily contains a three-term arithmetic progression, together with Szemerédi's extension to k-term progressions (1975).

It is helpful to explore Roth's result by considering the corresponding statement in a different setting, in which E is a subset of density δ of a vector space V over a finite field k. Here the answer is found by using discrete Fourier analysis: the key point is that either E is 'Fourier uniform' in the sense that the Fourier coefficients of its characteristic function 1_E are small, or else 1_E has a large Fourier coefficient. In the first case E has roughly the same number of three-term arithmetic progressions as a random subset of the same density; in the second, E has increased density on some affine hyperplane.

The idea is not sufficient to establish the analogous result for four-term progressions. One reason is that the count of four-term progressions is biased when the subset is the hypersurface $\{Q(x) = 0\}$ for some high-rank quadratic form Q. Here the Fourier coefficients are small, but the identity

$$Q(x) - 3Q(x+d) + 3Q(x+2d) - Q(x+3d) = 0$$

forces the fourth term of a progression to lie on the hypersurface whenever the first three do, so the hypersurface contains many more four-term progressions than might be expected. Instead it is necessary to adapt the argument by looking at 'quadratic Fourier coefficients'. Either these are all small or E has increased density on a quadratic hypersurface.



This raises the general question: if E has a 'biased' number of d-term progressions, does it necessarily have increased density on some hypersurface of degree less than $d-1$? The approach based on *Gowers norms* starts with the observation that, rather than count four-term progressions, it is better to count three-dimensional cubes, as on the left. If there is bias in the count of progressions, then there is also a bias in the count of cubes. The observation is coupled with another, that if $\mathrm{char}(k) > d$, then $P : V \to k$ is a polynomial of degree less that d if and only if it vanishes on every d-dimensional cube in the sense that the alternating sum of its values at the vertices vanishes.

In terms of cubes, the question becomes the following.

> (Q2) If the number of d-dimensional cubes in E is not as in a random set, is it necessarily true that E has increased density on a hypersurface of degree less than d ($\mathrm{char}(k) > d$)?

Before addressing this, Ziegler turned to the ergodic theory approach, which is based on *Furstenberg's correspondence principle*. Under this, the statement that there are k-term progressions in a set $E \subset \mathbb{N}$ is translated to one about multiple return times for a distinguished subset A of a measure space X with a measure-preserving map $T : X \to X$. If

$$\mu(A \cap T^{-n}A \cap \cdots \cap T^{-kn}A) > 0 \tag{1}$$

for some n, then E contains a $k+1$-term progression. Furstenbergs's idea was to study such questions about return times by using morphisms to simpler measure-preserving systems.

Ziegler illustrated how the idea works by outlining an alternative approach to Roth's theorem. The argument has the same general shape as the original, but now the two cases are distinguished by whether or not T has non-trivial eigenfunctions. If not, then

$$\frac{1}{N}\sum_{n\leq N}\mu(A\cap T^{-n}A\cap T^{-2n}A)\to\mu(A)^3$$

and we are done; otherwise there are non-trivial eigenfunctions ψ_i, with $\psi(Tx)=\lambda\psi(x)$. When normalized, one or more of these determine a morphism from X to a much simpler dynamical system,

$$\pi\,:\,X\to Z\,:=\prod_i S^1\,.$$

This is a *Kronecker system*: Z is an abelian group with Haar measure and multiplication by the eigenvalues λ_i determines a rotation $S\,:\,Z\to Z$. In the Kronecker system, the inequality (1) can be established by showing that the sum

$$\frac{1}{N}\sum_{n\leq N}\pi_*\mu(\pi_*A\cap S^{-n}\pi_*A\cap S^{-2n}\pi_*A)$$

is positive. By replacing A by its characteristic function, one is then led to consider the asymptotic behaviour of averages of the form

$$\frac{1}{N}\sum_1^N\int\pi_*f(z)\pi_*f(z+n\alpha)\pi_*f(z+2n\alpha)\,\mathrm{d}\pi_*\mu$$

for some positive function f. This is much more straightforward than the original problem.

To extend the picture to four-term and longer progressions, one needs to replace the eigenfunctions ψ_i, on which the action of T is linear, by 'quadratic' and higher degree 'polynomials'. So we now consider a space X with measure μ and a measure-preserving ergodic action $T_u\,:\,X\to X$, $u\in G$, of a countable abelian group G. For $f\,:\,X\to\mathbb{C}$, put

$$\Delta_u f(x)=f(T_u x)\overline{f(x)}\,.$$

A function P is said to be a *polynomial* of degree less than d if

$$\Delta_{u_d}\ldots\Delta_{u_1}P(x)=1\text{ a.e.}\quad\forall u_1,\ldots,u_d\in G\,. \tag{2}$$

A polynomial of degree less than one is constant, assuming ergodicity; and an eigenfunction is a polynomial of degree less than two.

The definition can be re-expressed in terms of values at the vertices of cubes. With $d=4$, the condition (2) can be restated as the 'vanishing' of P on cubes, in the multiplicative sense:

$$\prod_{\omega\in\{0,1\}^3}P^{(\omega)}(T_{\omega\cdot(u_1,u_2,u_3)})(x)=1\ \text{ a.e.} \tag{3}$$

where $P^{(\omega)}$ is equal to P or $\overline{P}$ as $|\omega|$ is even or odd.

Bergelson, Tao, and Ziegler combined this last idea with the cube construction to reduce the averaging problem for four-term progressions down to simpler *polynomial systems*, in an analogous way to the use of eigenfunctions for the three-term progressions.

By using polynomial systems of higher degree Tao and Ziegler answered question (Q2) positively and constructed bounds extending those found by Green, Tao, and Gowers in the case $d = 4$ by using additive combinatorics.

Ziegler then explained how these techniques can be used to answer the original question (Q1) in the case $G = W = V$, where W is infinite. The approximating homomorphism is found by making finite approximations, with $k = \mathbb{F}_p$ and $|W| = p^n$. Because ρ is close to linear, the function

$$f(w, x) = \exp(2\pi i \langle \rho(w)x, x \rangle / p)$$

is close to being a 'cubic' polynomial in w, x. An exact 'cubic' vanishes on four-dimensional cubes in the sense of (3), while the values of f are biased because ρ is close to being linear. The bias allows one to use the theory above to establish the existence of a cubic $Q(w, z)$ (in the standard sense) such that $\rho(w) - Q(w, \cdot)$ is of bounded rank.

Finally Ziegler outlined an 'approximate cohomology theory' that she had developed with David Kazhdan that provides a general framework for considering questions of his type.

Clay Research Conference Workshops

The plenary talks were drawn from the areas of the workshops that were held during the week of the conference.

Harmonic Analysis and Related Areas

September 24-28, 2017

Organizers

Larry Guth, Massachusetts Institute of Technology
Nets Katz, California Institute of Technology

Speakers

Jonathan Bennett, University of Birmingham, *Perturbed Brascamp—Lieb inequalities*
Ciprian Demeter, Indiana University, *Decoupling beyond uniform sets*
Polona Durcik, California Institute of Technology, *Entangled multilinear forms and applications*
Ben Green, University of Oxford, *The arithmetic Kakeya conjecture of Katz and Tao*
Izabella Laba, University of British Columbia, *Lower bounds for the maximal directional Hilbert transform*
Elon Lindenstrauss, Hebrew University of Jerusalem, *Bourgain's discretized projection theorem, revisited*
Stefanie Petermichl, Université de Toulouse III, *Higher order Journé commutators*
Tom Sanders, University of Oxford, *The cost of commuting*
Christopher Sogge, Johns Hopkins University, *On the concentration of eigenfunctions*
Christoph Thiele, Universität Bonn, *Some results on directional operators*
Trevor Wooley, University of Bristol, *Nested Efficient Congruencing and relatives of Vinogradov's mean value theorem*
Joshua Zahl, University of British Columbia, *Breaking the 3/2 barrier for unit distances in three dimensions*

Ergodic Theory: Numbers, Fractals, and Geometry

September 24-28, 2017

Organizers

Manfred Einsiedler, ETH Zürich
Tom Ward, University of Leeds
Tamar Ziegler, Hebrew University of Jerusalem

Speakers

Nalini Anantharaman, Université de Strasbourg, *Quantum ergodicity on graphs: spectral and spatial delocalization*
Tim Austin, University of California, Los Angeles, *Measure concentration and the weak Pinsker property*
Lewis Bowen, University of Texas at Austin, *A counterexample to the weak Pinsker conjecture for free group actions*
Emmanuel Breuillard, Université Paris Sud, *Homogeneous dynamics and the subspace theorem*
Giovanni Forni, University of Maryland, *Effective equidistribution by scaling, and number theory*
Alex Gorodnik, University of Bristol, *Discrepancy in Diophantine problems*
Alex Kontorovich, Rutgers University, *Geometry to arithmetic of crystallographic packings*
Elon Lindenstrauss, Hebrew University of Jerusalem, *Random walks on homogenous spaces*
Jens Marklof, University of Bristol, *Higher dimensional Steinhaus problems via homogeneous dynamics*
Hee Oh, Yale University, *Geometric prime number theorems and fractals*
Jean-Francois Quint, Université de Bordeaux, *Limit theorems for the spectral radius*
Pablo Shmerkin, Universidad Torcuata Di Tella, *Furstenberg's intersection conjecture and self-similar measures*
Corinna Ulcigrai, University of Bristol, *On Birkhoff sums and Roth type conditions for interval exchange transformations*
Péter Varjú, University of Cambridge, *Full dimension of Bernoulli convolutions*

Modern Moduli Theory

September 25-29, 2017

Organizers

Dominic Joyce, University of Oxford
Kevin McGerty, University of Oxford
Balázs Szendrői, University of Oxford

Speakers

Dario Beraldo, University of Oxford, *The center of the monoidal category of "higher differential operators"*
Chris Brav, Higher School of Economics, *Relative Calabi-Yau structures and shifted Lagrangians*
Yalong Cao, University of Oxford, *Gopakumar-Vafa type invariants for Calabi-Yau 4-folds*
Ben Davison, IST Vienna, *BPS sheaves and Lie algebras*
Chris Dodd, University of Illinois, *A k-valued cohomology theory for varieties over k*
Barbara Fantechi, SISSA, *DGLA and deformations of log orbifolds*
Elham Izadi, University of California, San Diego, *A uniformization of the moduli space of abelian sixfolds*
Frances Kirwan, University of Oxford, *Applications of non-reductive geometric invariant theory*
Kobi Kremnitzer, University of Oxford, *Differentiable chiral algebras*
Sven Meinhardt, University of Sheffield, *Integrality in Donaldson-Thomas theory*
Tom Nevins, University of Illinois, *Compactifications, cohomology, and categories associated to moduli spaces*
Georg Oberdieck, Massachusetts Institute of Technology, *Curve counting on elliptic Calabi-Yau threefolds: K3xE and the Schoen Calabi-Yau*
Andrei Okounkov, Columbia University, *Gauge theories and Bethe eigenfunctions*
Tony Pantev, University of Pennsylvania, *Gluing in nc geometry and applications*
Jørgen Rennemo, University of Oxford, *Homological projective duality and the birational Torelli problem for Calabi-Yau 3-folds*
Nick Rozenblyum, University of Chicago, *Quantization of moduli spaces and counting*
Yukinobu Toda, Kavli IPMU, *Gopakumar-Vafa invariants and wall-crossing*
Bertrand Toën, Université de Toulouse, *Shifted symplectic structures and exponential motives*

Nonlocal PDEs

September 25-29, 2017

Organizers

Luis Caffarelli, University of Texas at Austin
Ovidiu Savin, Columbia University

Speakers

Xavier Cabre, Universitat Politecnica de Catalunya, *Nonlocal minimal cones and surfaces with constant nonlocal mean curvature*
María del Mar Gonzáles, Universidad Autónoma de Madrid, *Non-local ode: an application to the singular fractional Yamabe problem in conformal geometry*
Cyril Imbert, ENS Paris, *The weak Harnack inequality of the Boltzmann equation without cut-off*
Tianling Jin, Hong Kong University of Science and Technology, *On the isoperimetric quotient over scalar-flat conformal classes*
Moritz Kassmann, Universität Bielefeld, *Nonlocal energy forms and function spaces*
Dennis Kriventsov, New York University, *Regularity in time for fully nonlinear, nonlocal (and local) parabolic equations*
Jean-Michel Roquejoffre, Université de Toulouse, *Front propagation driven by a line of fast diffusion: a property of the level sets*
Ovidiu Savin, Columbia University, *Rigidity results for non-local phase transitions*
Joaquim Serra, ETH Zürich, *The De Giorgi conjecture for the half-Laplacian in dimension 4*
Yannick Sire, Johns Hopkins University, *Asymptotic limits for the fractional Allen-Cahn equation and stationary nonlocal minimal surfaces*
Enrico Valdinoci, University of Melbourne, *Crystal dislocation, nonlocal equations and fractional dynamical systems*
Alexis Vasseur, University of Texas at Austin, *Regularity theory for non local in time operators*
Juan Luis Vazquez, Universidad Autónoma de Madrid, *Nonlinear fractional diffusion, from older to recent work*
Jun-cheng Wei, University of British Columbia, *Counter-examples to De Giorgi Conjecture for Fractional Allen-Cahn*

2017 CLAY RESEARCH AWARD

The Clay Research Awards, presented annually at the Clay Research Conference, celebrate the outstanding achievements of the world's most gifted mathematicians.

Aleksandr Logunov and Eugenia Malinnikova

The joint Award to Aleksandr Logunov (Tel Aviv University and Chebyshev Laboratory, St Petersburg State University) and Eugenia Malinnikova (NTNU) was made in recognition of their introduction of a novel geometric combinatorial method to study doubling properties of solutions to elliptic eigenvalue problems. This has led to the solution of long-standing problems in spectral geometry, for instance the optimal lower bound on the measure of the nodal set of an eigenfunction of the Laplace-Beltrami operator in a compact smooth manifold (Yau and Nadirashvili's conjectures).

Maryna Viazovska

The Award to Maryna Viazovska (Princeton University and École Polytechnique Fédérale de Lausanne) was made in recognition of her groundbreaking work on sphere-packing problems in eight and twenty-four dimensions. In particular, her innovative use of modular and quasimodular forms, which enabled her to prove that the $E8$ lattice is an optimal solution in eight dimensions. The result had been suggested by earlier work of Henry Cohn and Noam Elkies, who had conjectured the existence of a certain special function that would force the optimality of the $E8$ lattice through an application of the Poisson summation formula. Viazovska's construction of the function involved the introduction of unexpected new techniques and establishes important connections with number theory and analysis. Her elegant proof is conceptually simpler than that of the corresponding result in three dimensions. She subsequently adapted her method in collaboration with Henry Cohn, Abhinav Kumar, Stephen Miller, and Danylo Radchenko to prove that the Leech lattice is similarly optimal in twenty-four dimensions.

Jason Miller and Scott Sheffield

The Award to Jason Miller (University of Cambridge) and Scott Sheffield (MIT) was made in recognition of their groundbreaking and conceptually novel work on the geometry of the Gaussian free field and its application to the solution of open problems in the theory of two-dimensional random structures.

The two-dimensional Gaussian free field (GFF) is a classical and fundamental object in probability theory and field theory. It is a random and Gaussian generalized function h defined in a planar domain D. Despite its roughness and the fact that it is not a continuous function, it possesses a spatial Markov property that explains why it is the natural counterpart of Brownian motion when the time-line is replaced by the two-dimensional set D. Miller and Sheffield have studied what can be viewed as level-lines of h and more generally flow lines of the vector fields $\exp(iah)$, where a is any given constant. This framework, which they call imaginary geometry, allows them to embed many Schramm-Loewner Evolutions (SLE) within a given GFF. A detailed study of the way in which the flow lines interact and bounce off each other allowed Miller and Sheffield to shed light on a number of open

Maryna Viazovska, Aleksandr Logunov, Eugenia Malinnikova

questions in the area and to pave the way for further investigations involving new random growth processes and connections with quantum gravity.

Their award will be presented at the Clay Mathematics Institute's 20th anniversary conference in September 2018.

PROFILE

An Interview with Maryna Viazovska

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I was attracted to mathematics by the simplicity and elegance of the subject. In mathematics we can start with simple axioms and build complex theories explaining the world around us.

Could you talk about your mathematical education? What experiences and people were especially influential?

I started my education in Kiev. I received my Bachelor's degree from Kiev Taras Shevchenko University. Then I obtained a Master's degree from TU Kaiserslautern and did my PhD at the Max Planck Institute for Mathematics in Bonn.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I was lucky to have several great mentors. Probably the first person who showed me how interesting the science of mathematics is was my grandfather. My high school teachers in physics and mathematics were extremely enthusiastic about their subjects, they taught me to work hard. I am grateful to Igor Shevchuk, Sergiy Ovsienko, Gerhard Phister, and Don Zagier who have been my research advisors.

From your own experience at high school, are there any aspects of mathematics education that you would like to see changed?

I think of my own high school experience as very positive. I studied in a school which specialized in natural sciences. So, I spent a lot of time studying the subjects I liked the most and also worked with teachers and fellow students who shared my enthusiasm. I think learning can be a joyful and creative activity. When it is, both students and teachers will invest a lot of effort. A challenge for the educational system is to organize such a creative atmosphere and to include as many children as possible into it.

What attracted you to the particular problems you have studied?

A perfect research problem has to be in a sweet spot between what is challenging and what is feasible. I choose problems according to my taste, my interests, and also my abilities.

What research problems and areas are you likely to explore in the future?

In future, I will keep working on optimization problems. It would be interesting for me to find new connections between purely mathematical questions and applications.

How have your collaborators made a difference for you?

I have learned a lot from my collaborators.

What advice would you give lay persons who would like to know more about mathematics - what it is, what its role in our society has been and is, etc.? What should they read? How should they proceed?

I think mathematics is a perfect "mental yoga". There are many wonderful recreational books in mathematics, here are only few of them: *A Mathematician's Miscellany* by J.E. Littlewood, *Mathematical Puzzles* by M. Gardner, *Mathematics Can be Fun* by Y. Perelman, and *Proofs from the Book* by M. Aigner and G. Ziegler

How do you think mathematics benefits culture and society?

We live in exiting and frightening times, when new technologies can change society. Mathematical literacy has become extremely important now. Without it people cannot understand the processes happening in the world, cannot make rational and informed decisions, and cannot fully participate in a discussion about our future.

Summary of 2017 Research Activities

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2017:

Research Fellows, Research Awardees, Senior Scholars 26

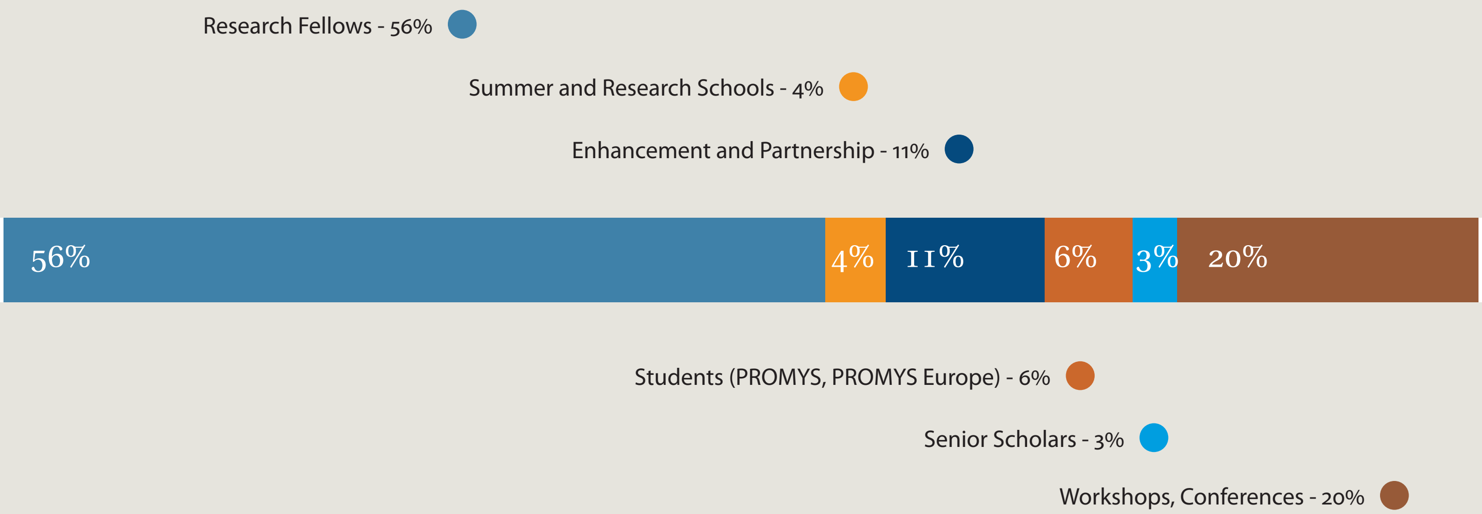
CMI Workshops 146

Research Schools 191

PROMYS/PROMYS Europe Faculty and Participant 55

Participants attending Conferences and Joint Programs 100+

Research Expenses for Fiscal Year 2017



CLAY RESEARCH FELLOWS

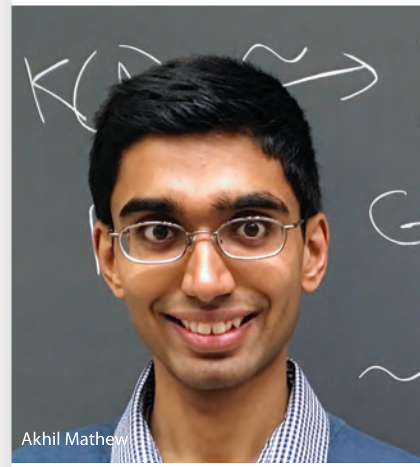
Peter Hintz

Peter Hintz studies hyperbolic partial differential equations arising in general relativity using methods from microlocal analysis, spectral and scattering theory, and dynamical systems. His recent work concerns the stability of black holes in expanding spacetimes. Born in Kassel, Germany, Peter received a BSc in Mathematics and BSc in Physics from the University of Göttingen in 2011 and a PhD in 2015 from Stanford University under the supervision of András Vasy. Peter is a Miller Research Fellow (2015-2017) at the University of California, Berkeley, mentored by Maciej Zworski. Peter has been appointed as a Clay Research Fellow for a term of three years beginning August 2017.


Peter Hintz

Akhil Mathew

Akhil Mathew will receive his PhD in 2017 from Harvard University under the supervision of Jacob Lurie. Previously he received an undergraduate degree from Harvard and studied at UC Berkeley for one year. His research focuses on homotopy theory, higher categories, and their applications, especially to derived algebraic geometry and algebraic K-theory. Some of his past work studied various generalizations of faithfully flat descent in stable homotopy theory and their role in describing certain invariants of structured ring spectra. Akhil has been appointed as a Clay Research Fellow for a term of five years beginning July 2017.


Akhil Mathew

Research Fellows

Semyon Dyatlov (2013-2018)
California Institute of Technology

Simion Filip (2016-2021)
Harvard University

Peter Hintz (2017-2020)
University of California, Berkeley

June Huh (2014-2019)
Princeton University

Akhil Mathew (2017-2022)
University of Chicago

James Maynard (2015-2018)
University of Oxford

John Pardon (2015-2020)
Princeton University

Aaron Pixton (2013-2018)
Massachusetts Institute of Technology

Jack Thorne (2013-2017)
University of Cambridge

Miguel Walsh (2014-2018)
University of Oxford

Alex Wright (2014-2019)
Stanford University

Tony Yue Yu (2016-2021)
Université Paris Sud

Senior Scholars

Dmitri Orlov (IAS, Spring 2017)
Homological Mirror Symmetry

Emmanuel Breuillard (INI, Spring 2017)
Positive Curvature Group Actions and Cohomology

Alexander Volberg (MSRI, Spring 2017)
Harmonic Analysis

Manjul Bhargava (MSRI, Spring 2017)
Analytic Number Theory

Roger Heath-Brown (MSRI, Spring 2017)
Analytic Number Theory

Craig Tracy (PCMI, Summer 2017)
Random Matrices

H. T. Yau (PCMI, Summer 2017)
Random Matrices

William Johnson (MSRI, Fall 2017)
Geometric Functional Analysis and Applications

Francisco Santos (MSRI, Fall 2017)
Geometric and Topological Combinatorics

CMI WORKSHOPS

Symplectic Geometry
A celebration of the work of Simon Donaldson

August 14-18, 2017 | Isaac Newton Institute, Cambridge

This meeting of the world's experts in symplectic geometry and neighbouring fields celebrated the 60th birthday of Simon Donaldson and his profound influence on the subject. A characteristic of both his work and this meeting is the influence of (and on) other fields, such as low dimensional topology, algebraic geometry, geometric analysis and theoretical physics.

Many of the speakers and participants were present at the 1994 programme on symplectic geometry, to which this meeting was nominally a follow-up. The spectacular progress in the subject since then was evident, as were the similarities and differences.

The talks covered large swathes of mathematics around symplectic geometry, from topology and algebraic geometry through geometric analysis and homological algebra to physics. Some talks discussed solutions, or progress toward solutions, to important old problems which were well known in 1994. Overall, the conference enforced the impression of a field which is both vibrant and mature, now a large mainstream subject in its own right which heavily influences topology, geometry, homological algebra and physics, as well as being influenced by them.

This was a joint CMI-INI workshop.

Organizers

Dusa McDuff, Barnard College
Dietmar Salamon, ETH Zürich
Paul Seidel, Massachusetts Institute of Technology
Richard Thomas, Imperial College London

Speakers

Mina Aganagic, University of California, Berkeley
Michael Atiyah, University of Edinburgh
Denis Auroux, University of California, Berkeley
Kenji Fukaya, Stony Brook University
Mikhail Gromov, IHÉS
Nigel Hitchin, University of Oxford
Eleny Ionel, Stanford University
Frances Kirwan, University of Oxford
Peter Kronheimer, Harvard University
Dusa McDuff, Barnard College
Tom Mrowka, Massachusetts Institute of Technology
Emmy Murphy, Northwestern University
Peter Ozsváth, Princeton University
John Pardon, Princeton University
Paul Seidel, Massachusetts Institute of Technology
Ivan Smith, University of Cambridge
Song Sun, Stony Brook University
Zoltán Szabó, Princeton University
Thomas Walpuski, Massachusetts Institute of Technology
Katrin Wehrheim, University of California, Berkeley

D-modules, Geometric Representation Theory and Arithmetic Applications

December 4-8, 2017 | University of Oxford

The fields of *p*-adic representation theory and of the theory of *p*-adic differential equations will particularly benefit from closer interaction with one another; the aim of this workshop was to facilitate this, both at the time and into the future.

While *D*-modules have played a major role in the representation theory of real reductive groups for a long time, the use of appropriate differential operators on the *p*-adic spaces to study representations of *p*-adic reductive groups is a more recent development.

The workshop brought together experts working in the fields of *D*-modules, the representation theory of real and *p*-adic reductive groups and the theory of *p*-adic differential equations to review the progress that has been made in the *p*-adic setting over the past four to five years. There were talks from those involved in each of these areas and connections between them were evident throughout. The workshop was also popular amongst the younger participants, who contributed a total of seven posters describing their research.

The workshop brought into sharper focus the fact that certain outstanding problems from *p*-adic representation theory, such as the construction of representations through the cohomology of Drinfeld coverings of *p*-adic symmetric spaces, may profitably be attacked using the techniques from *D*-modules on *p*-adic spaces such as formal schemes and rigid analytic spaces.

Organizers

Konstantin Ardakov, University of Oxford
Tobias Schmidt, Université de Rennes
Matthias Strauch, Indiana University
Simon Wadsley, University of Cambridge

Speakers

Tomoyuki Abe, University of Tokyo
Francesco Baldassarri, Università degli Studi di Padova
Dan Ciubotaru, University of Oxford
Richard Crew, University of Florida
Dmitry Gourevitch, Weizmann Institute
Christine Huyghe, Université de Strasbourg
Bernard Le Stum, Université de Rennes
Ruochuan Liu, Beijing International Center for Mathematical Research
Adriano Marmora, Université de Strasbourg
Dragan Milicic, University of Utah
Andrea Pulita, Université Grenoble Alpes
Peter Schneider, Universität Münster

Photo: Julie Banfield

PROMYS EUROPE

July 9–August 19, 2017

In July, pre-university students from across Europe arrived in Oxford to participate in PROMYS Europe. The PROMYS (Program in Mathematics for Young Scientists) summer school for high school students was set up by Glenn Stevens in Boston, MA, in 1989. A mirror program, PROMYS Europe, was launched in Oxford in 2015 as a partnership between PROMYS, the Clay Mathematics Institute, the Mathematical Institute at Oxford University, and Wadham College, with support from Oxford alumni and the Heilbronn Institute for Mathematical Research.

PROMYS Europe continued to grow in 2017, welcoming a strong group of 28 mathematically ambitious students from 13 countries, six of whom were returning for a second summer. The students were supported by eight counsellors, undergraduate mathematicians from top European universities, some of whom are themselves PROMYS Europe alumni.

First-year students focused primarily on a series of very challenging problem sets, daily lectures, and exploration projects in number theory. Returning students were offered courses in graph theory and algebraic number theory, as well as seminars and research projects mentored by professional mathematicians. There was also a program of talks by guest mathematicians and PROMYS Europe counsellors on a wide range of mathematical subjects.

Participants enjoyed six weeks of rigorous mathematical activity within a supportive community of students, counsellors, mentors, faculty and visiting mathematicians. While selection for PROMYS Europe is highly competitive, it is needs-blind with costs covered for those who would otherwise be unable to attend.

Lecturers

Henry Cohn, Microsoft Research and Massachusetts Institute of Technology
Vicky Neale, University of Oxford
Glenn Stevens, Boston University

Guest Lecturers

Ben Barber, University of Bristol
Edith Elkind, University of Oxford
Dan Kráľ, University of Warwick
Alan Lauder, University of Oxford
Owen Patashnick, University of Bristol
Claudia Scheimbauer, University of Oxford
Simon Singh
Balázs Szendrői, University of Oxford
Andrew Wiles, University of Oxford

Counsellors

Levi Borodenko, University of Oxford
Elizaveta Lokteva, Uppsala University
Lucas Mann, Humboldt University Berlin
Miroslav Marinov, University of Oxford
Anastasia Prokudina, Humboldt University Berlin
Julia Stadlmann, University of Oxford
Art Waeterschoot, KU Leuven
Wojciech Wawrów, Adam Mickiewicz University

New Trends in Representation Theory
The Impact of Cluster Theory in Representation Theory

June 19-23, 2017 | University of Leicester

Cluster algebras were introduced by Fomin and Zelevinsky in 2002 in the context of Lie theory. Since then, the subject has seen a rapid development and many exciting connections with other areas of mathematics have been established, including Teichmüller Theory, discrete integrable systems, KP equations, combinatorics of RNA secondary structures, and total positivity of matrices. Very quickly it became apparent that the theory had a vast impact on the representation theory of algebras.

This research school offered an overview of the recent advances that have emerged in representation theory through cluster theory. It consisted of three lecture courses, each supported by tutorial sessions.

- *n-Representation Theory* by Peter Jørgensen began with an introduction to *d*-abelian categories and *d*-cluster tilting, then moved on to describe higher Auslander-Reiten theory, and finished with the introduction of *(d+2)*-angulated categories.
- *Integrable systems and friezes* by Sophie Morier-Genoud opened with an introduction to integrable systems and then covered different types of frieze patterns. Her final lectures introduced generalized friezes and related these to representation theory.
- *Infinite dimensional representations* by Lidia Angeleri-Hügel focused on the newly developed silting theory. Her lectures developed the theory of silting complexes over rings and showed how these are in bijection with t-structures and co-t-structures in the derived category.

Organizers

Karin Baur, Universität Graz
Sibylle Schroll, University of Leicester

Lecturers

Peter Jørgensen, University of Newcastle
Sophie Morier-Genoud, Institut de Mathématiques de Jussieu, UPMC
Lidia Angeleri-Hügel, Università degli Studi di Verona

Guest Lecturers

Martin Herschend, Uppsala University
Pierre-Guy Plamondon, Université de Paris Sud
Mike Prest, University of Manchester

Tutors

Max Glick, University of Connecticut
Gustavo Jasso, Universität Bonn
Jorge Vitoria, Università degli Studi di Verona

Three guest lectures complemented the lecture courses, each connecting the topic of the course to an active area of research and presenting the newest results and ongoing work. Martin Herschend's talk *Higher preprojective algebras* illustrated new results on *d*-representation finite algebras. *Group action on quivers on triangulated surfaces with punctures* by Pierre-Guy Plamondon presented a geometric model encoding the representation theory of such surfaces, thereby solving a longstanding open problem. In *Definable categories*, Mike Prest gave an overview of how the material present by Angeleri-Hügel fits into a bigger context such as model theory.

Based on the workshop report prepared by the organizers.

Microlocal Analysis and Applications

June 26-30, 2017 | Cardiff University

Organizers

Suresh Eswarathasan, Cardiff University
Colin Guillarmou, CNRS, Université Paris-Sud
Roman Schubert, University of Bristol

Lecturers

Viviane Baladi, CNRS, Institut de Mathématiques de Jussieu, UPMC
Colin Guillarmou, CNRS, Université Paris-Sud
Andrew Hassell, Australian National University
Stéphane Nonnenmacher, Université Paris-Sud
Alexander Strohmaier, University of Leeds
Jared Wunsch, Northwestern University

Guest Lecturers

Nicolas Burq, Université Paris-Sud
Mark Pollicott, University of Warwick
Gunther Uhlmann, University of Washington

Tutors

Yannick Bonthonneau, Université Rennes
Maxime Ingremeau, Université Paris-Sud
Martin Vogel, Université Paris-Sud

Microlocal analysis originated in the study of partial differential equations through the lens of phase space methods by combining ideas from symplectic geometry and Fourier analysis to investigate qualitative and quantitive properties of PDEs. In particular, the philosophy has lead to major advances in the understanding of linear PDEs in the last 50 years.

More recently, the resolution of the Bethe-Sommerfeld Conjecture by Parnovski and Sobolev and the groundbreaking work of Anantharaman giving progress on the Quantum Unique Ergodicity Conjecture of Rudnick-Sarnak both utilised microlocal techniques. The field continues to develop and is constantly finding new applications in diverse areas of mathematics, such as the spectral theory of self adjoint and non-self adjoint operators, scattering theory, inverse problems in medical imaging, and mixing in dynamical systems.

This research school offered young researchers the opportunity to learn key ideas and techniques of the field together with some of the most important recent applications. Three mini-courses, led by internationally prominent experts in microlocal analysis and their applications, covered:

- *Basic ideas in microlocal analysis* by Alexander Strohmaier and Jared Wunsch
- *Spectral and scattering theory* by Stéphane Nonnenmacher and Andrew Hassell
- *Pollicott-Ruelle resonances, mixing in dynamical systems, and x-ray transforms* by Viviane Baladi and Colin Guillarmou

The lecture courses were supported by exercise/tutorial sessions as well as open problem sessions where current trends and potential open problems were discussed in detail. Three distinguished mathematicians who gave lectures which complemented the mini courses were Mark Pollicott, Gunther Uhlmann, and Nicolas Burq.

Based on the workshop report prepared by the organizers.

Introduction to Geometry, Dynamics, and Moduli in Low Dimensions

September 11-15, 2017 | University of Warwick

Low-dimensional geometry, topology and dynamics underwent a major renaissance starting in the mid-1970s, driven by work of Gromov, Thurston, and Sullivan, among others. Teichmüller space, which is a moduli space for hyperbolic structures on surfaces, played an important role in the development of all of these topics. This has motivated the study of analogous spaces, such as Outer space (a moduli space of metric graphs) and Strata, moduli spaces for flat structures on surfaces, which are now undergoing rapid development. Cube complexes were used as an essential tool for resolving some questions raised by Thurston, and moduli spaces of cube complexes are also being developed to study automorphisms of right-angled Artin groups. All of these spaces can be thought of as analogues of homogeneous spaces, but in contrast to the well-developed, coherent theory of homogeneous spaces, the various theories for these analogues have been treated in a less coherent, more ad hoc manner.

The aim of this research school was to tie together these various directions and introduce a new generation of researchers to the beautiful interplay between geometric, topological, algebraic, and dynamical study of Teichmüller space, of the mapping class group, of Outer space, and of $\mathrm{Out}(F_n)$. It featured a set of five minicourses covering:

- *The geometry of outer space* by Yael Algom-Kfir
- *Description of Teichmüller space in terms of hyperbolic geometry* by Tara Brendle
- *Methods for computation of geometric structures and invariants* by Nathan Dunfield
- *Teichmüller dynamics* by Erwan Lanneau
- *Geometric structures viewed in terms of representatives* by Julien Marché

Tutorial support for each course provided participants the opportunity to review the lectures, solve problems, and ask questions related to the material presented in the courses. There were also six workshop sessions in which lecturers, TAs and organizers were available to answer questions and offer hints on the many problems assigned.

Based on the workshop report prepared by the organizers.

Organizers

Javier Aramayona, Universidad Autónoma de Madrid
Saul Schleimer, University of Warwick
John Smillie, University of Warwick

Lecturers

Yael Algom-Kfir, University of Haifa
Tara Brendle, University of Glasgow
Nathan Dunfield, University of Illinois at Urbana-Champaign
Erwann Lanneau, Université Grenoble Alpes
Julien Marché, Université Pierre et Marie Curie

Tutors

Mark Bell, University of Illinois at Urbana-Champaign
Stergios Antonakoudis, University of Cambridge

Algebraic Topology of Manifolds

September 11-15, 2017 | University of Oxford

Organizer

Ulrike Tillmann, University of Oxford

Lecturers

Greg Arone, Stockholm University
Dan Freed, University of Texas at Austin
Oscar Randal-Williams, University of Cambridge
Nathalie Wahl, University of Copenhagen

Guest Lecturers

Soren Galatius, Stanford University
Graeme Segal, University of Oxford

Tutors

Sam Nariman, Northwestern University
Martin Palmer, Universität Bonn
Claudia Scheimbauer, University of Oxford

Manifolds are at the center of much of geometry and topology, and through the influence of axiomatic topological quantum field theory they have become an important organizing force in category and representation theory.

Classically, in the 1960s, algebraic topology was at the heart of their classification theory in form of characteristic classes and characteristic numbers, cobordism theory, surgery theory, and later Waldhausen's K-theory of manifolds. By the 1980s the machinery got heavy with diminishing returns. We are now experiencing a renaissance of the field as well as a paradigm shift where manifolds not only are the objects of study but become the tools.

The research school was designed to inspire the next generation with this exciting success story of interwoven ideas bouncing between different fields, and give the participants the tools to contribute to this lively research area.

The week comprised four lecture series, supplemented by exercises, tutorials and problem sessions:

- *Topological Quantum Field Theory* by Dan Freed covered basic concepts of TQFT and examples, cobordism theory, Baez-Dolan cobordism hypothesis, invertible theories and anomalies, extended and relative field theories.
- *Characteristic classes and moduli spaces of manifolds* by Oscar Randal-Williams addressed characteristic classes of manifolds, moduli spaces of smooth manifolds and the scanning map.
- *The Goodwillie-Weiss embedding calculus* by Greg Arone covered configuration and embedding spaces, homotopy limits, approximation and convergence theorems, and applications.
- *Homological stability* by Nathalie Wahl examined homological stability of groups and categorical frame work.

A Participants Forum provided an opportunity for students and tutors to present short talks introducing their own research topics. Guest lectures were presented by Graeme Segal and Soren Galatius.

Based on the workshop report prepared by the organizer.

ENHANCEMENT AND PARTNERSHIP

CMI's Enhancement and Partnership Program aims to add value to activities that have already been planned, particularly by increasing international participation. In accordance with CMI's mission and its status as an operating foundation, its funding is utilized to enhance mathematical activities organized by, or planned in partnership with, other organizations. In 2017, CMI partnered in 15 initiatives in eight countries, often by funding a distinguished international speaker or supporting participants from outside the host country.

2017: Fifteen initiatives in eight countries:

March 11-15 | Arizona Winter School | University of Arizona, Tucson, AZ

March 13 | STEM for Britain | London, UK

March 20-24 | Young Geometric Group Theory Meeting VI | University of Oxford, UK

March-July | Operator Algebras: Dynamics and Interactions | CRM, Barcelona, Spain

May 15-19 | Recent Developments in Harmonic Analysis | MSRI, Berkeley, CA

May 29-June 2 | Random Walks with Memory | CIRM, Luminy, France

July 10-14 | International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC) | Queen Mary University London, UK

July 17-21 | Harmonic Analysis and its Interactions | ICMS, Edinburgh, UK

August 13-18 | Women in Numbers 4 | BIRS, Banff, Canada

September 11-22 | Instruments of Algebraic Geometry | University of Bucharest, Romania

November 13-17 | Geometric Functional Analysis and Applications | MSRI, Berkeley, CA

November 29-December 1 | Women in Topology | MSRI, Berkeley, CA

December 14-19 | Transformation Groups 2017 | Independent University of Moscow, Russia

December 18-21 | V International Symposium on Nonlinear Equations and Free Boundary Problems | University of Buenos Aires, Argentina

December 18-22 | Classical and Quantum Motion in Disordered Environment | Queen Mary University of London, UK

PUBLICATIONS

Selected Articles by Research Fellows

Semyon Dyatlov

Fractal uncertainty for transfer operators, with Maciej Zworski. *International Mathematics Research Notices*, to appear. arXiv: 1710.05430

Semiclassical measures on hyperbolic surfaces have full support, with Long Jin. arXiv: 1705.05019

Simion Filip

Smooth and rough positive currents, with Valentino Tosatti, submitted. arXiv: 1709.05385

Notes on the multiplicative ergodic theorem, *Ergodic Theory and Dynamical Systems*, to appear. arXiv: 1710.10694

Peter Hintz

Reconstruction of Lorentzian manifolds from boundary light observation sets, with Gunther Uhlmann. arXiv: 1705.01215

A global analysis proof of the stability of Minkowski space and the polyhomogeneity of the metric, with András Vasy. arXiv: 1711.00195

June Huh

Combinatorial applications of the Hodge-Riemann relations, *Proceedings of the International Congress of Mathematicians*, 2018, to appear. arXiv: 1711.11176

Enumeration of points, lines, planes, etc., with Botong Wang. *Acta Mathematica*, 218 (2017), 297-317

Akhil Mathew

K-theory and topological cyclic homology of henselian pairs, with Dustin Clausen and Matthew Morrow. arXiv: 1803.10897

Kaledin's degeneration theorem and topological Hochschild homology. arXiv: 1710.09045

James Maynard

Sign changes of Kloosterman sums and exceptional characters, with Sary Drappeau. arXiv: 1802.10278

Long gaps in sieved sets, with Kevin Ford, Sergei Konyagin, Carl Pomerance and Terence Tao. arXiv: 1802.07604

John Pardon

Covariantly functorial wrapped Floer theory on Liouville sectors, with Sheel Ganatra and Vivek Shende, submitted. arXiv: 1706.03152

Contact homology and virtual fundamental cycles, submitted. arXiv: 1508.03873

Aaron Pixton

Gromov-Witten theory of elliptic fibrations: Jacobi forms and holomorphic anomaly equations, with Georg Oberdieck, submitted. arXiv: 1709.01481

Multiplicativity of the double ramification cycle, with David Holmes and Johannes Schmitt, submitted. arXiv: 1711.10341

Jack Thorne

On subquotients of the étale cohomology of Shimura varieties, with Christian Johansson. https://www.dpmms.cam.ac.uk/~jat58/Galois.pdf

On the arithmetic of simple singularities of type E, with Beth Romano. https://www.dpmms.cam.ac.uk/~jat58/bounded_selmer_E7_e8.pdf

Alex Wright

Billiards, quadrilaterals and moduli spaces, with Alex Eskin, Curtis McMullen and Ronen Mukamel. http://www.math.harvard.edu/~ctm/papers/home/text/papers/dm/dm.pdf

Full rank affine invariant submanifolds, with Maryam Mirzakhani. *Duke Mathematical Journal*, 167, 1 (2018), 1-40

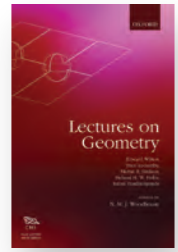
Tony Yue Yu

The non-archimedean SYZ fibration, with Johannes Nicaise and Chenyang Xu. arXiv:1802.00287

Derived Hom spaces in rigid analytic geometry, with Mauro Porta. arXiv:1801.07730

Books

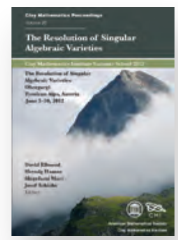
Lectures on Geometry



Editor: N. M. J. Woodhouse. Authors: Edward Witten, Martin Bridson, Helmut Hofer, Marc Lackenby, Rahul Pandharipande. CMI/OUP, 2017, 208 pp., hardcover, ISBN 97800-19-878491-3, List price: £40.00. Available from Oxford University Press.

This volume contains a collection of papers based on lectures delivered by distinguished mathematicians at Clay Mathematics Institute events over the past few years. It is intended to be the first in an occasional series of volumes of CMI lectures. Although not explicitly linked, the topics in this inaugural volume have a common flavor and a common appeal to all who are interested in recent developments in geometry. They are intended to be accessible to all who work in this general area, regardless of their own particular research interests.

The Resolution of Singular Algebraic Varieties



Editors: David Ellwood, Herwig Hauser, Shigefumi Mori and Josef Schicho. CMI/AMS, 2014, 340 pp., softcover, ISBN: 0-8218-8982-4. List price: $101. AMS Members: $80.80. Order Code: CMIP/20.

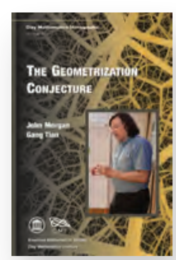
Resolution of singularities has long been considered a difficult to access area of mathematics. The more systematic and simpler proofs that have appeared in the last few years in zero characteristic now give us a much better understanding of singularities. They reveal the aesthetics of both the logical structure of the proof and the various methods used in it. This volume is intended for readers who are not yet experts but always wondered about the intricacies of resolution. As such, it provides a gentle and quite comprehensive introduction to this amazing field. The book may tempt the reader to enter more deeply into a topic where many mysteries—especially the positive characteristic case—await discovery.

The Poincaré Conjecture

Editor: James Carlson. CMI/AMS, 2014, 178 pp., softcover, ISBN: 0-8218-9865-5. List price: $69. AMS Members: $55.20. Order Code: CMIP/19.

The conference to celebrate the resolution of the Poincaré conjecture, one of CMI's seven Millennium Prize Problems, was held at the Institut Henri Poincaré in Paris. Several leading mathematicians gave lectures providing an overview of the conjecture—its history, its influence on the development of mathematics, and its proof. This volume contains papers based on the lectures at that conference. Taken together, they form an extraordinary record of the work that went into the solution of one of the great problems of mathematics.

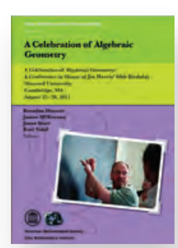
The Geometrization Conjecture



Authors: John Morgan and Gang Tian. CMI/AMS, 2014, 291 pp., hardcover, ISBN: 0-8218-5201-9. List price: $81. AMS Members: $64.80. Order Code: CMIM/5.

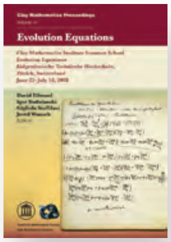
This book gives a complete proof of the geometrizaton conjecture, which describes all compact 3-manifolds in terms of geometric pieces, i.e., 3-manifolds with locally homogeneous metrics of finite volume. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. In the course of proving the geometrization conjecture, the authors provide an overview of the main results about Ricci flows with surgery on 3-dimension manifolds, introducing the reader to difficult material. The book also includes an elementary introduction to Gromov-Hausdorff limits and to the basics of the theory of Alexandrov spaces. In addition, a complete picture of the local structure of Alexandrov surfaces is developed.

A Celebration of Algebraic Geometry



Editors: Brendan Hassett, James McKernan, Jason Starr and Ravi Vakil. CMI/AMS, 2013, 599 pp., softcover, ISBN: 0-8218-8983-4. List Price: $149. AMS Members: $119.20. Order Code: CMIP/18.

This volume resulted from the conference held in honor of Joe Harris' 60th birthday. Harris is famous around the world for his lively textbooks and enthusiastic teaching, as well as for his seminal research contributions. The articles are written in this spirit: clear, original, engaging, enlivened by examples, and accessible to young mathematicians. The articles focus on the moduli space of curves and more general varieties, commutative algebra, invariant theory, enumerative geometry both classical and modern, rationally connected and Fano varieties, Hodge theory and abelian varieties, and Calabi-Yau and hyperkähler manifolds. Taken together, they present a comprehensive view of the long frontier of current knowledge in algebraic geometry.



Evolution Equations

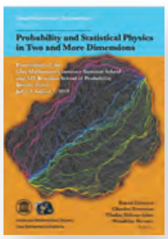
Editors: David Ellwood, Igor Rodnianski, Gigliola Staffilani and Jared Wunsch. CMI/AMS, 2013, 572 pp., softcover, ISBN: 0-8218-6861-6. List Price: $149. AMS Members: $119.20. Order Code: CMIP/17.

This volume is a collection of notes from lectures given at the 2008 Clay Mathematics Institute Summer School, held in Zurich, Switzerland. The lectures were designed for graduate students and mathematicians within five years of their PhD and the main focus of the program was on recent progress in the theory of evolution equations. Such equations lie at the heart of many areas of mathematical physics and arise not only in situations with a manifest time evolution (such as nonlinear wave and Schrödinger equations) but also in the high energy or semi-classical limits of elliptic problems.

Topics in Noncommutative Geometry

Editor: Guillermo Cortiñas. CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16.

This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.



Probability and Statistical Physics in Two and More Dimensions

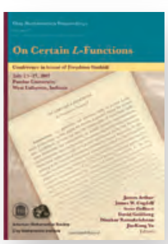
Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner. CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15.

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, "Probability and Statistical Physics in Two and More Dimensions" and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.

Grassmannians, Moduli Spaces and Vector Bundles

Editors: David A. Ellwood, Emma Previato. CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $58. AMS Members: $46.40. Order Code: CMIP/14.

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.



On Certain L-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu. CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $136. AMS Members: $108.80. Order Code: CMIP/13.

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their L-functions, and both local and global theory are addressed.

Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg. CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $94. AMS Members: $75.20. Order Code: CMIP/12.

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics

Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths

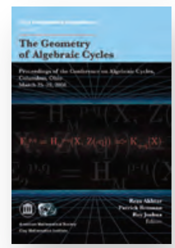
Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa. CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $136. AMS Members: $108.80. Order Code: CMIP/11.

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Kleinbock, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz . CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $104. AMS Members: $83.20. Order Code: CMIP/10.

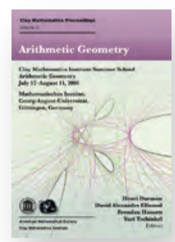
This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.



The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua. CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $55. AMS Members: $44. Order Code: CMIP/9.

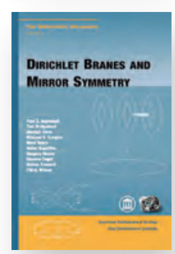
The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.



Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel. CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $125. AMS Members: $100. Order Code: CMIP/8.

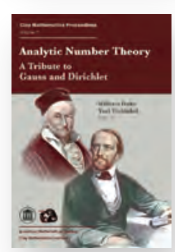
This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.



Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross. CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $115. AMS Members: $92. Order Code: CMIM/4.

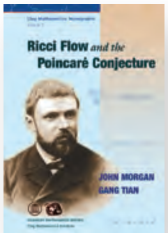
The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.



Analytic Number Theory: A Tribute to Gauss and Dirichlet

Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $53. AMS Members: $42.40. Order Code: CMIP/7.

This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture

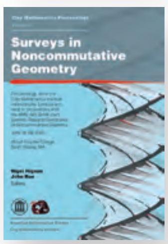
Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $75. AMS Members: $60. Order Code: CMIM/3.

This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $32. AMS Members: $25.60. Order Code: MPRIZE.

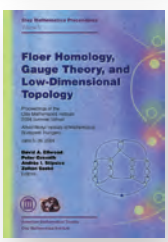
This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.



Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $53. AMS Members: $42.40. Order Code: CMIP/6.

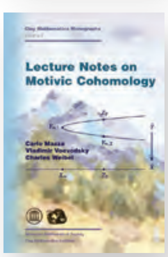
In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.



Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $70. AMS Members: $56. Order Code: CMIP/5.

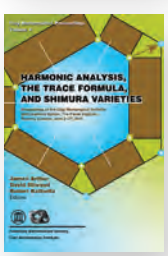
This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $50. AMS Members: $40. Order Code: CMIM/2.S.

This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

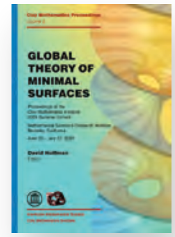


Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $138. AMS Members: $110.40. Order Code: CMIP/4.

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

Global Theory of Minimal Surfaces



Editor: David Hoffman. CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $138. AMS Members: $110.40. Order Code: CMIP/2

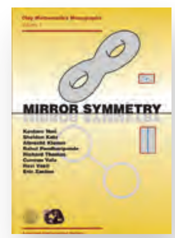
This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and thedouble-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

Strings and Geometry

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $80. AMS Members: $64. Order Code: CMIP/3.

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.

Mirror Symmetry



Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $144. AMS Members: $115.20. Order Code: CMIM/1

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $91. ASM Members: $72.80. Order Code: CMIP/1.

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

Unless otherwise indicated, to order print copies of these books please visit ***www.ams.org/bookstore****. PDF versions are posted on CMI's Online Library six months after publication and can be found at www.claymath.org/node/262.*

Digital Library

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Ada Lovelace Papers

Often called the first computer programmer, Ada Lovelace is celebrated for her pioneering work on programming Charles Babbage's Analytical Engine. These papers offer a rounded picture of the development of Lovelace's mathematical and scientific interests and include both sides of the extensive correspondence between Ada and the mathematician Augustus DeMorgan. CMI is very grateful to Ada's descendant, the Earl of Lytton, for his family's permission to undertake this project. High resolution images are available through the Bodleian Library, University of Oxford.
www.claymath.org/ada-lovelaces-mathematical-papers

Quillen Notebooks

Daniel Quillen obtained his PhD under the supervision of Raoul Bott at Harvard in 1961. He worked at MIT before moving to the University of Oxford in 1984. During his long mathematical career, Quillen kept a set of detailed notes which give a day-to-day record of his mathematical research.
www.claymath.org/publications/quillen-notebooks

Euclid's Elements

The manuscript MS D'Orville 301 contains the thirteen books of Euclid's Elements, copied by Stephen the Clerk for Arethas of Patras in Constantinople in 888 AD. It is kept in the Bodleian Library at the University of Oxford where high resolution copies of the manuscript are available for study.
www.claymath.org/euclids-elements

Riemann's 1859 Manuscript

Bernhard Riemann's paper, *Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse* (On the number of primes less than a given quantity), was first published in the *Monatsberichte der Berliner Akademie*, in November 1859. Just six manuscript pages in length, it introduced radically new ideas to the study of prime numbers.
www.claymath.org/publications/riemanns-1859-manuscript

Klein Protokolle

The "Klein Protokolle," comprising 8600 pages in 29 volumes, is a detailed handwritten registry of seminar lectures given by Felix Klein, his colleagues and students, and distinguished visitors in Göttingen for the years 1872-1912.
www.claymath.org/publications/klein-protokolle

James Arthur Archive

James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author's complete published work in an easily accessible set of searchable PDFs.
www.claymath.org/publications/collected-works-james-g-arthur

Notes of Talks at the I. M. Gelfand Seminar

The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin's notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education.
www.claymath.org/publications/notes-talks-imgelfand-seminar

NOMINATIONS, PROPOSALS AND APPLICATIONS

Research Fellowship Nominations

Nominations for Clay Research Fellows are considered once a year. The primary selection criteria for the Fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader. Selection decisions are made by the Scientific Advisory Board based on the nominating materials: letter of nomination, names and contact information for two other references, Curriculum Vitae, and publication list for the nominee.

Address all nominations to CMI President at president@claymath.org, copied to admin@claymath.org.

Workshops at the Mathematical Institute

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty people, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

Enhancement and Partnership

The Clay Mathematics Institute invites proposals under its Enhancement and Partnership Program. The aim is to enhance activities that are already planned and financially viable, particularly by funding international participation. The program is broadly defined, but subject to the general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board. For more information, visit www.claymath.org/programs/enhancement-and-partnership-program. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

Annual Deadlines

Research Fellowship nominations:
November 16

Workshop proposals:
March 1
June 1
September 1
December 1

Enhancement and Partnership proposals, including Senior Scholars nominations:
March 1
June 1
September 1
December 1

Graduate Summer School proposals:
March 1

2018 Institute Calendar

Date	Event	Location
Spring 2018	Senior Scholar Gunter Malle, *Group Representation Theory and Applications*	MSRI, Berkeley
Spring 2018	Senior Scholar Hiraku Nakajima, *Enumerative Geometry Beyond Numbers*	MSRI, Berkeley
Feb-July	Geometry, Topology and Group Theory in Low Dimensions	CIRM, Luminy
March 3-7	Arizona Winter School	University of Arizona
March 12	STEM for Britain 2018	London, UK
April 9-15	European Girls' Mathematical Olympiad	Florence, Italy
May 17-19	Mathematics and Science: in Honour of Sir John Ball	University of Oxford
May 21-25	New Methods in Finsler Geometry	DeGiorgi Center, Pisa
June 4-8	Perspectives on the Riemann Hypothesis	University of Bristol
June 4-15	International School on Extrinsic Curvature Flows	ICTP, Trieste
June 11-14	British Mathematical Colloquium	University of St Andrews
June 11-15	Arithmetic and Algebraic Geometry	IHÉS, Bures-sur-Yvette
June 18-22	String-Math 2018	Tohoku University
June 25-30	Geometric Aspects of Harmonic Analysis	Cortona, Italy
Summer	Al-Khwarizmi-Noether Institute	Bethlehem, Palestine
July 1-21	Senior Scholar Fang Hua Lin, *Harmonic Analysis*	PCMI, Salt Lake City
July 1-Aug 11	PROMYS	Boston University

Date	Event	Location
July 2-5	International Conference on Algebra and Related Topics (ICART 2018)	Mohammed V University, Rabat
July 9-13	LMS-CMI Research School: Homotopy Theory and Arithmetic Geometry: Motivic and Diophantine Aspects	Imperial College London
July 11-27	K-theory School and Workshops	University of La Plata
July 15-Aug 25	PROMYS Europe	University of Oxford
July 23-26	Complexity Theory Workshop	University of Oxford
Aug 13-17	LMS-CMI Research School: New Trends in Analytic Number Theory	University of Exeter
Aug 13-Dec 14	Senior Scholar Albert Fathi, *Hamiltonian Systems*	MSRI, Berkeley
Aug 27-Sept 1	Workshop in Algebraic Geometry	University of Nairobi
Sept 3-7	Groups, Representations and Geometry	University of Oxford
Sept 24-26	CMI at 20	University of Oxford
Sept 27-28	Discrete Structures: Harmonic Analysis and Probability	University of Oxford
Sept 27-28	Analysis and Probability	University of Oxford
Nov 5-9	Constructions and Obstructions in Birational Geometry	ICMS, Edinburgh
Nov 16-19	Reflections on Set Theoretic Reflection	Sant Bernat, Catalonia
Fall 2018	Thematic Program on Teichmüller Theory and its Connections to Geometry, Topology and Dynamics	Fields Institute, Toronto
Dec 3-7	Combinatorics Workshop	University of Oxford

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