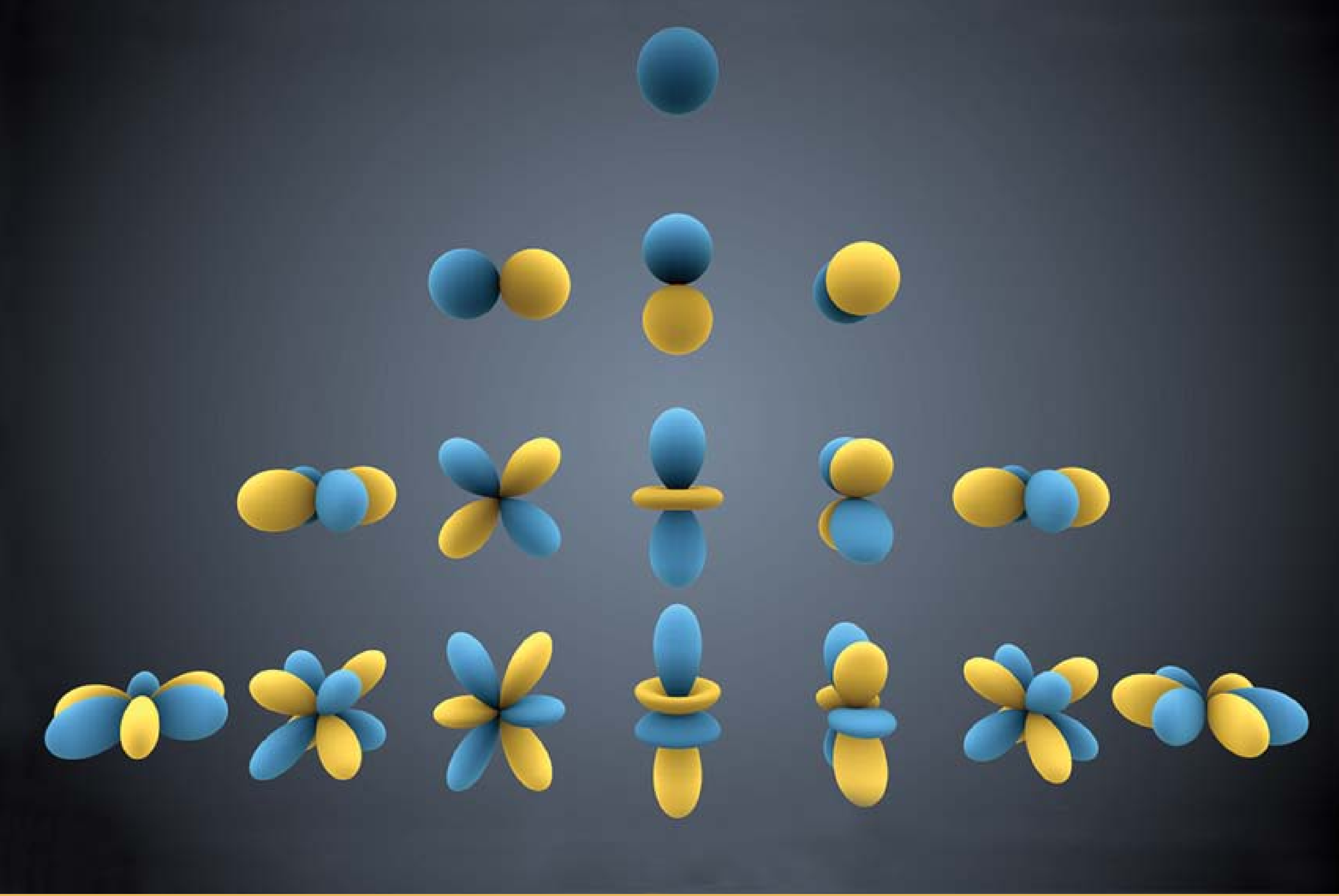

ANNUAL REPORT
2016

CMI

CLAY MATHEMATICS INSTITUTE

CLAY MATHEMATICS INSTITUTE

MISSION

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.

The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

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LETTER FROM THE PRESIDENT

In September, Lavinia Clay stepped down from the Board of Directors of the Clay Mathematics Institute and Richard Clay took her place. He is warmly welcomed: his willingness to take on this task reflects the continuing commitment of the whole Clay family to CMI.

Lavinia Clay has been an energetic member of the Board since the foundation of the Institute in 1998 and has been enormously influential in keeping Landon Clay's vision at the forefront of CMI's activities. Warm and generous in her praise, but helpfully clear, direct and courteous in her guidance, her influence has been profound.

In her interview in this report, she gives a vivid insight into the origin of CMI in Landon Clay's vision for mathematics, and into the family's engagement with the project.

Mathematicians are aware of the depth of the impact of their discipline, but they are not always good at articulating it. So it is refreshing to read of how Landon Clay reacted to an influential popular work from the 1960s that charted the development of Western Civilization over two thousand years, without touching on the contribution of mathematicians—not by making a resigned apology for the inaccessibility of the subject, but by realising an ambitious plan to celebrate the achievements of the mathematicians whose contributions were "amongst the greatest benefit conferred on mankind" and to promote dissemination of their ideas.

Lavinia Clay speaks of the motivation for posing the Millennium Prize Problems—not just the obvious one of stimulating research but also the wider aim of spreading awareness in popular culture of mathematics as a living and growing discipline. The success of the latter venture can be measured by the appearance of the problems in movies (most recently, *Gifted*) or by the huge volume of optimistic emails to CMI proposing solutions, but more quantitatively in the reaction online. CMI's webpage on the problems receives over 40,000 visits each month; a Google search for "Millennium Prize Problems" generates over a quarter of a million hits. It does not take much more than a little casual surfing to see the impact of the prizes beyond the mathematical research community.

The internet plays an important and expanding role in CMI's fulfillment of its mission. By posting online recordings of lectures given under CMI's auspices, we can reach a wide audience throughout the world, much more quickly than through the publication of traditional conference proceedings. The internet can also be used to good effect in widening access to iconic historical documents that allow a glimpse of the evolution of mathematical ideas alongside wider cultural and social developments, from a millennium-old text of Euclid's *Elements,* through Ada Lovelace's mathematical papers, notes from Israel Gelfand's seminar in the Soviet Union, to the 30,000 pages of notebooks left by the American Fields Medallist Dan Quillen. There will be many opportunities to do more in years to come.

N. M. J. Woodhouse

The 2016 Clay Research Conference

Nick Woodhouse

The 2016 Clay Research Conference was held in Oxford on September 28. The main part of the day was taken up with talks by David Ben-Zvi (Austin), Manjul Bhargava (Princeton), János Kollár (Princeton), and William Minicozzi (MIT). The talks are summarised below and are also available online in CMI's video library.[1]

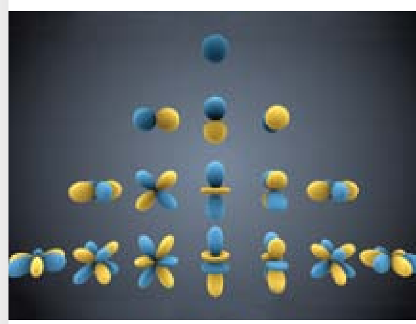
The day ended with presentations of the 2016 Clay Research Awards by Thomas Clay and laudations for the awardees. Alessio Corti spoke about the work of Mark Gross and Bernd Siebert, and Raphael Rouquier spoke about the work of Geordie Williamson.

David Ben-Zvi

David Ben-Zvi gave a very elegant and wide-ranging lecture on representation theory as seen through the lens of gauge theory, focusing on four themes:

- Harmonic analysis as the exploitation of symmetry;
- Commutative algebra as a signal of geometry;
- Topology as a source of commutativity;
- Gauge theory as a bridge between topology and representation theory.

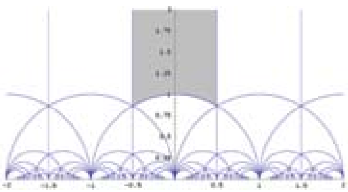
These he explored in three settings: representation theory, quantum field theory, and, bridging between them, gauge theory. The first centered on harmonic analysis and various generalizations of the Fourier transform. A basic example is the way in which Fourier series can be understood in terms of the action of the circle group $G = U(1)$ on $L^2(S^1)$. Here one can move back and forth between a representation theory picture and a geometric one in which the central object is a family of vector spaces over the dual $\hat{G}$—the set of irreducible unitary representations of G. The correspondence is through the spectral decomposition of $L^2(S^1)$ under the action of the ring $\mathbb{C}G$ of linear combinations of group elements. In line with the second theme, a geometric picture emerges because G is abelian and $\mathbb{C}G$ is commutative.



By looking at the $U(1)$ representations in this way one has a model of how to understand more complex cases. The extension to non-abelian groups is motivated by studying the action of $SO(3)$ on $L^2(S^2)$. Here the decomposition is the expansion in spherical harmonics, that is, in eigenfunctions of the Laplacian. The Laplacian in turn is picked out by the Casimir element $C = (\boldsymbol{i}^2 + \boldsymbol{j}^2 + \boldsymbol{k}^2)$ in the center of the enveloping algebra of $SO(3)$. In this case the unitary dual is $\widehat{SO}(3) = \mathbb{Z}_+$ and C determines a function $\ell \mapsto \ell(\ell + 1)$ on the dual.

The lesson from this example is that to see geometry one should look for commutative algebra. With that in mind, one seeks to access $\hat{G}$ more generally for a nonabelian group by exploiting Schur's lemma, that the center of $\mathbb{C}G$ acts by scalar multiplication in any unitary representation. So a representation determines a function on $\hat{G}$. Conversely functions on $\hat{G}$ give rise to operators that commute with the action of G and so determine symmetries of arbitrary representations of G.

[1] www.claymath.org/library/video-catalogue



David Ben-Zvi

Ben-Zvi moved on to explore this theme within the Langland's program, focusing on automorphic forms and the spectral decomposition of $L^2(X)$ when

$$X = SL_2\mathbb{Z}\backslash SL_2\mathbb{R}/SO_2$$

is the moduli space of elliptic curves. Here in addition to the Laplacian (and higher Casimirs), there are also the actions of the algebras of Hecke operators, one for each prime p. All Hecke operators for almost all p commute.

One can understand the Langlands program in this way: first, study the spectral decomposition of $L^2(X)$; second, identify the joint spectrum with a space of Galois representations.

Turning to the second topic, Ben-Zvi gave a brief outline of quantum theory and quantum field theory, as formalized by Atiyah and Segal, before focusing on topological field theories (TFTs). An n-dimensional TFT is to be thought of as a structure that associates vector spaces $\mathcal{Z}(M)$ of states with $(n-1)$-dimensional manifolds M. More formally a functor satisfying certain axioms, from the cobordism category, which has $(n-1)$-manifolds as objects and cobordisms as morphisms, to the category of vector spaces.

In constructing TFTs, it is simpler to focus on states attached to small patches of space. This leads to a picture in which states attached to a compact n-manifold M are determined by *local boundary conditions*; that is $(n-1)$-dimensional field theories on the $(n-2)$-dimensional boundaries of small regions in M. The boundaries themselves may have interfaces: one also has field theories on the interfaces of boundaries, and on the interfaces of interfaces. Proceeding in this way, one is led into higher category theory, with the interfaces between interfaces giving 2-morphisms and so on.

An important idea here is Lurie's version of the Baez-Dolan cobordism hypothesis, that *an n-dimensional topological field theory $\mathcal{Z}$ is uniquely determined by its higher category of boundary conditions.* This gives a deep geometric perspective on category theory: categories correspond to 2-dimensional quantum field theories and higher categories to higher dimensional theories.

Within this framework, one also has an elegant illustration of the theme 'topology as a source of commutativity'. Measurements at points of space-time are represented by *local operators* in TFT; these correspond to states on small spheres—the boundaries of balls containing the points. Composition of operators is determined by cobordism, with commutativity for $n \geq 2$ following from a topological argument. When $n = 1$ (the quantum mechanics case), the operators do not necessarily commute. The situation mirrors the behavior of homotopy groups: π_1 need not be abelian, while π_n is necessarily abelian for $n \geq 2$.

The final part of the lecture presented the gauge theory of a group G as a bridge between analogous structures that emerge in representation theory and quantum field theory.

Locally, connections are trivial, but they carry pointwise G symmetry. This suggests a focus not on the global picture but on boundary conditions on small pieces of space-time. Motivated by the cobordism hypothesis one is led to consider $(n-1)$-dimensional quantum field theories and representations of G on field theories. In a topological context, these should determine the whole theory.

The approach was illustrated first in the case of two-dimensional topological

Yang-Mills theories, in which the boundary conditions are one-dimensional TFTs, that is, quantum theories. The boundary conditions in this case correspond to representations of G.

A more significant realization of the connection is in the geometric Langlands program. Here one studies the moduli space of G-bundles on a Riemann surface C for a complex reductive group G and the action of Hecke operators at points of C. As suggested by Kapustin and Witten, a Hecke operator can be interpreted in terms of the creation of a monopole in the three-dimensional gauge theory on $C \times \mathbb{R}$, and hence, by using the same ideas as those underlying the cobordism hypothesis, as an operator on states on C. Inspired by the topological argument for commutativity of local operators in QFT, Beilinson and Drinfeld used the latter point of view to identify the source of commutativity for Hecke operators, in line with the third theme.

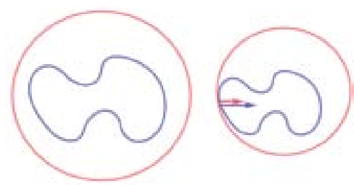
The commutativity of Hecke operators signals the presence of geometry, as in the second theme. The study of their spectrum leads to the identification of Hecke operators with representations of the Langlands dual group $\hat{G}$.

William Minicozzi

William Minicozzi's talk covered geometric and analytic aspects of the mean curvature flow problem. This is the problem of understanding the behavior of a family of hypersurfaces M_t evolving under the differential equation

$$\frac{\partial \boldsymbol{x}}{\partial t} = -H\boldsymbol{n}$$

where H is the mean curvature of M_t and $\boldsymbol{n}$ is the unit normal. Two examples of solutions in $\mathbb{R}^3$ are 2-spheres of radius $\sqrt{-4t}$ and cylinders of radius $\sqrt{-2t}$ $(t < 0)$. In these cases the evolution equations are ordinary differential equations. In both the surfaces are 'extinquished' in a finite time, in the first with the spheres contracting to a point, in the second, with the cylinders contracting to a line.

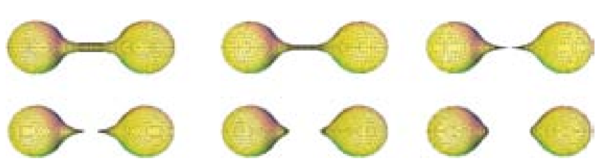


A general feature of mean curvature flow is that a hypersurface enclosing another cannot overtake the inner one, because at the point at which they first touched, the inner surface, with larger mean curvature, would have to be moving faster than the outer one. This is the 'avoidance principle': if hypersurfaces start disjoint, then they must remain disjoint. Any closed hypersurface must shrink to a point: it must remain inside any enclosing sphere, which must itself shrink to a point. Singularities must develop. The problem is to understand them.

William Minicozzi

In 1984, Huisken dealt with the convex case by proving that a closed convex surface remains convex and flows smoothly until it disappears to a point. Just before the extinction time, it looks completely round.

The non-convex case is less straightforward. A thin symmetrical torus, for example, collapses to a circle. A more involved example is 'Grayson's dumbell'—two large 'bells' connected by a thin bar. Under the flow, the bells shrink more slowly than the bar. The result is that the neck first pinches off; the bells then shrink to points. Because the flow continues past the first singularity, one needs the notion of a 'weak



solution' to understand examples of this sort. One such is given by focusing not a single hypersurface but on the level sets of a function, all evolving simultaneously under the flow. Because of the avoidance principle, they remain level sets of a function.

In the *mean convex case* (i.e. $H > 0$), an appropriate function is $v(x, t) = u(x) - t$, where $u(x)$ is the *arrival time*, the time at which M_t reaches x. If M_0 is mean convex then M_t moves monotonically inwards and $u(x)$ is well-defined in the domain swept out by M_t. It satisfies the degenerate elliptic equation

$$\Delta u - \mathrm{Hess}_u \left(\frac{\nabla u}{|\nabla u|}, \frac{\nabla u}{|\nabla u|} \right) = -1 \, .$$

The critical points of u are the singularities of the flow. By using viscosity solutions, it had been shown that there exists a solution, and that it is Lipschitz. In 2016, Colding and Minicozzi (CM) showed that u is twice differentiable everwhere and that at critical points the Hessian matches that of the flow of the cylinder or of the sphere. That left open the question of whether or not u was actually C^2. This CM resolved in 2016: the solution is C^2 if and only if the critical set is either a single point where the Hessian is spherical or a simple closed C^1 curve where the Hessian is cylindrical.

The key to understanding the detailed structure of the singularities is that they are self-similar under rescaling. This is clear in the cases of spheres and cylinders, but is true more generally. As we zoom in on a critical point, the level sets are spherical or cylindrical. For the second derivative to exist, the orientation of the cylinders must not jump around from point to point, in particular their axes must be aligned. This 'uniqueness' was established by CM in 2015, the essential ingredient being the application of a Lojasiewicz inequality inspired by algebraic geometry.


János Kollár

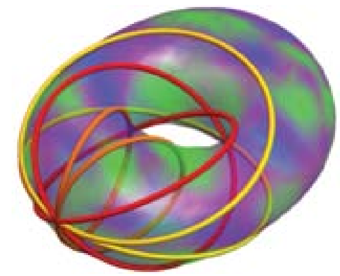
In the examples (the sphere, the cylinder, the torus, and the dumbbell), the singular set is made up of points and curves. It is never more than one dimensional. By using the same ideas White showed in 2000 that generally in the mean convex case it never exceeds one, in the sense of Hausdorff measure. Again in 2016, CM proved 'rectifiability' by showing that the singular set is in fact contained in a union of compact C^1 curves and a countable set of points.

The lecture concluded with a brief review of higher-dimensional results and of the tools used to prove them. There is a richer set of possibilities for the singularities in higher dimensions. For example, in the mean convex case it is possible to have the product of a sphere of any dimension with a plane of complementary dimension. But otherwise much of the three-dimensional theory extends in a natural way. So, for example, the singular set is rectifiable with finite measure and contained in the finite union of compact C^1 $(n-1)$-manifolds together with a set of dimension not greater than $n-2$. The lower strata are themselves countable unions of C^1 manifolds of lower dimension.

János Kollár

János Kollár spoke about classical and modern work on the classification of *celestial surfaces*. To a first approximation a celestial surface is a *cyclide*, a class of surfaces much studied in the 19th century. More precisely, Kollár defined a celestial surface to be a surface that contains at least two circles through a general point. Despite their definition having a metric character, celestial surfaces fall within the realm of algebraic geometry because it can be shown that a surface that contains at least two algebraic curves through a general point is locally algebraic.

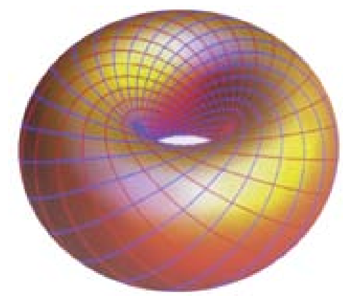
Basic examples are the plane, the sphere, and quadric surfaces—except that the two families of circles on a general quadric degenerate into a single family in the presence of circular symmetry. Kollár explained how the geometry in the quadric case can be understood by introducing the *ideal quadric* at infinity in $\mathbb{R}^n \subset \mathbb{P}^n$, defined by $x_0 = \sum_1^n x_i^2 = 0$, and by characterizing circles as conics that meet the ideal quadric in two conjugate point pairs.



A further example is a torus formed from a circle as a surface of revolution. The method of construction immediately gives two families of circles through every point. But there are also two other families that wind around the torus in opposing senses. They are attributed to Villarceau, who wrote about them in the mid 19th century, although they were known to the masons who built Strasbourg cathedral around 1300 AD.

In this example of a torus there are four families through each point. For a more general torus, there are in fact six: the reduction to four is a consequence of the special symmetry of a surface of revolution.

The underlying algebraic geometry is illustrated by starting with a quadric $Q \subset \mathbb{P}^3$ and blowing up four points. The resulting surface has 10 families of conics, but only six of these can be real.



The circle families on the torus can be understood in a similar way by noting that, in algebraic-geometric terms, a torus is a degree-four Del Pezzo surface with four double points.

Other examples of surfaces with two families of circles have been exploited in architecture. A general classification program was begun by Dupin, Kummer, and Darboux. Examples that were not known to them have been found in more recent years in all dimensions.

In 2001, Josef Schicho listed all the pairs (S, L) (S a normal projective surface, L an ample Cartier divisor) which give rise to surfaces embedded in higher dimensional spaces with at least two conics through each point. The way in which this works is illustrated in two particular cases:

- In the Veronese case $L = 2H$ and $\mathbb{P}^2$ is embedded in $\mathbb{P}^5$ by presenting $\mathbb{P}^5$ as the projective form of the six-dimensional space of homogeneous degree-two polynomials in three variables.
- In the double Segre case $L = (2, 2)$ and the embedding is $\mathbb{P}^1 \times \mathbb{P}^1 \hookrightarrow \mathbb{P}^8$ given by thinking of $\mathbb{P}^8$ as the projective space of homogeneous polynomials of degree two in two sets of variables.

Under these degree-two embeddings the images of lines are conics. In the second case, there are two conics through each point. In the first, there are infinitely many conics through each point. The key question is: can one choose the embedding more carefully so that these conics actually become circles?

Part of the answer is provided by applying a (genuinely) classical theorem that stereographic projection $S^n \to \mathbb{R}^n$ preserves circles. In consequence, there is an equivalence between (i) a circle contained in a surface in $\mathbb{R}^n$ and (ii) a conic contained in a surface in S^n, because a conic in S^n, being a plane section of the sphere, must be a circle. So the problem translates to the algebraic-geometric one of classifying real algebraic surfaces

$$F \subset S^n := (x_0^2 + \cdots + x_n^2 = 1) \subset \mathbb{R}^{n+1}$$

that admit at least two families of conics.

More generally, one has the problem of classifying the algebraic surfaces over a field k,

$$F \subset Q^n := \big(q(x_0, \ldots, x_{n+1}) = 0\big) \subset \mathbb{P}_k^{n+1},$$

that admit at least two families of conics, where q is a quadratic. In the two particular cases above, it is the problem of describing the spaces

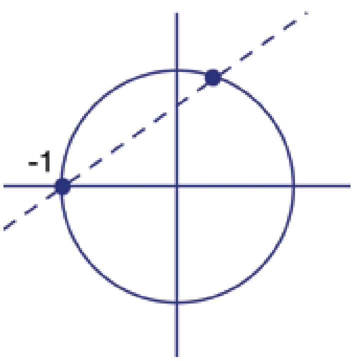
- $\mathrm{Map}_2(\mathbb{P}^2, Q^n)$—surfaces with a 2-dimensional family of circles and
- $\mathrm{Map}_2(\mathbb{P}^1 \times \mathbb{P}^1, Q^n)$—surfaces with two 1-dimensional families of circles,

where Map_2 denotes maps of degree two. It is generally hard to describe such spaces, but in these cases it reduces to a tractable algebraic problem. So, for example, if Q^4 is a smooth quadric 4-fold in $\mathbb{P}^5$ over $k = \bar{k}$, then $\mathrm{Map}_2(\mathbb{P}^2, Q^4)$ has five irreducible components, each birational to $\mathbb{P}^{20}$ and each understandable in terms of Veronese surfaces or quadruple planes.

The lecture concluded by considering the Veronese case over an arbitrary field. Here there is a one-to-one correspondence over any field between four-dimensional quadrics containing the Veronese surface and rational normal curves in $\mathbb{P}^4$. Again this can be formulated in entirely algebraic terms.

Images by courtesy of Neils Lubbes (first image) and Daniel Dreibelbis (second image).

Manjul Bhargava



Manjul Bhargava spoke about elliptic curves and the Birch and Swinnerton-Dyer Conjecture (BSD), one of the most fundamental problems in number theory. He explained its origin and its statement in elementary terms, as well as summarising progress towards a proof, including results that were emerging in the workshop taking place during week of the Research Conference.

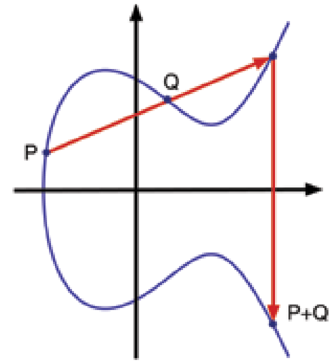
A central theme of number theory is the search for rational solutions of polynomial equations. For a polynomial in one variable with integer coefficients, the problem is easily solved by using the Rational Root Theorem. Two variable equations are more interesting.

Equations of degree one in two variables are easy. The degree two case is not much more difficult. A simple geometric argument, for example, puts the rational points on the circle $x^2 + y^2 = 1$ (the points with rational coordinates) into correspondence with $\mathbb{Q} \cup \{\infty\}$, by noting that the rational points are the intersections with the circle of lines through $(-1, 0)$ with slope $s \in \mathbb{Q}$. The point $(-1, 0)$ itself is labelled by $s = \infty$.

The Hasse-Minkowski theorem gives an algorithm for determining whether or not a general conic contains any rational points. Provided that there is at least one rational point, the same procedure as for the circle works for a general conic, with the rational point replacing $(-1, 0)$

For two-variable equations of degree three, the problem is much more difficult. A given equation may have no rational solutions, a positive finite number of solutions, or infinitely many solutions, but it is already an unsolved problem to determine which. This is where the BSD conjecture comes in: if true, it would provide a method to determine all the rational solutions, and in particular to determine whether there are finitely or infinitely many such solutions. So BSD is the key to understanding degree three equations.

What then is the Birch and Swinnerton-Dyer conjecture? Uniquely in the cubic case, the set of rational points on a cubic curve has the structure of an abelian group (the *divisor class group*). Assume that we are given a smooth cubic equation $f(x, y) = 0$ (i.e. the curve is smooth) with at least one rational solution $P = (x_0, y_0)$. By a rational change of variables, P can be sent to infinity and the equation reduced to



$$y^2 = x^3 + Ax + B \quad (*)$$

where, for smoothness, the discriminant $\Delta = -4A^3 - 27B^2$ is nonzero. We then have an *elliptic curve* $E = E_{A,B}$ *in Weierstrass form.*

Given two rational points P, Q on E, the line through them intersects E in a third point. The group law is defined by taking the sum $P + Q$ to be the third point, reflected in the x-axis. The identity is the point at infinity.

A lot is known about the group $E(\mathbb{Q})$ of rational points on E. To begin with, we have Mordell's theorem that $E(\mathbb{Q})$ is finitely generated. Since it is also abelian, it must be of the form

$$E(\mathbb{Q}) \cong \mathbb{Z}^r \oplus T$$

for some $r \geq 0$ and some finite abelian group T. A theorem of Mazur's establishes that $|T|$ cannot exceed 16.

The invariant r is called the *rank* of E. It measures the size of $E(\mathbb{Q})$. In particular if $r = 0$ then E has only finitely many rational points while if $r > 0$, there are infinitely many.

A lot of questions about ranks of elliptic curves are unsolved: it is not known if there is an upper bound (in the most extreme known example, found by Elkies, the rank is at least 28); very little is known about the distribution of r as A, B vary; most mathematicians believe that 100% of elliptic curves have rank 0 or 1 in an appropriate probabilistic sense—this has not been proved, although it has been shown recently that at least 1% do; it is not known whether or not there is an algorithm to determine the rank that provably terminates with the correct answer. The BSD conjecture addresses this last question.

The formulation of the BSD conjecture begins with consideration of the number of solutions to $(*)$ mod p, a prime. An arbitrary choice for x mod p will give a solution mod p if the $x^3 + Ax + B$ is a square mod p. This will happen in general for about half the choices made for x, and each time it does happen there are two possible values of y. So there should be about p solutions mod p.

If E contains many rational points then by reducing their coordinates mod p, one obtains lots of solutions to $(*)$ (mod p). Birch and Swinnerton-Dyer hypothesized that if the rank of E is large then on average E should have more than p points (mod p). After many computations to check this, they arrived at

> **Conjecture (Birch and Swinnerton-Dyer)**. Let E be an elliptic curve over $\mathbb{Q}$, let r be its rank, and let N_p denote the number of points on E (mod p). Then
>
> $$\prod_{p \leq X} \frac{N_p}{p} \sim c \cdot (\log X)^r \qquad (**)$$
>
> for some constant c.

The strong form of the conjecture includes an expression for c in terms of arithmetic invariants of E.


Manjul Bhargava

A modern form of the conjecture involves the behavior of the *incomplete L-function* of E, which is defined by

$$L(E, s) = \prod_{p \nmid 2\Delta} \frac{1}{1 - a_p p^{-s} + p^{1-2s}}$$

where $a_p = p + 1 - N_p$. The partial products at $s = 1$ are precisely the expressions on the left-hand side of $(**)$. A heuristic argument suggests that if the rank of E is large then p/N_p should be small and the product should vanish. Although the product itself only converges for $\mathrm{Re}(s) > 3/2$ one can still make sense of this: it was conjectured by Hasse and proved by Wiles and others that L extends to an entire function of s, as a consequence of the modularity theorem. In terms of L, the BSD conjecture is that $r = r'$, where r' is the order of the zero of L at $s = 1$ (in fact this is weaker than Birch and Swinnerton-Dyer's original formulation).

The interger r' is called the *analytic rank* of E. The conjecture is remarkable in equating r', which is defined analytically, to r (sometimes called the 'algebraic rank'), which is defined algebraically.

What is known about BSD?

- In 1977, Coates and Wiles showed that BSD is true when E is of the form $y^2 = x^3 + Ax$ or $y^2 = x^3 + B$, and $r' = 0$.
- In 1989, Gross, Zagier, and Kolyvagin showed that BSD is true when $r' = 0$ or $r' = 1$.
- In 2013, Skinner, Urban, and Zhang showed that if $r = 0$ or $r = 1$ and if E satisfies some further conditions, then BSD is true; the list of further conditions has since been reduced, not least during the 2016 Clay workshop.

Do any elliptic curves satisfy these conditions? Define the *height* of $E = E_{A,B}$ to be $\max\{4|A|^3, 27B^2\}$ and list elliptic curves $E/\mathbb{Q}$ in order of increasing height. One can then ask statistical questions about the probability that BSD holds.

- In 2013, Bhargava and Shankar showed that at least 83% of elliptic curves have $r = 0$ or $r = 1$.

Because a proportion of the 83% satisfy the further conditions, it follows that *a positive proportion of elliptic curves satisfy BSD*. In fact Bhargava, Skinner, and Wei Zhang have shown that BSD is true for 66% of all elliptic curves. This percentage has risen during the workshop.

A lot remains to be done. The 'further conditions' are still to be eliminated to establish BSD for elliptic curves of rank zero or one. These are conjectured to be 100% of all curves in a probability sense. But there remain infinitely many elliptic curves of higher rank. For them, essentially nothing is known. There are also many beautiful generalizations of the conjecture: extensions to higher dimensions, to p-adic analogues, and to curves over number fields.

Clay Research Conference Workshops

Mean Curvature Flow

September 26-30, 2016

Mean curvature flow (MCF) is the negative gradient flow of volume, so any hypersurface flows through hypersurfaces in the direction of steepest descent for volume and eventually becomes extinct in finite time. It has been studied in material science to model things such as cell, grain, and bubble growth. It also plays an important role in image processing and has been widely studied numerically. The flow is well-defined classically until the first singularity. The level set method was invented to study the flow, even past singularities, and it is now widely used in a variety of settings.

Before the flow becomes extinct, changes occur as it goes through singularities. To understand the flow, it is crucial to understand these. This can be done completely for embedded curves, but there are infinitely many possible types of singularities in all higher dimensions. Recent years have seen major progress on some of the most natural questions: Is it possible to bring the hypersurface into general position so that there is a smaller set of "generic" singularities? What does the flow look like near these singularities? What is the structure of the singular set itself?

This workshop explored recent developments in MCF, focusing on which singularities can occur, which of these are generic, what the flow looks like near these singularities, and the structure of the singular set.

The workshop brought together a very diverse group of people covering many aspects of the field and related areas. Both pure and applied mathematics were represented with many people having made, and continuing to make, major contributions to the field. The range was extraordinarily broad, ranging from numerics to pure mathematicians working in analysis or geometry. There were also talks on recent work in other fields whose ideas and techniques originated in mean curvature flow. This was a unique opportunity to connect with people working on similar programs but from a very different perspective.

Organizers

Toby Colding, Massachusetts Institute of Technology
Bill Minicozzi, Massachusetts Institute of Technology

Speakers

Sigurd Angenent, University of Wisconsin, Madison
Jacob Bernstein, Johns Hopkins University
Yoshikazu Giga, University of Tokyo
Gerhard Huisken, University of Tübingen
Bruce Kleiner, New York University
Bob Kohn, New York University
Felix Otto, MPIM Leipzig
Felix Schulze, University College London
James Sethian, University of California, Berkeley
Peter Topping, University of Warwick
Lu Wang, University of Wisconsin, Madison
Mu-Tao Wang, Columbia University
Brian White, Stanford University

Algebraic Geometry: Old and New

September 26-30, 2016

The aim of this workshop was to focus on related old topics in algebraic geometry that are transformed by a recent new point of view, as well as new topics that have applications to old problems, including:

- Fano manifolds, special metrics and moduli
- stable objects in derived categories
- applications of mirror symmetry

The workshop featured at least three dominant recurring themes: mirror symmetry (Gross, Siebert), the application of ideas from mirror symmetry to the algebraic geometry of Fano varieties (Coates), and the quest for an algebra-geometric criterion for K-stability (covered by several speakers, but notably Fujita).

These are different areas of geometry and it is rare to see two of them represented at a workshop, and rarer still three. The organizers believe that each of the three areas will benefit a great deal from meaningful future interaction with the other two, and believe that their workshop set some of the foundation for this future interaction.

Organizers

Alessio Corti, Imperial College London
János Kollár, Princeton University
Miles Reid, University of Warwick
Nick Shepherd-Barron, Kings College London

Speakers

Christian Boehning, University of Warwick
Tom Coates, Imperial College London
Kento Fujita, Kyoto University
Mark Gross, University of Cambridge
Dominic Joyce, University of Oxford
Anne-Sophie Kaloghiros, Brunel University
Conan Leung, Chinese University of Hong Kong
Yuchen Liu, Princeton University
Eduard Looijenga, Utrecht University and Tsinghua University
Emanuele Macri, Northeastern University
Yuji Odaka, Kyoto University
Bernd Siebert, Universität Hamburg
Cristiano Spotti, Aarhus University
Jacopo Stoppa, SISSA Trieste
Song Sun, University of Stony Brook
Alessandro Verra, Università Roma Tre
Xiaowei Wang, Rutgers University
Jaroslaw Wisniewski, University of Warsaw

Geometric Representation Theory and Beyond

September 26-30, 2016

There are powerful new tools and ideas at the forefront of geometric representation theory, particularly categorical incarnations of ideas from mathematical physics, algebraic, arithmetic and symplectic geometry, and topology: perhaps geometry is no longer the future of geometric representation theory.

Speakers at this workshop were invited to present their own vision for the future of the field.

Organizers

Raphael Rouquier, University of California, Los Angeles
Iain Gordon, University of Edinburgh
Kobi Kremnitzer, University of Oxford

Speakers

David Ben-Zvi, University of Texas, Austin
Alexander Braverman, University of Toronto and Perimeter Institute for Theoretical Physics
Laurent Fargues, Jussieu
Dennis Gaitsgory, Harvard University
Victor Ginzburg, University of Chicago
Michael Harris, Columbia University
Tamas Hausel, IST Austria
David Nadler, University of California, Berkeley
Andrei Okounkov, Columbia University
Vera Serganova, University of California, Berkeley
Wolfgang Soergel, Universität Freiburg
Catharina Stroppel, Universität Bonn
Constantin Teleman, University of Oxford
Geordie Williamson, Universität Bonn

Recent Developments on Elliptic Curves

September 26-30, 2016

The last few years have witnessed a number of developments in the arithmetic of elliptic curves, notably the proof that there are positive proportions of elliptic curves of rank zero and rank one for which the Birch–Swinnerton-Dyer conjecture is true. The proof of this landmark result relies on an appealing mix of diverse techniques arising from the newly resurgent field of arithmetic invariant theory, Iwasara theory, congruences between modular forms, and the theory of Heegner points and related Euler systems.

This workshop surveyed the proof of this theorem and described the new perspectives on the Birch–Swinnterton-Dyer conjecture which it opens up.

Organizers

Manjul Bhargava, Princeton University
Henri Darmon, McGill University
Chris Skinner, Princeton University

Speakers

Manjul Bhargava, Princeton University
Mirela Çiperiani, University of Texas, Austin
Henri Darmon, McGill University
Ellen Eischen, University of Oregon
Benedict Gross, Harvard University
Wei Ho, University of Michigan
Antonio Lei, Université Laval
Chao Li, Columbia University
Kartik Prasanna, University of Michigan
Victor Rotger, Universitat Politècnica de Catalunya
Arul Shankar, Harvard University
Chris Skinner, Princeton University
Ye Tian, Chinese Academy of Sciences
Eric Urban, Columbia University
Rodolfo Venerucci, Universität Duisberg-Essen
Xin Wan, Columbia University
Xiaoheng Jerry Wang, Princeton University
Andrew Wiles, University of Oxford
Shou-Wu Zhang, Princeton University

Geordie Williamson, Bernd Siebert, Mark Gross, Thomas Clay

RECOGNIZING ACHIEVEMENT

2016 Clay Research Award

The Clay Research Awards, presented annually at the Clay Research Conference, celebrate the outstanding achievements of the world's most gifted mathematicians. Although perhaps less well known outside the mathematical world than the Clay Millennium Prize Problems, the Clay Research Award is widely appreciated within it.

Mark Gross and Bernd Siebert

The joint Award to Mark Gross and Bernd Siebert was made in recognition of their groundbreaking contributions to the understanding of mirror symmetry, in joint work generally known as the 'Gross-Siebert Program.' It has its origins in surprising predictions of non-perturbative dualities in string theory: that the properties of certain interesting geometries, notably Calabi-Yau manifolds, are reflected in counter-intuitive ways in partner geometries ('mirror manifolds').


Mark Gross


Bernd Siebert


Geordie Williamson

The Gross-Siebert program builds on an earlier, differential-geometric, proposal of Strominger, Yau, and Zaslow, in which the Calabi-Yau manifold is fibred by special Lagrangian tori, and the mirror by dual tori. The program's central idea is to translate this into an algebro-geometric construction in an appropriate limit, involving combinatorial data associated with a degenerating family of Calabi-Yau manifolds. It draws on many areas of geometry, analysis and combinatorics and has made a deep impact on fields such as tropical and non-archimedean geometry, logarithmic geometry, the calculation of Gromov-Witten invariants, the theory of cluster algebras and combinatorial representation theory. Remarkable results independent of mirror symmetry are now emerging, notably in the geometric compactification of moduli spaces of K3 surfaces, in the construction of theta functions on Fano and Calabi-Yau varieties, and in proofs of Looijenga's conjecture on the smoothability of certain surface cusps and of the positivity of Laurent coefficients conjecture.

Geordie Williamson

The Award to Geordie Williamson was made in recognition of his groundbreaking work in representation theory and related fields. In particular, the award recognises two major breakthroughs. First, his proof, with Ben Elias, of Soergel's conjecture on bimodules associated to Coxeter groups. This established the combinatorial result that the coefficients of the Kazhdan-Lusztig polynomials are non-negative, as well as yielding a new proof of Kazhdan and Lusztig's conjectured character formula for representations of complex semi-simple Lie algebras. The second is the construction (building on earlier work with Ben Elias and Xuhua He) of counterexamples to the expected bounds in Lusztig's conjectured character formula for rational representations of algebraic groups in positive characteristics that grow exponentially with the rank of the group.

Clay Research Awardees

2016
Mark Gross & Bernd Siebert
Geordie Williamson

2015
Larry Guth & Nets Katz

2014
Maryam Mirzakhani
Peter Scholze

2013
Rahul Pandharipande

2012
Jeremy Kahn & Vladimir Markovic

2011
Yves Benoist & Jean-François Quint
Jonathan Pila

2009
Ian Agol, Danny Calegari & David Gabai
Jean-Loup Waldspurger

2008
Cliff Taubes
Claire Voisin

2007
Alex Eskin
Christopher Hacon & James McKernan
Michael Harris & Richard Taylor

2005
Manjul Bhargava
Nils Dencker

2004
Ben Green
Gérard Laumon & Ngô Bảo Châu

2003
Richard Hamilton
Terence Tao

2002
Manindra Agrawal
Oded Schramm

2001
Stanislav Smirnov
Edward Witten

2000
Alain Connes
Laurent Lafforgue

1999
Andrew Wiles

PROFILE

PROFILE

An Interview with Lavinia D. Clay

Lavinia Clay served as a Director of CMI from its foundation in 1998 until she stepped down in September.

The Clay Mathematics Institute has had a huge impact on the mathematical community. How did the idea for the Institute arise?

Landon has always pursued long-term objectives, in business and personal life as well as in philanthropic dedication. He chose objectives meaningful to him. Early in life Landon admired the beauty and clarity of math, teaching himself calculus to divert his mind while stationed on Tinian in the last months of World War II. While he chose to concentrate in English literature in college, his respect for mathematics deepened through the years. The omission of mathematics from the Nobel Prize categories gnawed at him. Landon believed mathematicians should be included among those who confer the "greatest benefit on mankind." With each annual announcement of the Nobel Prizes, he would again recall that mathematicians' works were insufficiently appreciated, as if it were an as-yet-unaddressed task. I would say that Landon's initial impetus for the Clay Mathematics Institute was to redress the omission of the field of mathematics from the Nobel Prizes almost one hundred years before.

Kenneth Clark published an extensive study of Western civilization from the fall of Rome to the then-present year of 1969. Shortly after publication I was reading Landon's copy of Clark's *Civilization*. As in many of his books, Landon's comments in the margins converted the text to a wide-ranging discussion. Clark dwelt on art, philosophy, history, and, to some extent, technology. Landon's comments in the margins of this book about the contributions of mathematical developments made me realize the depth and sincerity of his affection for the field. Landon was also a businessman, contributing with purpose in mind. He preferred to donate where he could encourage research and achievement. A baseball player as a student and baseball fan as an adult, he knew the importance of developing farm teams. Above all, Landon valued independence and concentration on mathematics rather than administrative structure. All these objectives coalesced into the founding of a free-standing institute to further mathematicians' research and offer a stage for their works.

You and Landon Clay have been enormously generous in supporting a number of important philanthropic projects. How do they reflect the family's interests and objectives?

I'd say our whole family got on board with Landon's interests just because that's where the action lay—in books, travel, friendships. We participated as a family, whether it was visiting an archeological dig in the South Pacific, experiencing first light at the Clay telescope in Chile, or embracing more easily scheduled pursuits at Boston's museums, academic, and health organizations.

Do you see a connection between mathematics and your own background in art?

From my senior citizen's point of view, I see a prodigious overlap that would

Photos courtesy of University of Oxford

have been tremendous fun to explore, but the two worlds were quite disparate for me at a younger age and perspective.

The Millennium Prize Problems continue to attract a great deal of attention, not just within the mathematical community but in popular culture. Did you anticipate this reaction when the problem list was announced?

While the intention of the Millennium Prize Problems was to stimulate research, we did hope they might also attract attention in popular culture, and that appears to be the case. There were two occasions—once at the John F. Kennedy Center for the Performing Arts in Washington, when we co-hosted the International Mathematical Olympiad, and once on a stage at MIT—when, upon the introduction of Sir Andrew Wiles as the man who solved Fermat's last theorem, I witnessed young mathematicians erupting with the exuberance one normally associates with rock stars. When the students went wild, yes, then I knew it had legs.

A high point for CMI was the conference in Paris in 2010 to celebrate Perelman's proof of the Poincaré conjecture. What are your recollections of that event?

Multiple: I was struck by the level of animated discussion among diverse attendees and the vitality that represented; I was still concerned about Perelman's having chosen not to accept the Prize; but more than anything else, I remember, almost in slow motion, the look of incredulity, followed by a quick smile, coming over Sir Andrew's face, when I turned to him and said I thought that CMI should move to UK/Europe.

What have been the other most memorable events in your time as a Director of CMI?

Without a doubt, it has been conversing with the individuals involved, whether mathematicians, administrators, staff, or host institutions, all driven by a singularity of purpose.

What do you see as the distinctive contribution that philanthropic foundations can make in the funding of research?

A philanthropic foundation that remains small, independent, and focused, can be nimble and flexible in identifying and addressing needs and opportunities as they arise.

What achievements of CMI have given you most satisfaction?

Most satisfying is seeing the growth and maturation of CMI as an institution, especially as managed currently from the President's office in Oxford, United Kingdom.

What are your thoughts on how CMI should evolve?

Stay global and continue to support the most promising research—with a very small staff!

"A PHILANTHROPIC FOUNDATION THAT REMAINS SMALL, INDEPENDENT, AND FOCUSED, CAN BE NIMBLE AND FLEXIBLE IN IDENTIFYING AND ADDRESSING NEEDS AND OPPORTUNITIES AS THEY ARISE."

PROGRAM OVERVIEW

Summary of 2016 Research Activities

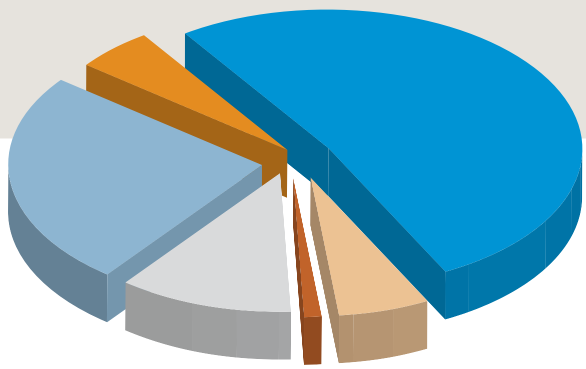
The activities of CMI researchers and research programs are sketched on the following pages. Researchers and programs are selected by the Scientific Advisory Board.

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2016:

18	Research Fellows, Research Awardees, Senior Scholars
141	CMI Workshops
108	Research Schools
57	PROMYS/PROMYS Europe/PRIMES Faculty and Participants
>240	Participants attending Conferences and Joint Programs

Research Expenses for Fiscal Year 2016



Clay Research Fellows

Simion Filip

Simion Filip received his PhD in June 2016 from the University of Chicago under the supervision of Alex Eskin. He is interested in the connections between dynamical systems and algebraic geometry, in particular between Teichmüller dynamics and Hodge theory. His recent interests also involve K3 surfaces and their special geometric properties. Simion has been appointed as a Clay Research Fellow for a term of five years beginning 1 July 2016.


Simion Filip

Tony Yue Yu

Tony Yue Yu received his PhD in 2016 from Université Paris Diderot under the supervision of Maxim Kontsevich and Antoine Chambert-Loir. He works on non-archimedean geometry, tropical geometry and mirror symmetry. He aims to build a theory of enumerative geometry in the setting of Berkovich spaces. Such a theory will give us a new understanding of the enumerative geometry of Calabi-Yau manifolds, as well as the structure of their mirrors. It is also intimately related to the theory of cluster algebras and wall-crossing structures. Tony has been appointed as a Clay Research Fellow for a term of five years beginning 1 September 2016.


Tony Yue Yu

Research Fellows

Ivan Corwin
2012-2016
Columbia University

Semyon Dyatlov
2013-2018
Massachusetts Institute of Technology

Simion Filip
2016-2021
Harvard University

June Huh
2014-2019
Princeton University

James Maynard
2015-2018
University of Oxford

John Pardon
2015-2020
Princeton University

Aaron Pixton
2013-2018
Massachusetts Institute of Technology

Peter Scholze
2011-2016
Universität Bonn

Jack Thorne
2013-2017
University of Cambridge

Miguel Walsh
2014-2018
University of Oxford

Alex Wright
2014-2019
Stanford University

Tony Yue Yu
2016-2021
Université Paris Sud

Senior Scholars

Tobias Colding (MSRI)
Spring 2016
Differential Geometry

Karen Vogtmann (MSRI)
Fall 2016
Geometric Group Theory

Pham Huu Tiep (EPFL)
Fall 2016
Local Representation Theory and Simple Groups

Photo credits:
Simion Filip / Eduard Duryev
Tony Yue Yu / Jindie Mi

Photo: Jennifer Balakrishnan

CMI WORKSHOPS

CMI conducts a program of workshops at the Mathematical Institute in Oxford, UK which bring together a small set of researchers quickly, outside the usual grant and application cycle, when this is likely to result in significant progress.

Explicit p-Adic Methods in Number Theory, Sage Days 71

March 21-24, 2016

This workshop illuminated a cluster of ideas centered around p-adic methods in number theory. Beginning with an introduction to Sage and Sage development, the daily talks covered six main themes for the week:

- p-adic algorithms for zeta functions
- Explicit computations in Iwasawa theory
- Overconvergent modular symbols
- Motivic integration
- Variations on the Chabauty-Coleman method
- p-adic precision and Sage

There were also several "lightening" talks by graduate students and post-doctoral fellows. Afternoon working sessions provided valuable time for participants to collaborate with and learn from each other.

Organizers

Jennifer Balakrishnan, University of Oxford
David Roe, University of Pittsburgh

Speakers

Jennifer Balakrishnan, University of Oxford
Francesca Bianchi, University of Oxford
Xavier Caruso, Université de Rennes 1
Raf Cluckers, Université Lille 1
Edgar Costa, Dartmouth College
Alyson Deines, Center for Communications Research
Victor Flynn, University of Oxford
Immanuel Halupczok, University of Leeds
Maurizio Monge, Universidad Federal do Rio de Janeiro
Fernando Rodriguez Villegas, ICTP Trieste
David Roe, University of Pittsburgh
Ander Steele, University of California, Santa Cruz
Jan Tuitman, KU Leuven
Tristan Vaccon, Rikkyo University
Jeanine Van Order, Universität Bielefeld
Chris Wuthrich, University of Nottingham

The participants, ranging from first-year graduate students to established researchers, contributed expertise in a broad range of number theory and Sage know-how. In all, at least 46 Sage trac tickets were modified, fixing bugs in existing code or contributing new code.

Generalized Geometry and Noncommutative Algebra

December 5-9, 2016

There are several striking similarities between the structure of noncommutative graded algebras and the geometry of generalized complex manifolds. At present, these parallels are mostly phenomenological in nature; a more precise formulation of the relationship is expected to lead to significant insights into both subjects.

This workshop aimed to clarify these links by bringing together experts from both noncommutative algebra and generalized geometry, as well as nearby subjects such as mirror symmetry, deformation quantization and mathematical physics. It resulted in a stimulating exchange of ideas between groups of researchers who don't often have the opportunity to interact.

The 24 speakers presented talks that summarized the basic ideas of their subjects, with a particular focus on:

- Structure of generalized Kähler 4-manifolds and Artin's conjecture on noncommutative surfaces
- Quantization of generalized complex and Poisson manifolds
- Noncommutative D-branes and mirror symmetry

The strong links between these subjects were particularly apparent in the talks of Henrique Bursztyn, who explained the relationship between geometric and algebraic notions of Morita equivalence; Daniel Chan, who described the construction of noncommutative Mori contractions; Marco Gualtieri, who gave a generalized geometry interpretation of noncommutative projective planes; and Justin Sawon, who highlighted a number of results and open problems on mirror symmetry for generalized and noncommutative K3 surfaces.

Organizers

Marco Gualtieri, University of Toronto
Brent Pym, University of Oxford
Michel Van den Bergh, Hasselt University

Speakers

Michael Bailey, Utrecht University
Pieter Belmans, University of Antwerp
Henrique Bursztyn, IMPA
Daniel Chan, University of New South Wales
Ryushi Goto, Osaka University
Marco Gualtieri, University of Toronto
Chris Hull, Imperial College London
Colin Ingalls, University of New Brunswick
Ulf Lindström, Uppsala University
Wendy Lowen, University of Antwerp
Ruxandra Moraru, University of Waterloo
Shinnosuke Okawa, Osaka University
Martin Rocek, University of Stony Brook
Justin Sawon, University of North Carolina
Geoffrey E. Schneider, Temple University
Pavol Ševera, University of Geneva
Susan Sierra, University of Edinburgh
Paul Smith, University of Washington
Toby Stafford, University of Manchester
Jeffrey Streets, University of California, Irvine
Joey van der Leer Durán, Utrecht University
Daniel Waldram, Imperial College London
Chelsea Walton, Temple University
Maxim Zabzine, Uppsala University

Photo courtesy of Wadham College

PROMYS EUROPE

In July, pre-university students from across Europe once again arrived in Oxford to participate in PROMYS Europe. The PROMYS (Program in Mathematics for Young Scientists) summer school for high school students was set up by Glenn Stevens in Boston, MA, in 1989. It has been the starting point for many a stellar career in mathematics. A mirror program, PROMYS Europe, was launched in Oxford in 2015 as a partnership between PROMYS, the Clay Mathematics Institute, the Mathematical Institute at Oxford University, and Wadham College, and with support from Oxford alumni.

Lecturers

Henry Cohn, Microsoft Research and Massachusetts Institute of Technology
Vicky Neale, University of Oxford
Glenn Stevens, Boston University

Guest Lecturers

Jennifer Balakrishnan, University of Oxford
Thomas Bloom, University of Bristol
Kevin Buzzard, Imperial College London
David Conlon, University of Oxford
Kevin Hughes, University of Bristol
Owen Patashnick, University of Bristol
Andrew Wiles, University of Oxford
Sarah Zerbes, University College London

The program ran very successfully for the second time in July and August 2016: 24 students and seven counsellors from 15 countries spent six weeks tackling challenging number theory problems, formulating and proving their own conjectures, and really getting a taste for what it is to work as a mathematician.

First-year students focused primarily on a series of very challenging problem sets, daily lectures, and exploration projects in number theory. There was also a program of talks by guest mathematicians and PROMYS Europe counsellors, on a wide range of mathematical subjects, as well as courses aimed primarily at students who are returning to PROMYS for a second or third time. Participants found a richly stimulating and supportive community of fellow first-year students, returning students, undergraduate counsellors, research mentors, faculty, and visiting mathematicians.

While selection for PROMYS Europe is highly competitive, it is needs-blind, with costs covered for those who would otherwise be unable to attend.

CMI again partnered with the London Mathematical Society to deliver week-long research schools at various locations in the United Kingdom. The schools attracted a very strong pool of applicants from around the world, with over 100 participants from 18 countries attending.

Developments in Contact and Symplectic Topology

June 20-24, 2016
University of Glasgow

Organizers

Brendan Owens, University of Glasgow
Andy Wand, University of Glasgow
Liam Watson, University of Glasgow

Lecturers

Vincent Colin, Université de Nantes
Emmy Murphy, Massachusetts Institute of Technology
András Stipsicz, Alfréd Rényi Institute of Mathematics

Guest Lecturers

Patrick Massot, École Polytechnique
Gordana Matic, University of Georgia
Ivan Smith, University of Cambridge

Symplectic manifolds arise as phase spaces in the Hamiltonian formulation of classical dynamics. In this setting, fixed energy levels naturally give rise to contact structures on odd-dimensional manifolds. The global topology of symplectic and contact manifolds has been a central area of study since the groundbreaking work of Gromov, Donaldson, Floer, and others in the 1980s and 1990s. Contact topology has steadily increased in importance as a subject in its own right, and has been an essential ingredient in some of the biggest results in low-dimensional topology this century, including Kronheimer-Mrowka's proof of Property P and the deep links exhibited by Ozsváth-Szabó and others between Floer-theoretic invariants such as Heegaard Floer homology and the topology of 3-manifolds.

Several key themes have emerged in three dimensional contact topology. These include: Gromov's h-principle, and the resulting flexible/rigid dichotomy; the importance of open book decompositions of manifolds; fillability questions concerning symplectic manifolds bounded by a given contact manifold; and the intimate relationship between symplectic and contact topology and gauge-theoretic invariants of smooth manifolds.

A key result of Borman-Eliashberg-Murphy in 2014 gave a dramatic advance in our understanding of contact topology in higher dimensions: using an h-principle argument, they extended Eliashberg's classification of flexible contact structures in three dimensions to arbitrary dimension. This result generated a great deal of activity in high-dimensional contact topology, leading to a rapidly growing understanding of which aspects of the theory in three dimensions can be generalised to higher dimensions, and of new phenomena which arise in high dimensions.

This research school provided students and early-career researchers with a comprehensive and accessible introduction to key aspects of contact topology in three dimensions and to the new frontier of high-dimensional contact topology. Three lecture courses, led by foremost experts in this research area, covered:

- *Open book decompositions and applications* by Vincent Colin
- *Applications of flexibility in high dimensional contact geometry* by Emmy Murphy
- *Contact topology and Heegaard Floer theory* by András Stipsicz

The lecture courses were supported by seven hours of problem and discussion sessions and supplemented by guest lectures from Partick Massot, Gordana Matic, and Ivan Smith. Together they provided the participants with a unique opportunity to get a hands-on guided tour of an exciting and fast-developing area of research from a world-leading team of experts.

Modern Topics in Nonlinear PDE and Geometric Analysis

July 4-8, 2016
University of Reading

This research school was designed to introduce graduate students to recent research topics in partial differential equations, including calculus of variations, nonlinear elliptic PDE, PDE in symplectic geometry, general theory of relativity, fluid dynamics, and inverse PDE problems.

The school consisted of six mini-courses in three central areas of mathematical analysis. Each of the three-hour lecture courses was supplemented by an hour-long question and answer session.

Course 1: Nonliner PDE and calculus of variations
Adjoint methods for nonlinear PDE by Lawrence C. Evans
Convexity notions in the calculus of variations by Jan Kristensen

Course 2: Geometric hyperbolic PDE, general relativity and fluid dynamics
The stability problem for black holes by Mihalis Dafermos
The formation of shocks in three dimensional fluid flows by Gustav Holzegel

Course 3: Geometric nonlinear PDE
The pull-back equation for differential forms by Bernard Dacorogna
Geometric inverse problems and PDE by Spyros Alexakis

The week was enhanced by two guest lecturers. Robert Jensen spoke about *Maximum principle techniques for fully nonlinear elliptic PDEs* and Juan Manfredi about *Obstacle problems for the p-Laplacian via optimal stopping of Tug-of-War games*. Two contributed lectures, given by the organizers, addressed *Higher order L∞ variational problems and the ∞-Polylaplacian* (Katzourakis) and *Decay for the wave equation on curved backgrounds (Aretakis)*.

Organizers

Stefanos Aretakis, Princeton University
Nikos Katzourakis, University of Reading

Lecturers

Spyros Alexakis, University of Toronto
Bernard Dacorogna, EPFL
Mihalis Dafermos, Princeton University and University of Cambridge
Lawrence C. Evans, University of California, Berkeley
Gustav Holzegal, Imperial College London
Jan Kristensen, University of Oxford

Guest Lecturers

Robert Jensen, Loyola University of Chicago
Juan Manfredi, University of Pittsburgh

ENHANCEMENT AND PARTNERSHIP

CMI's Enhancement and Partnership Program aims to add value to activities that have already been planned, particularly by increasing international participation. In accordance with CMI's mission and its status as an operating foundation, its funding is utilized to enhance mathematical activities organized by, or planned in partnership with, other organizations. In 2016, CMI partnered in 25 initiatives in 13 countries, often by funding a distinguished international speaker or supporting participants from outside the host country.

January 4-8 | AIMS Conference on Quadratic Differentials | AIMS, Cape Town, South Africa

January 25-29 | Analysis, PDEs and Geometry | Princeton University, Princeton, NJ

March 7 | SET for Britain | London, UK

March 21-24 | British Mathematical Colloquium | University of Bristol, UK

March 21-25 | Kähler Geometry, Einstein Metrics and Generalizations | MSRI, Berkeley, CA

April 5-8 | British Applied Mathematics Colloquium | University of Oxford, UK

April 10-16 | European Girls Mathematical Olympiad | Bușteni, Romania

May 2-6 | Geometric Flows in Riemannian and Complex Geometry | MSRI, Berkeley, CA

May 23-June 3 | Geometric Group Theory and Low-Dimensional Topology | ICTP, Trieste, Italy

May 30-June 4 | Constructive Approximation and Harmonic Analysis | CRM, Barcelona, Spain

June 13-16 | L-functions and Arithmetic | Harvard University, Cambridge, MA

June 13-17 | Dynamics, Geometry and Number Theory | Institut Henri Poincaré, Paris, France

June 13-17 | Leuca 2016 | Patù, Italy

June 27-July 1 | EMS/EWM Summer School: Geometric and Physical Aspects of Trudinger-Moser Type | Institute Mittag-Leffler, Djursholm, Sweden

June 27-July 2 | String-Math 2016 | Collège de France, Paris, France

July 10-20 | Summer School in Rational Homotopy Theory and its Interactions | Rabat, Morocco

July 17-20 | EMS/EWM Survey Lectures 2016 | TU Berlin, Germany

July 18-29 | Summer School on Non-linear Waves | IHÉS, Bures-sur-Yvette, France

July 18-December 9 | Combinatorial Algebraic Geometry | Fields Institute, Toronto, Canada

September 5-16 | Hitchin 70 | Aarhus University, Denmark, University of Oxford, UK, and ICMAT, Spain

September 5-December 16 | Large Cardinals and Strong Logics | CRM, Barcelona, Spain

September 12-23 | Quantum Integrable Systems | Institute d'Études Scientifiques de Cargèse, Corsica

September 25-October 2 | Women in Numbers Europe 2 | Lorentz Center, Leiden, Netherlands

December 6-9 | Amenability, Coarse Embeddability and Fixed Point Properties | MSRI, Berkeley, CA

December 5-9 | Deformation Theory, Completed Cohomology, Leopoldt Conjecture and K-theory | CIRM, Luminy, France

PUBLICATIONS

Selected Articles by Research Fellows

Ivan Corwin

Random-walk in Beta-distributed random environment, with Guillaume Barraquand. *Probability Theory and Related Fields*, to appear. arXiv: 1503.04117

KPZ equation limit of higher-spin exclusion processes, with Li-Cheng Tsai. *Annals of Probability*, to appear. arXiv: 1505.04158

Semyon Dyatlov

Spectral gaps without the pressure condition, with Jean Bourgain. arXiv: 1612.09040

Resonances for open quantum maps and a fractal uncertainty principle, with Long Jin. arXiv: 1608.02238

Simion Filip

The algebraic hull of the Kontsevich–Zorich cocycle, with Alex Eskin and Alex Wright. arXiv: 1702.02074

Counting special Lagrangian fibrations in twistor families of K3 surfaces. arXiv: 1612.08684

June Huh

Enumeration of points, lines, planes, etc., with Botong Wang. *Acta Mathematica*, to appear. arXiv: 1609.05484

Lefschetz classes on projective varieties, with Botong Wang, submitted. arXiv: 1609.08808

James Maynard

Primes with restricted digits. arXiv: 1604.01041

Sieve weights and their smoothings, with Andrew Granville and Dimitris Koukoulopoulos. arXiv: 1606.06781

John Pardon

A local criterion for generating the wrapped Fukaya category, with Sheel Ganatra and Vivek Shende.

Contact homology and virtual fundamental cycles, submitted. arXiv: 1508.03873

Aaron Pixton

Double ramification cycles on the moduli space of curves, with Felix Janda, Rahul Pandharipande and Dimitri Zvonkine, submitted. arXiv: 1602.04705

Tautological relations via r-spin structures, with Rahul Pandharipande and Dimitri Zvonkine, submitted. arXiv: 1607.00978

Peter Scholze

Integral p-adic Hodge theory, with Bhargav Bhatt and Matthew Morrow. arXiv: 1602.03148

On the generic part of the cohomology of compact unitary Shimura varieties, with Ana Caraiani. arXiv: 1511.02418

Jack Thorne

$\hat{G}$-local systems on smooth projective curves are potentially automorphic, with Gebhard Böckle, Michael Harris and Chandrashekhar Khare, submitted. arXiv: 1609.03491

On the average number of 2-Selmer elements of elliptic curves over ?q(X) with two marked points, submitted. arXiv: 1607.00997

Miguel Walsh

Bounded rational points on curves, International Mathematics Research Notes, 2015, no. 14, 5644-5658.

The algebraicity of ill-distributed sets, Geometric and Functional Analysis, 24 (2014), 959-967.

Alex Wright

Cubic curves and totally geodesic subvarieties of moduli space, with Curtis McMullen and Ronen Mukamel. *Annals of Mathematics*, to appear.

The boundary of an affine invariant submanifold, with Maryam Mirzakhani. *Inventiones Mathematicae*, to appear. arXiv: 1508.01446

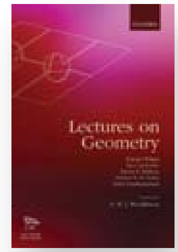
Tony Yue Yu

Enumeration of holomorphic cylinders in log Calabi-Yau surfaces. II. Positivity, integrality and the gluing formula, submitted. arXiv: 1608.07651

Derived non-archimedean analytic spaces, with Mauro Porta, *Selecta Mathematica*, to appear. arXiv: 1601.00859

Books

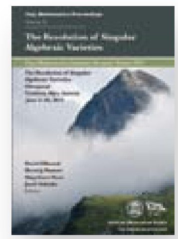
Lectures on Geometry



Editor: N. M. J. Woodhouse. Authors: Edward Witten, Martin Bridson, Helmut Hofer, Marc Lackenby, Rahul Pandharipande. CMI/OUP, 2017, 208 pp., hardcover, ISBN 97800-19-878491-3, List price: £40.00. Available from Oxford University Press.

This volume contains a collection of papers based on lectures delivered by distinguished mathematicians at Clay Mathematics Institute events over the past few years. It is intended to be the first in an occasional series of volumes of CMI lectures. Although not explicitly linked, the topics in this inaugural volume have a common flavour and a common appeal to all who are interested in recent developments in geometry. They are intended to be accessible to all who work in this general area, regardless of their own particular research interests.

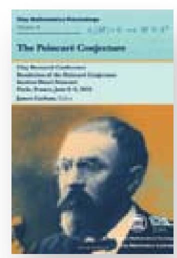
The Resolution of Singular Algebraic Varieties



Editors: David Ellwood, Herwig Hauser, Shigefumi Mori and Josef Schicho. CMI/AMS, 2014, 340 pp., softcover, ISBN: 0-8218-8982-4. List price: $101. AMS Members: $80.80. Order Code: CMIP/20.

Resolution of singularities has long been considered a difficult to access area of mathematics. The more systematic and simpler proofs that have appeared in the last few years in zero characteristic now give us a much better understanding of singularities. They reveal the aesthetics of both the logical structure of the proof and the various methods used in it. This volume is intended for readers who are not yet experts but always wondered about the intricacies of resolution. As such, it provides a gentle and quite comprehensive introduction to this amazing field. The book may tempt the reader to enter more deeply into a topic where many mysteries—especially the positive characteristic case—await discovery.

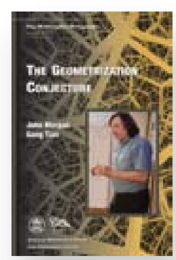
The Poincaré Conjecture



Editor: James Carlson. CMI/AMS, 2014, 178 pp., softcover, ISBN: 0-8218-9865-5. List price: $69. AMS Members: $55.20. Order Code: CMIP/19.

The conference to celebrate the resolution of the Poincaré conjecture, one of CMI's seven Millennium Prize Problems, was held at the Institut Henri Poincaré in Paris. Several leading mathematicians gave lectures providing an overview of the conjecture—its history, its influence on the development of mathematics, and its proof. This volume contains papers based on the lectures at that conference. Taken together, they form an extraordinary record of the work that went into the solution of one of the great problems of mathematics.

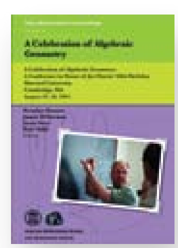
The Geometrization Conjecture



Authors: John Morgan and Gang Tian. CMI/AMS, 2014, 291 pp., hardcover, ISBN: 0-8218-5201-9. List price: $81. AMS Members: $64.80. Order Code: CMIM/5.

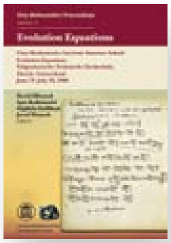
This book gives a complete proof of the geometrizaton conjecture, which describes all compact 3-manifolds in terms of geometric pieces, i.e., 3-manifolds with locally homogeneous metrics of finite volume. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. In the course of proving the geometrization conjecture, the authors provide an overview of the main results about Ricci flows with surgery on 3-dimension manifolds, introducing the reader to difficult material. The book also includes an elementary introduction to Gromov-Hausdorff limits and to the basics of the theory of Alexandrov spaces. In addition, a complete picture of the local structure of Alexandrov surfaces is developed.

A Celebration of Algebraic Geometry



Editors: Brendan Hassett, James McKernan, Jason Starr and Ravi Vakil. CMI/AMS, 2013, 599 pp., softcover, ISBN: 0-8218-8983-4. List Price: $149. AMS Members: $119.20. Order Code: CMIP/18.

This volume resulted from the conference held in honor of Joe Harris' 60th birthday. Harris is famous around the world for his lively textbooks and enthusiastic teaching, as well as for his seminal research contributions. The articles are written in this spirit: clear, original, engaging, enlivened by examples, and accessible to young mathematicians. The articles focus on the moduli space of curves and more general varieties, commutative algebra, invariant theory, enumerative geometry both classical and modern, rationally connected and Fano varieties, Hodge theory and abelian varieties, and Calabi-Yau and hyperkähler manifolds. Taken together, they present a comprehensive view of the long frontier of current knowledge in algebraic geometry.



Evolution Equations

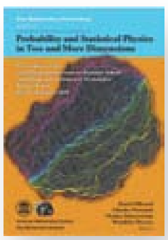
Editors: David Ellwood, Igor Rodnianski, Gigliola Staffilani and Jared Wunsch. CMI/AMS, 2013, 572 pp., softcover, ISBN: 0-8218-6861-6. List Price: $149. AMS Members: $119.20. Order Code: CMIP/17.

This volume is a collection of notes from lectures given at the 2008 Clay Mathematics Institute Summer School, held in Zurich, Switzerland. The lectures were designed for graduate students and mathematicians within five years of their PhD and the main focus of the program was on recent progress in the theory of evolution equations. Such equations lie at the heart of many areas of mathematical physics and arise not only in situations with a manifest time evolution (such as nonlinear wave and Schrödinger equations) but also in the high energy or semi-classical limits of elliptic problems.

Topics in Noncommutative Geometry

Editor: Guillermo Cortiñas. CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16.

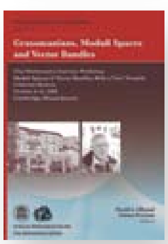
This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.



Probability and Statistical Physics in Two and More Dimensions

Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner. CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15.

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, "Probability and Statistical Physics in Two and More Dimensions" and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.



Grassmannians, Moduli Spaces and Vector Bundles

Editors: David A. Ellwood, Emma Previato. CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $58. AMS Members: $46.40. Order Code: CMIP/14.

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

On Certain *L*-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu. CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $136. AMS Members: $108.80. Order Code: CMIP/13.

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their *L*-functions, and both local and global theory are addressed.

Motives, Quantum Field Theory, and Pseudodifferential Operators

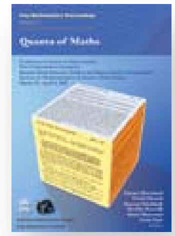
Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg. CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $94. AMS Members: $75.20. Order Code: CMIP/12.

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa. CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $136. AMS Members: $108.80. Order Code: CMIP/11.

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.



Homogeneous Flows, Moduli Spaces and Arithmetic

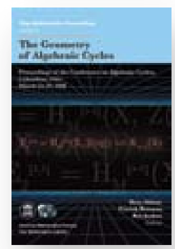
Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Kleinbock, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz. CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $104. AMS Members: $83.20. Order Code: CMIP/10.

This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.

The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua. CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $55. AMS Members: $44. Order Code: CMIP/9.

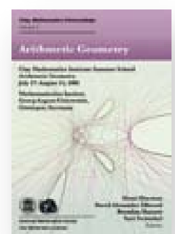
The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.



Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel. CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $125. AMS Members: $100. Order Code: CMIP/8.

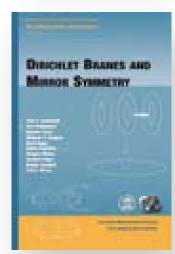
This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.



Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross. CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $115. AMS Members: $92. Order Code: CMIM/4.

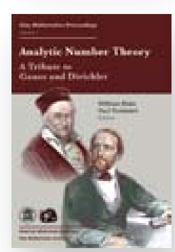
The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.

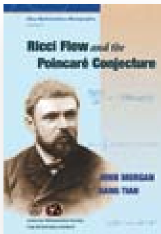


Analytic Number Theory: A Tribute to Gauss and Dirichlet

Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $53. AMS Members: $42.40. Order Code: CMIP/7.

This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.

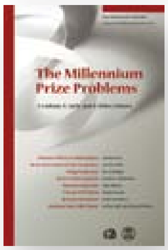




Ricci Flow and the Poincaré Conjecture

Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $75. AMS Members: $60. Order Code: CMIM/3.

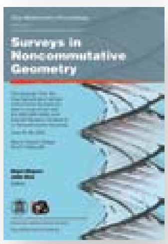
This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.



The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $32. AMS Members: $25.60. Order Code: MPRIZE.

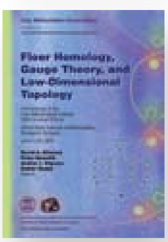
This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.



Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $53. AMS Members: $42.40. Order Code: CMIP/6.

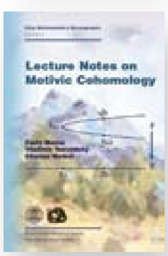
In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.



Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $70. AMS Members: $56. Order Code: CMIP/5.

This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $50. AMS Members: $40. Order Code: CMIM/2.S.

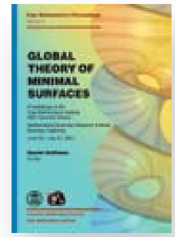
This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $138. AMS Members: $110.40. Order Code: CMIP/4.

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

Global Theory of Minimal Surfaces



Editor: David Hoffman. CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $138. AMS Members: $110.40. Order Code: CMIP/2

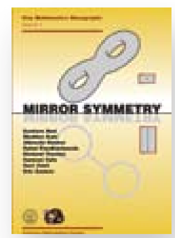
This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and thedouble-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

Strings and Geometry

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $80. AMS Members: $64. Order Code: CMIP/3.

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.

Mirror Symmetry



Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $144. AMS Members: $115.20. Order Code: CMIM/1

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $91. ASM Members: $72.80. Order Code: CMIP/1.

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

Unless otherwise indicated, to order print copies of these books please visit ***www.ams.org/bookstore****. PDF versions are posted on CMI's Online Library six months after publication and can be found at www.claymath.org/node/262.*

Digital Library

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Ada Lovelace Papers

Often called the first computer programmer, Ada Lovelace is celebrated for her pioneering work on programming Charles Babbage's Analytical Engine. These papers offer a rounded picture of the development of Lovelace's mathematical and scientific interests and include both sides of the extensive correspondence between Ada and the mathematician Augustus DeMorgan. CMI is very grateful to Ada's descendant, the Earl of Lytton, for his family's permission to undertake this project. High resolution images are available through the Bodleian Library, University of Oxford.
www.claymath.org/ada-lovelaces-mathematical-papers

Quillen Notebooks

Daniel Quillen obtained his PhD under the supervision of Raoul Bott at Harvard in 1961. He worked at MIT before moving to the University of Oxford in 1984. During his long mathematical career, Quillen kept a set of detailed notes which give a day-to-day record of his mathematical research.
www.claymath.org/publications/quillen-notebooks

Euclid's Elements

The manuscript MS D'Orville 301 contains the thirteen books of Euclid's Elements, copied by Stephen the Clerk for Arethas of Patras in Constantinople in 888 AD. It is kept in the Bodleian Library at the University of Oxford where high resolution copies of the manuscript are available for study.
www.claymath.org/euclids-elements

Riemann's 1859 Manuscript

Bernhard Riemann's paper, Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse (On the number of primes less than a given quantity), was first published in the Monatsberichte der Berliner Akademie, in November 1859. Just six manuscript pages in length, it introduced radically new ideas to the study of prime numbers.
www.claymath.org/publications/riemanns-1859-manuscript

Klein Protokolle

The "Klein Protokolle," comprising 8600 pages in 29 volumes, is a detailed handwritten registry of seminar lectures given by Felix Klein, his colleagues and students, and distinguished visitors in Göttingen for the years 1872-1912.
www.claymath.org/publications/klein-protokolle

James Arthur Archive

James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author's complete published work in an easily accessible set of searchable PDFs.
www.claymath.org/publications/collected-works-james-g-arthur

Notes of Talks at the I. M. Gelfand Seminar

The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin's notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education.
www.claymath.org/publications/notes-talks-imgelfand-seminar

NOMINATIONS, PROPOSALS AND APPLICATIONS

Research Fellowship Nominations

Nominations for Clay Research Fellows are considered once a year. The primary selection criteria for the Fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader. Selection decisions are made by the Scientific Advisory Board based on the nominating materials: letter of nomination, names and contact information for two other references, Curriculum Vitae, and publication list for the nominee.

Address all nominations to Nick Woodhouse at president@claymath.org, copied to admin@claymath.org.

Workshops at the Mathematical Institute

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty people, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

Enhancement and Partnership

The Clay Mathematics Institute invites proposals under its Enhancement and Partnership Program. The aim is to enhance activities that are already planned, particularly by funding international participation. The program is broadly defined, but subject to the general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board.

The Enhancement and Partnership Program can support graduate summer schools, or similar events, run in partnership with other organizations. CMI can provide the bulk of the funding for a short (week-long) school for up to 30 participants, or enhancement funding for a larger event. The intention is to enable the stars of the next generation of professional mathematicians to take significant steps along the road to becoming international leaders in their field.

Nominations for Senior Scholars will be considered within the scope of the Enhancement and Partnership Program. The aim of the Senior Scholar program is to foster mathematical research and the exchange of ideas by providing support for senior mathematicians who will play a leading role as "senior scientist" in a topical program at an institute or university. Senior Scholars will be in residence throughout the program and are expected to interact extensively with the other participants.

For more information, visit www.claymath.org/programs/enhancement-and-partnership-program. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

Annual Deadlines

Research Fellowship nominations:
November 16

Workshop proposals:
March 1
June 1
September 1
December 1

Enhancement and Partnership proposals, including Senior Scholars nominations:
March 1
June 1
September 1
December 1

Graduate Summer School proposals:
March 1

Nominations and proposals may also be mailed to:

Clay Mathematics Institute
Office of the President
Andrew Wiles Building
Radcliffe Observatory Quarter
Woodstock Road
Oxford OX2 6GG
United Kingdom

Institute Calendar 2017

Date	Event	Location
Jan-March	Senior Scholar Dmitry Orlov, *Homological Mirror Symmetry*	IAS, Princeton
Jan 3-June 23	Senior Scholar Emmanuel Breuillard, *Non-Positive Curvature Group Actions and Cohomology*	Isaac Newton Institute
Jan 17-May 26	Senior Scholar Alexander Volberg, *Harmonic Analysis*	MSRI, Berkeley
Jan 17-May 26	Senior Scholars Manjul Bhargava and Roger Heath-Brown, *Analytic Number Theory*	MSRI, Berkeley
March 11-15	Arizona Winter School	University of Arizona
March 13	STEM for Britain	London, UK
March 20-24	Young Geometric Group Theory Meeting VI	University of Oxford
March-July	Operator Algebras: Dynamics and Interactions	CRM, Barcelona
May 15-19	Recent Developments in Harmonic Analysis	MSRI, Berkeley
May 29-June 2	Random Walks with Memory	CIRM, Luminy
June 19-23	LMS-CMI Research School: New Trends in Representation Theory: The Impact of Cluster Theory in Representation Theory	University of Leicester
June 25-July 15	Senior Scholars Craig Tracy and H. T. Yau, *Random Matrices*	PCMI, Salt Lake City
June 26-30	LMS-CMI Research School: Microlocal Analysis and Applications	Cardiff University
July 3-Aug 11	PROMYS	Boston University
July 9-Aug 19	PROMYS Europe	University of Oxford
July 10-14	International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC)	Queen Mary University London
July 17-21	Harmonic Analysis and its Interactions	ICMS, Edinburgh
Aug 13-18	Women in Numbers 4	BIRS, Banff
Aug 14-18	Symplectic Geometry Workshop	Isaac Newton Institute
Sept 11-15	LMS-CMI Research School: Algebraic Topology of Manifolds	University of Oxford

Date	Event	Location
Sept 11-15	LMS-CMI Research School: Introduction to Geometry, Dynamics, and Moduli in Low Dimensions	University of Warwick
Sept 11-22	Instruments of Algebraic Geometry	University of Bucharest
Sept 24-28	Ergodic Theory: Numbers, Fractals and Geometry	University of Oxford
Sept 24-28	Harmonic Analysis and Related Areas	University of Oxford
Sept 25-29	Modern Moduli Theory	University of Oxford
Sept 25-29	Nonlocal PDEs	University of Oxford
Sept 27	Clay Research Conference	University of Oxford
Nov 13-17	Geometric Functional Analysis and Applications	MSRI, Berkeley
Nov 29-Dec 1	Women in Topology	MSRI, Berkeley
Fall 2017	Senior Scholar William Johnson, *Geometric Functional Analysis and Applications*	MSRI, Berkeley
Fall 2017	Senior Scholar Francisco Santos, *Geometric and Topological Combinatorics*	MSRI, Berkeley
Dec 4-8	*D*-modules, Geometric Representation Theory, and Arithmetic Applications Workshop	University of Oxford
Dec 14-19	Transformation Groups 2017	Independent University of Moscow
Dec 18-21	V International Symposium on Nonlinear Equations and Free Boundary Problems	University of Buenos Aires

CMI

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CLAY MATHEMATICS INSTITUTE

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