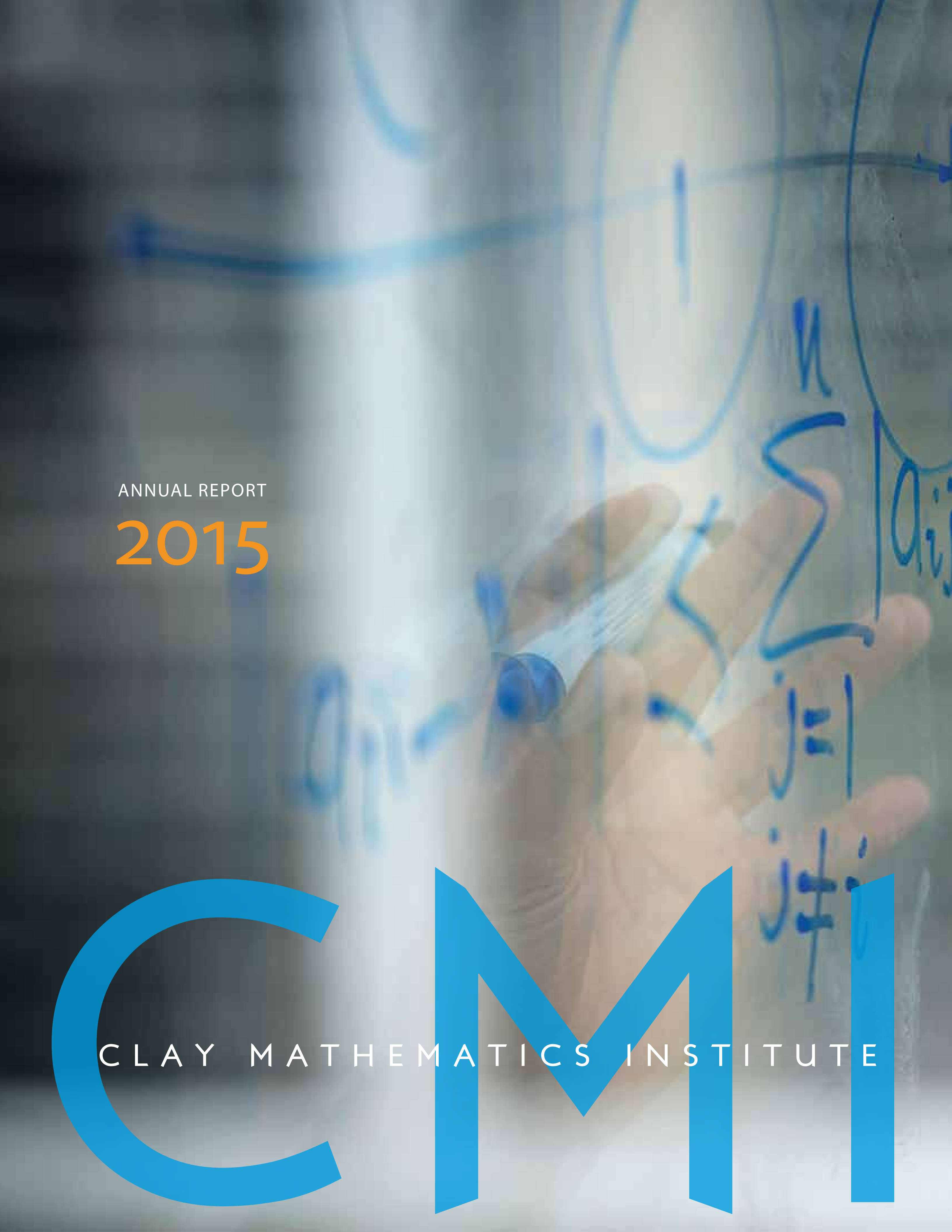


ANNUAL REPORT

2015

CMI

CLAY MATHEMATICS INSTITUTE

CLAY MATHEMATICS INSTITUTE

Mission

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.

The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

Cover photo: John Cairns
Design: Joyce Hempstead | Boston, Massachusetts

Contents

Letter from the President

The central mission of the Clay Mathematics Institute is the support of mathematical research, internationally and at the very highest level. Our report this year contains many accounts of the Institute's work in pursuit of this, through the appointment of Clay Research Fellows, the annual Clay Research Awards, our own conferences and workshops, and by promoting mathematical activity across the globe through the Enhancement and Partnership Program. Sometimes the amounts involved are small, sometimes much larger, but the accumulative impact has been huge. The pages that follow contain ample evidence of CMI's continuing work to 'increase mathematical knowledge', in the simple and direct wording of its founding mission statement.

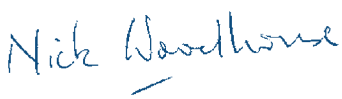
But that is not all with which we were tasked when Landon Clay launched the Institute in 1998. We are also to 'disseminate mathematical knowledge', 'educate mathematicians and other scientists about new discoveries in mathematics', and 'encourage gifted students to pursue mathematical careers'. The year saw the launch of two significant projects under these headings.

The first was the creation of the new Clay Award for Dissemination of Mathematical Knowledge. This is not simply an award for popularization. Rather it recognizes outstanding achievement in two intertwined strands—in the recipients' personal contributions to research at the highest level and in their distinction in explaining recent advances in mathematics. The award is intended to be a rare honor, made not on a regular schedule but only as it becomes clear that exceptional contributions should be recognized and celebrated. The first award was presented to Étienne Ghys, following a public lecture that he delivered during the 2015 Clay Research Conference.

The second was the launch of PROMYS Europe. For many years, CMI has supported PROMYS, the summer school founded by Glenn Stevens at Boston University. Each summer this brings together high school mathematicians from across America and beyond for six weeks of intense mathematical activity. Selection is highly competitive, the focus and achievements of the participants astonishing. Many of its alumni in the international community can credit PROMYS with providing the initial impetus for their careers in mathematics. As Henry Cohn explains (p 29), the goal is neither acceleration nor contest preparation, but rather to provide the deep experience of mathematical discovery. For many participants, the experience is not a single event: there are opportunities to return for a second year or later on as counselors.

The success of the Boston Program inspired CMI to develop a sister program in Oxford for European students, in partnership with Wadham College and the Mathematical Institute at the University of Oxford. After two trial years in which students were sent across the Atlantic to participate in the Boston summer school, and then return via a week of masterclasses in Oxford, PROMYS Europe ran for the first time at Wadham College in the summer of 2015. Glenn Stevens himself, together with Henry Cohn, a long-standing contributor to PROMYS, led the teaching, with Vicky Neale from the Oxford department. PROMYS Europe runs again in 2016, with a larger group and with very welcome additional support from Oxford and Wadham alumni.

Sincerely,



N. M. J. Woodhouse

Annual Meeting

The 2015 Clay Research Conference

Nick Woodhouse

MIKE HOPKINS

Mike Hopkins spoke about his recent work with Aravind Asok and Jean Fasel on a long-standing problem in algebraic topology: which (topological) vector bundles on an algebraic variety admit algebraic structures? It is a hard problem—for example, a complete answer would resolve the Hodge conjecture. It comes within the scope a general programme identified by J. F. Adams, under which one takes results from algebraic topology as particular cases of statements about finitely generated modules over rings and then attempts to prove them under suitable conditions.

A key property of topological vector bundles is that they are homotopy invariant, so that a rank-k vector bundle on $X \times [0,1]$ is entirely determined by its restriction to $X \times \{t\}$ for any $t \in [0,1]$. That is

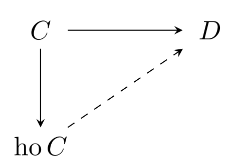
$$\mathsf{Vect}_k^{\mathrm{top}}(X \times [0,1]) \simeq \mathsf{Vect}_k^{\mathrm{top}}(X)\,.$$

The lecture focused on the key question: does the analogous statement hold for algebraic bundles? That is, is it true that

$$\mathsf{Vect}_k^{\mathrm{alg}}(X \times A^1) \simeq \mathsf{Vect}_k^{\mathrm{alg}}(X)\,,$$

where A^1 is the affine line? Can one study algebraic vector bundles by using homotopy theory?

The question can be rephrased in the language of Quillen and Kan's abstract homotopy theory, in which the ingredients are a category C and a collection of morphisms $W \subset C$ called *weak equivalences*. In the topological context, C is the category of topological spaces and weak equivalences are homotopy equivalences.



In the general abstract setting,a *homotopy functor* is a functor $D \to C$ taking weak equivalences to isomorphisms. There is a *universal homotopy functor* $C \to \mathrm{ho}\,C$ characterised by the universal property that, given any homotopy functor $C \to D$, there exists a unique functor $\dashrightarrow$ such the diagram on the left commutes.

Hopkins explained how Ken Brown had used the abstract theory to define algebraic homotopy theory. In that case, C is an enlargement of the category of (simplicial) presheaves on the category on smooth varieties (over $\mathbb{C}$) and weak equivalence captures the idea that the same variety can be assembled in different ways from local pieces.

In between the topological and algebraic, there is *motivic homotopy theory*, where maps $X \times A^1 \to X$ are added to the weak equivalences of the algebraic construction. So with all the varieties defined over $\mathbb{C}$, there are three homotopy theories to consider

$$C^{\mathrm{alg}} \to C^{\mathrm{mot}} \to C^{\mathrm{top}}\,.$$

The arrows are 'realization functors' determined by the underlying spaces. In this language, the problem is to take a topological vector bundle—an object on the right—and lift it all the way to the left to get an algebraic vector bundle.

In the topological case one can study G-vector bundles as maps into the classifying space BG. For k-dimensional vector bundles, one has

$$[X, \mathrm{BGL}_k]_{\mathrm{top}} \simeq [X, \mathrm{BU}(k)]_{\mathrm{top}} \simeq \mathsf{Vect}_k^{\mathrm{top}}(X)\,.$$

Mike Hopkins

In algebraic homotopy theory, one similarly has

$$[X, \mathrm{BGL}_k]_{\mathrm{alg}} \simeq \mathsf{Vect}_k^{\mathrm{alg}}(X)\,,$$

which comes down to stating that one can assemble vector bundles from charts and transition functions. In the motivic case, one uses this idea to introduce motivic vector bundles by taking

$$\mathsf{Vect}_k^{\mathrm{mot}}(X) \simeq [X, \mathrm{BGL}_k]_{\mathrm{mot}}$$

as a definition. Then we have

$$\mathsf{Vect}_k^{\mathrm{alg}}(X) \to \mathsf{Vect}_k^{\mathrm{mot}}(X) \to \mathsf{Vect}_k^{\mathrm{top}}(X)\,,$$

so breaking the problem into two stages, with the intermediate problem of finding a motivic vector bundle given the corresponding topological vector bundle.

A key step forward was Fabien Morel's proof in 2012 that for smooth affine X, the first map is an isomorphism. The proof was simplified and improved by Schlichting and by Asok, Hoyois, and Wendt in 2015.

When X is *not* affine, however, $\mathsf{Vect}_k^{\mathrm{alg}}(X)$ is not an invariant of motivic homotopy theory, as one can see from an example involving bundles on $\mathbb{P}^1$. But that leaves open the possibility that

$$\mathsf{Vect}_k^{\mathrm{mot}}(X) = \mathsf{Vect}_k^{\mathrm{alg}}(X)/\text{homotopy}\,.$$

The answer is not known, but the emerging consensus is that it is probably not true.

Hopkins explained how the question becomes a purely algebraic one by using a construction involving Jouanolous' device; and also that obstruction theory is not up to the task of answering it. He then turned to a more specific question, first investigated in the 1970s: which vector bundles on projective space are algebraic? In particular asking this of the rank-2 vector bundles over $\mathbb{CP}^n$ with zero Chern classes constructed by Elmer Rees.

By transcribing work of Suslin on vector bundles over spheres to a motivic context, Hopkins has been able to show that the Rees bundles lift to motivic bundles: this is one step towards answering the large question.

He ended by describing a 'proof in progress' that, under the assumption that the Wilson space hypothesis is true, the second realization functor, $\mathsf{Vect}_k^{\mathrm{mot}}(X) \to \mathsf{Vect}_k^{\mathrm{top}}(X)$ is a bijection in the case of projective and related spaces, such as Grassmannians.

Peter Scholze's lecture addressed the problem of defining 'the' cohomology of an algebraic variety. He described the background to a new result he had proved with Bhatt and Morrow, that part of the solution lies in a cohomology theory with values in a 'two-dimensional complete local ring' (Fontaine's $\mathbb{A}_{\text{inf}}$).

He started with a smooth projective variety $X/\mathrm{Spec}(\mathbb{Z})$. Then $X(\mathbb{C}) \subset \mathbb{CP}^n$ is a compact complex manifold defined by polynomial equations with integer coefficients. In thinking about the cohomology of X, there is a choice of coefficients—for example $\mathbb{Z}$, $\mathbb{F}_\ell$, $\mathbb{Z}_\ell$ (the ℓ-adic integers) for prime ℓ, and so on. Also one can regard $X \to \mathrm{Spec}(\mathbb{Z})$ as a family of varieties, the fibres of a projection. For each prime p, one has the fibre $X_{\mathbb{F}_p}/\mathrm{Spec}\,\mathbb{F}_p$; at infinity, the fibre is $X(\mathbb{C})$.

Scholze presented the problem as that of filling in the diagram in such a way that a cohomology group of $X_{\mathbb{F}_p}$ with coefficients labelled by ℓ is attached to each point (p, ℓ), with p and ℓ prime—the p-coordinate labels the fibre and the ℓ-coordinate the choice of coefficients.

Existing theories can be used at various locations in the diagram:

- Singular cohomology of $X(\mathbb{C})$ can be used on the line $p = \infty$.
- Grothendieck's étale cohomology fills in much of the diagram, attaching $H^i_{\text{ét}}(X_{\mathbb{F}_p}, \mathbb{Z}_\ell)$ to each point at which $p \neq \ell$.
- The diagonal is tackled by using de Rham cohomology; that is, the cohomology of the algebraic de Rham complex on X. By the Poincaré lemma, one has

  $$H^i_{\mathrm{dR}}(X) \otimes_{\mathbb{Z}} \mathbb{C} \simeq H^i_{\mathrm{dR}}\big(X(\mathbb{C})\big) \simeq H^i_{\mathrm{sing}}(X(\mathbb{C}), \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{C}\,,$$

  although, H^i_{dR} and H^i_{sing} are very different $\mathbb{Z}$-lattices. If one reduces the coefficients on the left to $\mathbb{F}_p$, then under torsion-freeness assumptions you get

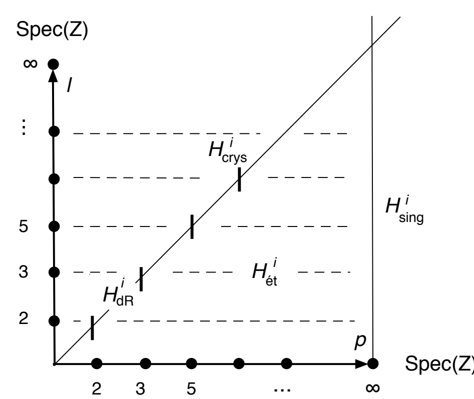
  $$H^i_{\mathrm{dR}}(X) \otimes_{\mathbb{Z}} \mathbb{F}_p \simeq H^i_{\mathrm{dR}}(X_{\mathbb{F}_p})\,.$$

  So in this cohomology theory, reducing the coefficients gives the same result as reducing the space mod p. That is

  $$H^i_{\mathrm{dR}}(X) \otimes_{\mathbb{Z}} \mathbb{F}_p \simeq H^i_{\mathrm{dR}}(X_{\mathbb{F}_p})\,,$$

  giving diagonal additions to the diagram by defining $H^*(X_{\mathbb{F}_p}, \mathbb{F}_p)$ to be $H^*_{\mathrm{dR}}(X_{\mathbb{F}_p})$.

There remains the problem of giving a meaning to $H^*(Y, \mathbb{Z}_p)$ for a scheme $Y/\mathbb{F}_p$. This is solved by Grothendieck's 'crystalline cohomology', which is hard to understand at an intuitive level.



We are then left with the question 'can one fill in more of the diagram by explicit cohomology theories?' For example what happens as étale cohomology 'degenerates' into crystalline cohomology? The new result allows one to address such questions.

One surprising consequence is the inequality

$$\#H^i_{\mathrm{dR}}(X)_{\mathrm{tor}} \geq \#H^i_{\mathrm{sing}}(X(\mathbb{C}), \mathbb{Z})_{\mathrm{tor}}\,.$$

The lecture ended by presenting a conjecture that the new cohomology theory can be undestood in terms of a 'q-deformation' of de Rham cohomology.

Peter Scholze

Charles Fefferman

CHARLES FEFFERMAN

Charles Fefferman spoke about two cases in which singularlities form in incompressible fluids with boundaries. The phenomena are remarkable: although there are many cases in which singularities are known in compressible fluids, these are the first physically interesting cases in which it is possible to prove rigorously that singularities form in incompressible fluids.

The first case, of water waves in two dimensions, was covered in work with Castro, Cordoba, Gancedo, and Gomez. Here he considered the case of a body of water separated from a vacuum region above by a moving interface $\partial\Omega$ given by a parameterized curve

$$\partial\Omega(t) = \{z(\alpha,t) \, : \, \alpha \in \mathbb{R}\} \qquad (*)$$

in $\mathbb{R}^2$. The fluid velocity and pressure are governed by the incompressible Euler equations

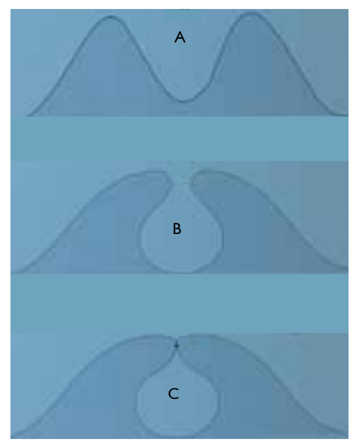
$$(\partial_t + u \cdot \nabla_x)u = -\nabla_x p - \begin{pmatrix} 0 \\ g \end{pmatrix}, \quad \operatorname{div} u = 0, \quad \operatorname{curl} u = 0\,.$$

At the interface, $p = 0$ and

$$\partial_t z(\alpha,t) = u(z(\alpha,t),t) + c(\alpha,t)\partial_\alpha z(\alpha,t)\,.$$

The second term on the right governs the change in the (unphysical) parameterization of the boundary.

The question is: given smooth initial velocity can a singularity form in finite time? Most recent work on water waves shows that singularities do not arise from small initial data. Castro *et al.* show that singularities can and do arise from large initial data. The mechanism involves an initially smooth interface (A) on which two portions of the boundary turn over (B) and then collide after a finite time in a single point—a 'splash' (C). The results that have been established rigorously are that it is possible to start with a smooth boundary as in (A) with no turn-over and then for a turn-over to form, as in (B), after a finite time; second, that a water wave can start as in (B) and then form a splash in a finite time. Computer simulations suggest that both steps can occur starting from A, but this does yet have a rigorous proof.



The next part of the lecture concerned the *Muskat problem*, in which there is an interface between oil (in sand) and water (in sand). Here the work was done with Castro, Cordoba, Gancedo, and Lopez. By coincidence, the equations are the same as for a Hele-Shaw cell, with Euler's equation $(*)$ replaced by *Darcy's law*,

$$u = -\nabla_x p - \begin{pmatrix} 0 \\ g\rho \end{pmatrix},$$

where ρ is equal to the density of water in its region and to the density of oil in the other. The boundary conditions are that the pressure must be continuous across the interface and the difference in the fluid velocities must be parallel to the interface.

The case in which the oil is above the water is better behaved than the reverse. As with the heat equation, the behaviours are interchanged when time is run backwards. The case in which the interface turns over is also badly behaved.

The initial configuration that forms a singulairity in this problem starts with the oil above the water (the stable case), but the interface subsequently turns over (so entering the 'bad regime'), and then goes on to form a singularity at a single point. Above and below this point, the interface is real analytic. In any neighbourhood of the singular point itself, the interface is C^3 but not C^4. This scenario is established rigorously.

Fefferman outlined the proof. The principal steps are:

- Write down the equations that determine the behaviour of the interface.
- Find a real analytic solution $Z_0(\alpha, t)$ that turns over.
- Pick the 'correct' norm.
- Perturb Z_0 in the correct norm to obtain a solution that turns over and then breaks down.

The final step is the subtle one. One finds a solution $Z(\alpha, t)$ that is close to Z_0, but has a singularity of the above form at $\alpha = 0$, $t = t_0$, and then shows that it has the turn-over behaviour by propagating it backwards in time from t_0. It has a singularity because that is prescribed; and it turns over because it is close to Z_0.

ANDREI OKOUNKOV

Andrei Okounkov spoke about a theorem he had proved with Smirnov that explains the remarkable relationship between two 'genetically distant' parts of the tree of mathematics, which become linked through the work of Bezrukavnikov and Kaledin. The focus was on the quantization of algebraic symplectic varieties N in characteristic $p >> 0$, where the 'distant branches' are on the one hand actions by geometric operators and, on the other, representations of geometrically defined algebras.

The link is made through identifying the action of Bezrukavnikov's groupoid on $K(N)$ first with the monodromy of the 'quantum connection' and second with the action of a 'Weyl group' of a 'quantum loop algebra' constructed by Maulik and Okounkov.

The main part of the lecture gave the background to this deep but technical result, and explained how the various ingredients emerge from earlier work. The story begins with the Deligne-Mumford moduli space $\overline{\mathcal{M}_{g,n}}$ of genus-g pointed curves $(C, p_1, p_2, \ldots, p_n)$ and the study of the intersections of Chern classes of various natural bundles on $\overline{\mathcal{M}_{g,n}}$, such as the Hodge bundle (with fibre the holomorphic differentials on C) or the line bundles with fibres $\mathcal{L}_i = T^*_{p_i}C$. A celebrated result of some 25 years standing is Witten's KdV-Virasoro formula for the intersections of the divisors of these line bundles.

Andrei Okounkov

Even when the interest is simply in the moduli space of curves, it is helpful to study curves with extra structure; in particular to study, modulo equivalence, pointed curves together with a map f from C to a variety X. The moduli space is $\overline{\mathcal{M}_{g,n}}(X, d)$, where $d \in H_2(X, \mathbb{Z})$ is the degree of f; it has a virtual fundamental class $[\overline{\mathcal{M}_{g,n}}(X, d)]^{\mathrm{vir}}$. These behave nicely in the case $\dim X = 3$ because of the cancellation in the formula

$$\dim [\overline{\mathcal{M}_{g,n}}(X, d)]^{\mathrm{vir}} = (3 - \dim X)(g - 1) + (c_1(X), d) + n\,.$$

The Gromov-Witten counts are defined by integrating natural cohomology classes on $\overline{\mathcal{M}_{g,n}}(X, d)$ over the homology class on the left.

The Witten classes—the Chern classes of the line bundles $\mathcal{L}_i$—are constructed by considering curves that intersect a given divisor D in X with given multiplicites; while the Hodge bundle is brought into the picture by replacing X by $X \times \mathbb{C}$ and imposing equivariance under the action of $\mathrm{GL}\,(1)$ on $\mathbb{C}$. In this extended picture, an equivariant map to $X \times \mathbb{C}$ is simply a map $C \to X = X \times \{0\}$, so nothing seems to have been added. But that is not quite true: the Chern polynomial of the dual Hodge bundle appears by considering trivial deformations of the curve in the normal direction along $\mathbb{C}$.

In the same spirit, one can consider the case in which $X = \mathbb{C}^2 \times \mathbb{P}^1$ and $D = \mathbb{C}^2 \times \{\infty\}$. Here there are three commuting actions of $\mathrm{GL}(1)$ and three normal directions. Information about the intersections of the Witten classes with cubic polynomials of the Chern classes of the Hodge bundle is contained in the vertex functions

$$V_g(\mu, t) = \int_{\overline{\mathcal{M}}_{g,n}} \frac{\prod_1^3 c(\text{Hodge}, -t_i^{-1})}{\prod_i^n (t_1 - \mu_i c_1(\mathcal{L}_i))}$$

where the variables t_1, t_2, t_3 correspond to three normal directions and $\{\mu_i\}$ is a partition of d.

A unique feature of three-dimensional varieties X is the special relationship between Gromov-Witten and Donaldson-Thomas counts, in the form

$$\sum_g u^{2g-2} \int_{[\overline{\mathcal{M}}_{g,n}]^{\mathrm{vir}}} \cdots \propto \sum_\chi [-z]^\chi \int_{[\mathrm{Hilb}(X,d,\chi)]^{\mathrm{vir}}} \cdots$$

where $z = \exp(\mathrm{i}u)$ and the integrals on the right are over the moduli spaces of subschemes C in X such that $[\mathcal{O}_C] = (0, 0, d, \chi)$.

In the problem at hand, by intersecting the subschemes with D, one can replace

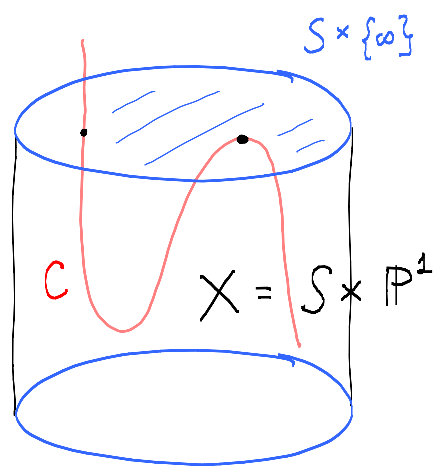
$$\text{Hilb}(X, \text{curves}) \text{ by } \text{Hilb}(D, \text{points})$$

and thence identify the generating function for the vertex functions as an element of $H^*(\text{Hilb}(D, \text{points}))[[z]]$with z regarded as a coordinate on

$$\mathrm{Pic}(\text{Hilb(D)}) \otimes \mathbb{C}^* .$$

The central point is that V is the product of U^{-1} and a fixed fundamental class, where U is the solution of a linear differential equation with regular in z; that is, it is constant with respect to the so-called 'quantum connection'. It is easier to find the connection than to compute Hodge intergals.The calculation of the quantum connection was orginally done by Maulik and Okounkov in the case that D is an ADE surface

$$S = S \times \{0\} \subset X = S \times \mathbb{C} .$$



More generally, Hilb(S) is replaced by a general Nakajima variety N. Here the connection is

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} - c_1(\lambda) \cup + \hbar \sum_{\alpha > 0} (\lambda, \alpha) \frac{z^\alpha}{1 - z^\alpha} e_\alpha^{(i)} e_{-\alpha}^{(i)} ,$$

where $\lambda \in \mathrm{Pic}(N)$, the cup product by $c_1(\lambda)$ acts on $H^*(N)$, $\alpha \in H_2(N)_{\text{effective}}$ ranges over the roots of a certain Lie algebra $\mathfrak{g}$ acting by correspondences and $e_\alpha^{(i)} e_{-\alpha}^{(i)}$ is the invariant element of $\mathfrak{g}_\alpha \mathfrak{g}_{-\alpha}$.

The final part of the lecture explained how these elements underpinned Okounkov's ongoing work with Smirnov, in which the action of Bezrukavnikov's groupoid on $K(N)$ can be understood on the one hand as the monodromy of the quantum connection, and on the other in terms of the action of a generalised 'Weyl group' of the quantum loop algebra of $\mathfrak{g}$ constructed by Maulik and Okounkov.

Maryam Mirzakhani

MARYAM MIRZAKHANI

(Summary by Howard Masur)

Maryam Mirzakhani's talk was about counting problems for the number of closed curves on a closed surface S of genus $g \geq 2$, equipped with a metric X of constant curvature -1, or equivalently a hyperbolic metric. It is a basic fact in hyperbolic geometry, that given a (free) homotopy class of curves, there is a unique geodesic in the homotopy class. Thus it makes sense to try to count the number of closed geodesics β of a certain length. It is a classical theorem that if one counts all closed curves β of length $\ell_X(\beta) \leq L$ then the number $N_X(L)$ grows asymptotically like $\frac{e^L}{L}$. Here the exponential growth rate does not depend on the genus or on X. Counting closed curves with restrictions is more difficult. In her celebrated earlier work, Mirzakhani counted the number of *simple* closed curves β with $\ell_X(\beta) \leq L$ and found this number to grow asymptotically like $B(X)L^{6g-6}$. Here $B(X)$ is the volume in the Thurston space of measured laminations $\mathcal{ML}$ of those laminations that have length at most 1 on X. The growth rate is polynomial. What is striking about this result is that there is an asymptotic formula at all and that the constant depends on X.

In the talk in Oxford, Mirzakhani greatly expanded on this previous work. She fixes a curve α which *fills* the surface. This means that every component of the complement is simply connected, or equivalently every closed curve not isotopic to α must intersect α. Now the mapping class group $\mathrm{Mod}(S)$ acts on the set of isotopy classes of closed curves. If we set $N_X(\alpha, L)$ to be the number of $g \in \mathrm{Mod}(S)$ such that $\ell_X(g(\alpha)) \leq L$ then her theorem is

$$N_X(\alpha, L) \sim B(X)c_\alpha L^{6g-6}.$$

Here $B(X)$ is volume as above and c_α is rational. In fact c_α is the volume in $\mathcal{ML}$ of those laminations λ such that $i(\lambda, \alpha) \leq 1$.

She suggested something much more general holds. Let γ, γ' be filling currents and $N(\gamma, \gamma', L)$ to be the cardinality of the set of $g \in \mathrm{Mod}(S)$ such that $i(g(\gamma), \gamma') \leq L$. Then

$$N(\gamma, \gamma', L) \sim c_\gamma c_{\gamma'} L^{6g-6}.$$

This is more general in that if one takes γ to be the Liouville current defined by a hyperbolic metric X and γ' a filling curve, then one gets the theorem stated above.

Clay Research Awards

The 2015 Research Conference concluded with Landon Clay's presentation of two Research Awards. Maryam Mirzakhani, who had been unable to travel to Oxford in 2014, received the 2014 Award, and Larry Guth and Nets Katz received the 2015 Award.

Howard Masur gave the laudation for Maryam Mirzakhani. He described three of her spectacular 'breakthrough' results on the geometry of Riemann surfaces and their moduli spaces.

- The first is that the number of simple closed geodesics of length at most L on a Riemann surface of genus g is asymptotic to a constant multiple of L^{6g-6} as $L \to \infty$.
- The second concerns *translation surfaces*—surfaces obtained by identifying opposite sides of a polygon which has opposite sides parallel. Mirzakhani, with A. Eskin and A. Mohammadi, showed that the closure of every orbit under a natural action of $\mathrm{SL}\,(2, \mathbb{R})$ on the moduli space an affine submanifold.

Gil Kali spoke by video from Jerusalem about some of the important and groundbreaking contributions of Larry Guth and Nets Katz, under the title *Mathematics without borders*.

- First, on the *Erdős distinct distances problem*: given n points in plane, what is the minimum number $f(n)$ of distnct distances between pairs of points? In 2010, Guth and Katz proved that $f(n)$ is at least $\Omega(n/\log n)$ for large n.
- Second, on the dimension of Kakeya sets (sets containing a unit interval in every direction). In 2002, Katz and Tao proved the lower bound $(2-\sqrt{2})(n-4)+3$.
- Third, on the *cup set problem*: what is maximum size of a set $A \subset (\mathbb{Z}/3\mathbb{Z})^n$ not containing three distinct elements summing to zero. In 2011, Bateman and Katz established the upper bound $3^n/n^{1+\varepsilon}$.

Finally, Kali described a sum-product theorem for finite fields, proved by Bourgain, Katz and Tao, work by Katz and Pavlovic on Navier-Stokes and related equations, Guth's work on systolic and isometric inequalities, and Guth's work with Lubotzky on quantum error correcting codes and 4-dimensional arithmetic hyperbolic manifolds.

Landon Clay with Nets Katz and Larry Guth

Landon Clay with Maryam Mirzakhani

ALGEBRAIC TOPOLOGY: MANIFOLDS UNLOCKING HIGHER STRUCTURES

September 28 – October 2

From one point of view, algebraic topology is the study of higher structures which are ubiquitous in mathematics and nature. Much development of effective machinery in algebraic topology has been motivated by the study of manifolds. While these powerful tools continue to solve deep problems in manifold theory, such as the Mumford conjecture and the Kervaire invariant problem, the tables have also been turned around with manifolds, via the cobordism hypothesis, defining the higher structures of interest and entirely new fields have emerged exploiting these higher structures in topology, geometry, and algebra.

This workshop exposed and explored recent developments and advances in the study of topological moduli spaces, topological field theories, infinity categories, derived algebraic geometry and related topics. Highlights included computations of families of stable and unstable characteristic classes for manifolds bundles for highly connected and aspherical manifolds; homology stability results and the proof of an old conjecture by Bass; a description of extended field theories in dimension three and reflection positivity in such theories.

Photo courtesy of Wadham College

Organizers

Mike Hopkins (Harvard University)
Ulrike Tillmann (University of Oxford)

Speakers

Clark Barwick (Massachusetts Institute of Technology)
Julie Bergner (University of California, Riverside)
Andrew Blumberg (University of Texas, Austin)
John Francis (Northwestern University)
Dan Freed (University of Texas, Austin)
Soren Galatius (Stanford University)
Kathryn Hess (EPFL)
Lars Hesselholt (Nagoya University and University of Copenhagen)
Nick Kuhn (University of Virginia)
Wolfgang Lück (Universität Bonn)
Ib Madsen (University of Copenhagen)
Oscar Randal-Williams (University of Cambridge)
Marco Schlichting (University of Warwick)
Chris Schommer-Pries (MPIM Bonn)
Neil Strickland (University of Sheffield)
Peter Teichner (University of California, Berkeley and MPIM Bonn)
Constantin Teleman (University of Oxford)
Nathalie Wahl (University of Copenhagen)

GEOMETRY AND DYNAMICS ON MODULI SPACES

September 28 – October 2, 2015

The moduli space of curves is important in both algebraic geometry and geometric topology. It also supports some interesting dynamical systems. The dynamics of the SL(2,R)-action and of the Teichmüller geodesic flow on the moduli space of curves, and the asymptotic monodromy of the Hodge bundle along this flow have numerous applications to the dynamics and geometry of measured foliations, to the dynamics of billiards in polygons and of interval exchange transformations, and to the geometry of flat surfaces.

The dynamics on the moduli space and its geometry interact on many levels, some of which are only now emerging. This workshop offered a survey of some of the recent developments in the area. The speakers and participants represented various adjacent domains, including homogeneous dynamics, hyperbolic geometry and dynamics, spaces of representations, integrable systems, and algebraic geometry. The interactions between ideas and techniques coming from these various domains promise to produce further fundamental advances in the core subject of the workshop.

Organizers

Alex Eskin (University of Chicago)
Giovanni Forni (University of Maryland)
Anton Zorich (Université Paris 7)

Speakers

Yves Benoist (Université Paris Sud)
Simion Filip (University of Chicago)
Pascal Hubert (Université Marseille)
Igor Krichever (Columbia University)
François Labourie (Université Paris Sud)
Erwan Lanneau (Institut Fourier)
Elon Lindenstrauss (Hebrew University of Jerusalem)
Howard Masur (University of Chicago)
Carlos Matheus (Université Paris 13)
Martin Möller (Göthe Universität)
Kasra Rafi (University of Toronto)
Caroline Series (University of Warwick)
Corinna Ulcigrai (University of Bristol)
Amie Wilkinson (University of Chicago)
Scott Wolpert (University of Maryland)
Alex Wright (Stanford University)
Jean-Christophe Yoccoz (College de France)
Peter Zograf (Steklov Institute of Mathematics)
Dmitri Zvonkine (Jussieu)

MOTIVES AND AUTOMORPHIC FORMS

September 28 – October 2

The inter-related study of motives and automorphic forms comprises some of the most central ideas and problems in number theory of our times. The arithmetic geometry of Diophantine equations eventually leads, following the well-known philosophy of Grothendieck, to the investigation of their constituent motives which, in turn, should be built from automorphic forms via the Langlands program. This workshop surveyed the latest developments in this research program, especially the interplay between the influence of Archimedean and non-Archimedean geometry.

This workshop was a gathering of the leading experts in Galois representations and automorphic forms, as speakers as well as among the other participants. There was a high degree of diversity in areas of expertise, including the arithmetic geometry of Shimura varieties, deformation theory and lifting problems, the construction of Galois representations, and Euler systems. The workshop provided good opportunities for interaction between senior researchers such as Harris, Ramakrishnan, and Kisin, and early career researchers like Boxer, Viehmann, and Caraiani.

The Oxford meeting is already leading to follow-up events, such as a workshop on the Langlands correspondence in arithmetic and geometry at the Korea Institute of Advanced Study planned for August, 2016, organised by Kim and Shin.

Organizers

Minhyong Kim (University of Oxford)
Peter Scholze (Universität Bonn)

Speakers

Massimo Bertolini (Universität Duisberg-Essen)
George Boxer (Harvard University)
Ana Caraiani (Princeton University)
Laurent Fargues (Jussieu)
Michael Harris (Jussieu)
Mark Kisin (Harvard University)
George Pappas (Michigan State University)
Vytautas Paskunas (Universität Duisberg-Essen)
Vincent Pilloni (ENS Lyon)
Dinakar Ramakrishnan (California Institute of Technology)
Sug Woo Shin (University of California, Berkeley)
Jack Thorne (University of Cambridge)
Bertrand Toën (Université de Toulouse)
Eric Urban (Columbia University)
Eva Viehmann (Technische Universität München)
Jared Weinstein (Boston University)
Sarah Zerbes (University College London)

WATER WAVES AND RELATED FLUID MODELS

September 28 – October 2, 2015

In the past five years, new methods have emerged in the study of global solutions of quasilinear evolutions, inspired by the advances in semilinear theory. The basic idea is to combine the classical energy and vector-fields methods with a new ingredient, namely refined analysis of the Duhamel formula using the Fourier transform method. These new methods led to major progress in understanding the global dynamics of small-data solutions of many physically relevant quasilinear models in 3 and 2 dimensions, including construction of the first nontrivial global solutions in several classical water wave models.

At the same time, new methods have been developed to study the dynamics of large-data solutions, and the formation of certain types of interface singularities, such as the so-called splash singularity, wherein a locally smooth interface self-intersects in finite time.

The workshop brought together senior and junior researchers from the US, UK, France, and Spain. The aim of the workshop was to survey recent developments in the theory of water waves and related fluid models and to discuss further research directions. The main topics included local and global regularity of water wave models, dynamical formation of singularities in interface problems, control theory, and numerical methods. The lectures sparked discussions between researchers in different areas, for example of the subtle role of pseudo-locality in many of these models and on the possible formation and control of curvature singularities in interface problems.

Photo courtesy of Wadham College

Organizers

Alex Ionescu (Princeton University)
Steve Shkoller (University of California, Davis)

Speakers

Thomas Alazard (ENS)
Peter Constantin (Princeton University)
Diego Cordoba (ICMAT)
Daniel Coutand (Heriot Watt University)
Javier Gomez Serrano (Princeton University)
Alex Ionescu (Princeton University)
Alex Kiselev (Rice University)
David Lannes (Université de Bordeaux)
Victor Lie (Purdue University)
Nader Masmoudi (Courant Institute)
Benoit Pausader (Princeton University)
Fabio Pusateri (Princeton University)
Gregory Seregin (University of Oxford)
Jalal Shatah (Courant Institute)
Steve Shkoller (University of California, Davis)
Chongchun Zeng (Georgia Institute of Technology)

Recognizing Achievement

Clay Research Award

The annual Clay Research Awards celebrate the outstanding achievements of the world's most gifted mathematicians. Although perhaps less well known outside the mathematical world than the Clay Millennium Prize Problems, the Clay Research Award is widely appreciated within it.

In 2015 the Clay Research Award was presented jointly to Larry Guth and Nets Katz in recognition of their solution of the Erdős distance problem and for other joint and separate contributions to combinatorial incidence geometry. Their work is an important contribution to the understanding of the interplay between combinatorics and geometry.

Maryam Mirzakhani was awarded a 2014 Clay Research Award for her many and significant contributions to geometry and ergodic theory, in particular to the proof of an analogue of Ratner's theorem on unipotent flows for moduli of flat surfaces. Professor Mirzakhani was presented with her award at the 2015 Clay Research Conference.

Clay Award for Dissemination of Mathematical Knowledge

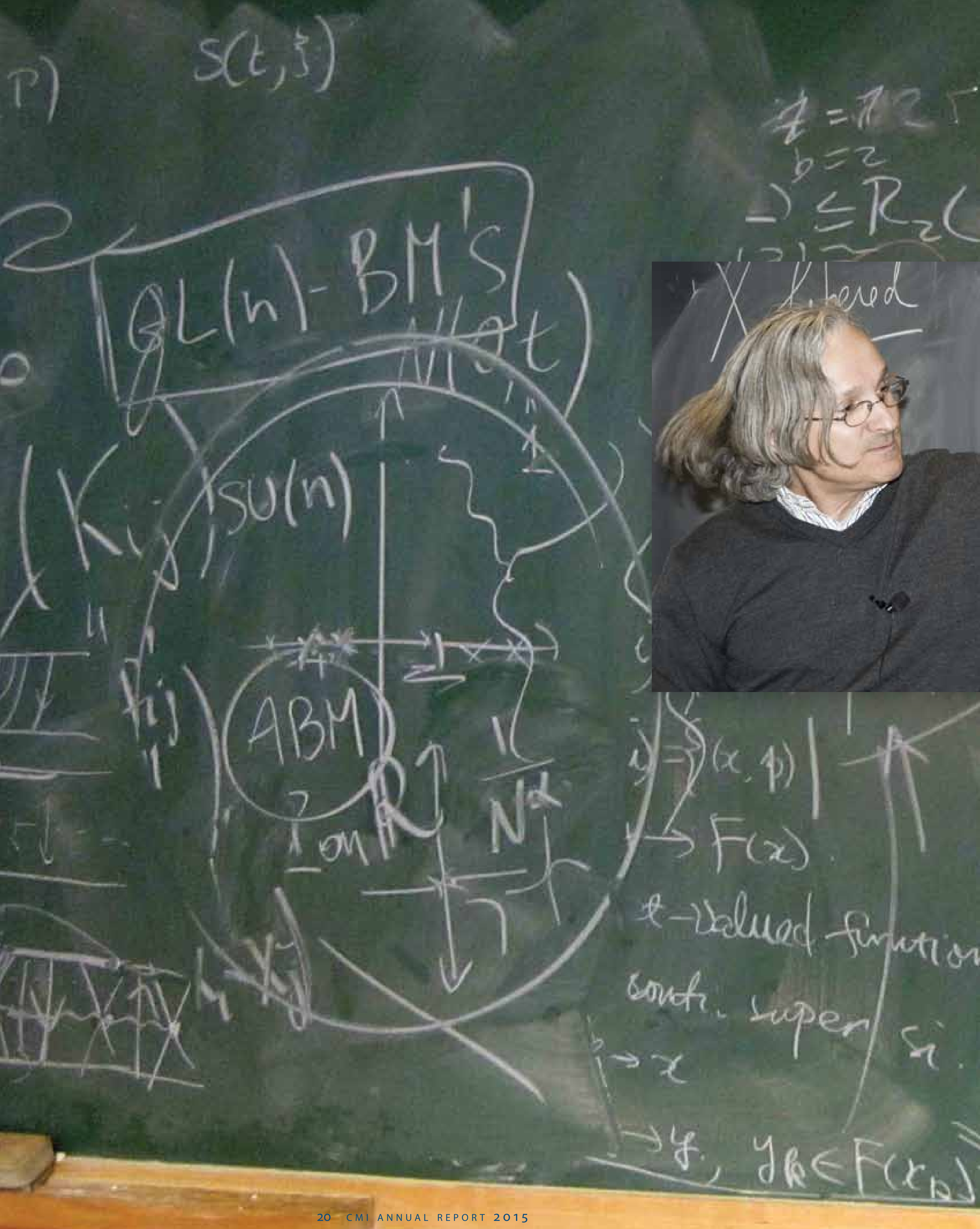
The first Clay Award for Dissemination of Mathematical Knowledge was awarded in 2015 to Étienne Ghys in recognition of his important contributions to mathematical research and for his distinguished work in the promotion of mathematics.

Étienne Ghys

Étienne Ghys is a CNRS Directeur de Recherche at ENS Lyon. He has published outstanding work in his own fields of geometry and dynamics, both under his own name and under the collaborative pseudonym Henri Paul de Saint Gervais. But it is through his work in the promotion of mathematics in France and elsewhere that he has become a legend. He has given numerous carefully crafted lectures to audiences ranging from school children to delegates at the International Congress in 2006. He has enthusiastically embraced modern technology to aid the exposition of deep ideas, for example during his editorship of *Images des mathématiques*, which he transformed to an online publication in 2009, and which received more that five million visits over his five-year term of office. He himself has written more than 980 articles for *Images* as well as a monthly column in *Le Monde*. His series of films, produced with Aurélien Alvarez and Jos Leys and published as DVDs and online in many languages, has had a huge impact on high school students. The first, *Dimensions*, has been downloaded more than a million times.

CLAY RESEARCH AWARDEES

2015	Larry Guth and Nets Katz
2014	Maryam Mirzakhani
	Peter Scholze
2013	Rahul Pandharipande
2012	Jeremy Kahn and Vladimir Markovic
2011	Yves Benoist and Jean-François Quint
	Jonathan Pila
2009	Ian Agol, Danny Calegari and David Gabai
	Jean-Loup Waldspurger
2008	Cliff Taubes
	Claire Voisin
2007	Alex Eskin
	Christopher Hacon and James McKernan
	Michael Harris and Richard Taylor
2005	Manjul Bhargava
	Nils Dencker
2004	Ben Green
	Gérard Laumon and Ngô Bảo Châu
2003	Richard Hamilton
	Terence Tao
2002	Manindra Agrawal
	Oded Schramm
2001	Stanislav Smirnov
	Edward Witten
2000	Alain Connes
	Laurent Lafforgue
1999	Andrew Wiles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SU(n)
ABM
F(x)

Profile

Interview with Étienne Ghys

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I was a very shy boy. Somehow, mathematics offered the possibility to create my own mental world where I could hide, without any need to share it with others. I did not like playing soccer with friends. Amazingly, today my feeling is completely different and I could not do maths without sharing with others (but I continue not playing soccer). Even though, from time to time, I still like to have the possibility of escape from the real world when I need it.

It is very difficult to describe these "earliest memories" since we all have a tendency to reconstruct the past. However, I do remember very clearly a moment in primary school, I was nine or 10 years old, when the teacher asked the pupils for the name of the regular polygon with the biggest number of sides. Complicated answers, like decagon, dodecagon, etc., did not satisfy the teacher. I remember his great smile when I said, without thinking too much, "the circle!" To think of a circle as a polygon with an infinite number of sides is a beautiful mathematical idea—of course, not original at all—but it is unclear to me whether I felt happy because I had a beautiful mathematical idea or because I pleased my teacher. Primary school teachers are so important, and not only for mathematics.

Could you talk about your mathematical education? What experiences and people were especially influential?

Teachers are fundamental. I was very lucky to have at least four great maths teachers, each playing a very important role, at different moments. Libraries are fundamental too. At least they used to be, since today one can easily get information on the internet. I lived in a small city, but with a great public library and a great librarian who helped me choose the right books.

Ultimately, I understood that friends are important, too. I discovered the pleasure of explaining maths to others and helping everybody with their homework, since most of the time it was easy for me. That also helped me to understand better. Later, I had the opportunity to meet people who understood maths much better than me, and I also discovered that it is easier and more pleasurable to learn maths from others than from a library.

Teachers are fundamental. I was very lucky to have at least four great maths teachers, each playing a very important role, at different moments.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I did not have a mentor until very late, not before my postdoc. I really don't know why I was attracted to mathematics, and my parents did not understand either...

From your own experience at high school, are there any aspects of mathematics education that you would like to see changed?

I was in high school at the exact moment of the transition to mathématiques modernes. I believe that this presentation of mathematics was terribly bad for the general population. We were studying set theory, equivalence relations, and a great quantity of formal things, which are of little use in everyday life. Most teachers had no previous training in this "modern way" of presenting mathematics and they were completely lost. Fortunately, this extreme way of teaching mathematics has disappeared today. I still remember the definition of a line (for pupils aged 13): a line is a set X equipped with a family of bijections $f_i : X \rightarrow R$ *such that for any i, j there exists a,b such that* $f_i \circ f_j^{-1}(x) = ax + b$. Totally incomprehensible! This was a terrible experience for almost every pupil—but not for me, I liked that!

What should be changed today? I can only speak about the French system. I wish that the mathematical education could be 1) closer to everyday life and 2) less competitive.

What attracted you to the particular problems you have studied?

Nothing in particular! As a young undergraduate I did not have a clear view of mathematics as a whole, and my choice was more related to personal opportunities. I was looking for the right advisor more than the right topic. I remember my professor falling asleep during the oral presentation of my master's thesis (!) and I decided immediately to choose another professor, and therefore another topic. Of course, later I chose my own path according to my own taste. Very early I began reading papers and books by Poincaré. It was, and it still is, difficult to understand, but it was clear to me that he was a master, and I liked his approach. "C'est par la logique qu'on démontre, c'est par l'intuition qu'on invente."[1] I like that quote!

Can you describe your research in accessible terms?

The honest answer is probably, no. I can't describe it in a few accessible sentences. Speed science (and speed everything) is fashionable today. For instance, I am not a great fan of the competition *My thesis in 180 seconds*. Sometimes you need time to explain things. One could make the same comment on almost every intellectual activity. Mathematics is not always easy, and this difficulty is one of its charms. I would certainly not say to a beginner, "Go and enjoy maths, this is very easy, let me explain everything in five minutes."

Nevertheless, when asked this question, I usually try to explain that I like qualitative mathematics. Most people believe that mathematicians are computers and produce calculations expressed in numbers. Sometimes this is not the case, and these are the situations I like most. For instance, the question of the stability of the Solar System is very interesting and is not related to any precise calculation of the positions of the planets.

Most people believe that mathematicians are computers and produce calculations expressed in numbers. Sometimes this is not the case, and these are the situations I like most.

Another keyword for me is examples. Unlike the French mathematician Bourbaki, I don't like going from the general to the particular, but on the contrary, I like to look at examples first.

What research problems and areas are you likely to explore in the future?

Well, I am a member of the French CNRS (National Center for Scientific Research) and as such I am supposed to write very frequently research projects. Looking back, I see that I (almost) never accomplished what I was planning to accomplish and that I always did something else. Some mathematicians have lifelong projects, but I don't. I do what I can, when I can, and that's not always easy.

Hardy once wrote, "Young men should prove theorems and old men should write books." I am certainly not old (yet) and I usually disagree with Hardy's points of view on mathematics, but he may be right on that specific point.

How did you become involved in dissemination of mathematics?

It was a long process, but there was an acceleration in 2006 after I gave a talk at the ICM in Madrid. Following that talk, which apparently was a success, I received many invitations for "popular lectures" and I enjoyed that a lot.

How important is it to tailor your talk to a specific audience?

I consider this a very important question. You have to know your audience as precisely as possible. The best is when you know them personally. I believe that it is a big mistake to prepare talks "ready to listen by a general audience." I don't like the concept of "general public," which I believe has no precise meaning. Adults, teenagers, children, senior citizens, mathematical colleagues, university colleagues, university students—all these publics deserve a different attention. This implies a close collaboration between the speaker and the organiser of the meeting, which unfortunately is not always the case.

What advice would you give lay persons who would like to know more about mathematics—what it is, what its role in our society has been and is, etc.? What should they read? How should they proceed?

Again, "lay person" is too vague and the answer to this question depends on the person. Today, the problem is not to find information, since the internet has completely changed the landscape. The more important problem is to be able to select the information that is relevant for a given person. The best is to get help from a teacher, a librarian, a friend, or a mathematician who can help you to select the right website or the right books.

How do you think mathematics benefits culture and society?

Good question, but the margin of this page is too small for me to say, as would have said Fermat!

Please tell us about things you enjoy when not doing mathematics.

Not doing mathematics? What do you mean? I don't understand!

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[1] It is through logic that we demonstrate, through intuition that we invent.

Program Overview

Summary of 2015 Research Activities

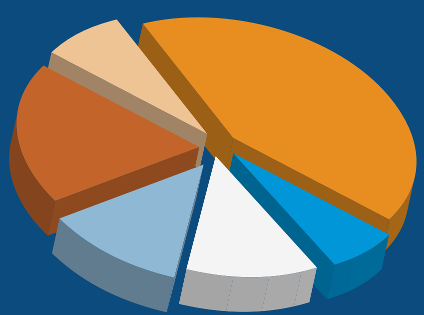
The activities of CMI researchers and research programs are sketched on the following pages. Researchers and programs are selected by the Scientific Advisory Board.

PROGRAM ALLOCATION

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2015:

22	Research Fellows, Research Awardees, Senior Scholars, Research Scholars
160	CMI Workshops
265	Research Schools
55	PROMYS/PROMYS Europe/Ross/PRIMES Faculty and Participants
>200	Participants attending Conferences and Joint Programs

RESEARCH EXPENSES FOR FISCAL YEAR 2015



Research Fellows **42%**
Students (Ross, PROMYS, PRIMES) **7%**
Senior & Research Scholars **12%**
Enhancement and Partnership **12%**
Workshops, Conferences **19%**
Summer and Research Schools **8%**

Researchers

RESEARCH SCHOLARS

Roman Travkin 2012-2015, Harvard University

SENIOR SCHOLARS

Rick Durrett (MBI, Ohio State University) September 2014 – June 2015, *Cancer and its Environment*
Wendelin Werner (INI) January – July 2015, *Random Geometry*
Marc Burger (MSRI) Spring 2015, *Dynamics on Moduli Spaces of Geometric Structures*
Elon Lindenstrauss (MSRI) Spring 2015, *Geometric Arithmetic Aspects of Homogeneous Dynamics*
Jeff Lagarias (ICERM) Spring 2015, *Phase Transitions and Emergent Properties*
Ngô Bảo Châu (PCMI) June 28 – July 18, *Geometry of Moduli Spaces and Representation Theory*
Andrei Okounkov (PCMI) June 28 – July 18, *Geometry of Moduli Spaces and Representation Theory*
Martin Hairer (MSRI) Fall 2015, *New Challenges in PDE*
Pierre Raphaël (MSRI) Fall 2015, *New Challenges in PDE*

John Pardon received his PhD in 2015 from Stanford University under the supervision of Yakov Eliashberg. His most recent work concerns the construction of virtual fundamental cycles on moduli spaces of holomorphic curves in symplectic geometry. He is also interested in geometry and low-dimensional topology. John received his AB in Mathematics from Princeton University in 2011. John has been appointed as a Clay Research Fellow for a term of five years beginning 1 July 2015.

James Maynard received his PhD in 2013 from the University of Oxford under the supervision of Roger Heath-Brown and is currently a Fellow by Examination at Magdalen College, Oxford. James is primarily interested in classical number theory, in particular the distribution of prime numbers. His research focuses on using tools from analytic number theory, particularly sieve methods, to study primes. James has been appointed as a Clay Research Fellow for a term of three years beginning 1 July 2015.

RESEARCH FELLOWS

Ivan Corwin 2012 – 2016, Columbia University and IHP

Semyon Dyatlov 2013 – 2018, Massachusetts Institute of Technology

June Huh 2014 – 2019, Princeton University

James Maynard 2015 – 2018, University of Oxford

John Pardon 2015 – 2020, Stanford University

Aaron Pixton 2013 – 2018, Massachusetts Institute of Technology

Peter Scholze 2011 – 2016, Universität Bonn

Jack Thorne 2013 – 2017, University of Cambridge

Miguel Walsh 2014 – 2018, University of Oxford

Alex Wright 2014 – 2019, Stanford University

CMI Workshops

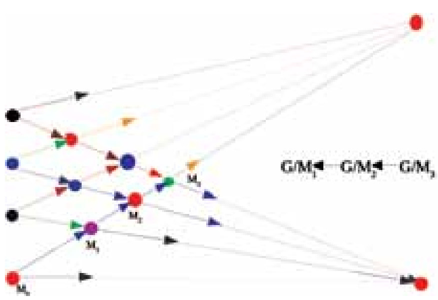
CMI conducts a program of workshops at the Mathematical Institute in Oxford, UK which bring together a small set of researchers quickly, outside the usual grant and application cycle, when this is likely to result in significant progress.

ASYMPTOTIC INVARIANTS OF GROUPS

April 13 – 17, 2015

There has been a great deal of interest among group theorists, topologists, and analysts in invariants of groups which are defined in terms of their finite approximations. This topic is a meeting ground for group theory, topology, ergodic theory and operator algebras. Classical examples include L^2-Betti numbers, cost of measurable group actions and their connections with recently developed notions such as rank gradient, invariant random subgroups, and study of sofic group approximations. There are many open questions, including the various forms of the fixed price conjecture and the modular analogues of Lück's approximation.

Recent progress in this area has been the development of the theory of invariant random subgroups (IRS), which is equivalent to the study of the random stabilizers of probability measure preserving action of a given group. One can also view this notion as the probabilistic generalization of a normal subgroup of a group. This notion has been applied to the study of invariant random subgroups in various classes of groups. A key tool in the case of Lie groups is the Stuck-Zimmer-Nevo theorem which can be viewed as a very strong form of the Margulis rigidity theory for higher rank semisimple Lie groups. This implies in particular that an IRS on a higher rank simple Lie group is induced from the lattice.



This workshop brought together some of the leading experts in these topics and, by presenting the latest advances in this rapidly developing field, stimulated further interactions spanning this diverse range of mathematics. Highlights included talks by A. Lubotzky on topological expenders, W. Lück on universal torsion and the Thurston norm, J. Peterson on solution of Connes' character rigidity conjecture, and A. Jaikin-Zapirain on a proof of the Hanna Neumann conjecture for pro-p groups. The workshop was well-attended by young mathematicians: graduate students and postdocs who benefited by direct contact with the experts.

Organizers

Martin Bridson (University of Oxford)
Nikolay Nikolov (University of Oxford)

Speakers

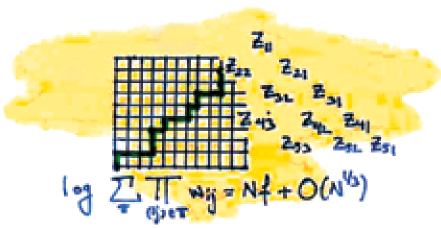
Miklos Abert (Renyi Institute Hungary)
Goulnara Arzhantseva (University of Vienna)
Nir Avni (Northwestern University)
Yiftach Barnea (Royal Holloway University of London)
Ian Biringer (Boston College)
Henry Bradford (University of Oxford)
Pierre-Emmanuel Caprace (UC Louvain)
Gabor Elek (University of Lancaster)
Mikhail Ershov (University of Virginia)
Elisabeth Fink (ENS Paris)
Daniel Franz (University of Virginia)
Tsachik Gelander (Weizmann Institute)
Yair Glasner (Ben Gurion University)
Gili Golan (Bar Ilan University)
Andrei Jaikin-Zapirain (Universidad Autonoma Madrid)
Urban Jezernik (University of Ljubljana)
Martin Kassabov (Cornell University)
Dessislava Kochloukova (Campinas)
François Le Maître (UC Louvain)
Alex Lubotzky (Hebrew University Jerusalem)
Wolfgang Lück (Universität Bonn)
Denis Osin (Vanderbilt University)
Jesse Peterson (Vanderbilt University)
Nicolas Radu (UC Louvain)
Mark Sapir (Vanderbilt University)
Andreas Thom (Leipzig University)
Balint Virag (University of Toronto)

RANDOM POLYMERS AND ALGEBRAIC COMBINATORICS

May 25 – 29, 2015

Random polymers models are the subject of intense research within probability theory and mathematical physics, with increasingly strong connections to algebraic combinatorics, representation theory, and integrable systems. Physically, the models seek to describe phenomena such as interface cracking, turbulence and pinning of magnetic flux lines. Recast in terms of random walks in random potentials, they have also been applied to study stochastic optimization problems such as arise in queuing theory and bioinformatics. The free energy of these models can be mapped onto problems of stochastic growth within the Kardar-Parisi-Zhang universality class.

Recently, there have been breakthroughs to this end as scaling exponents and limit theorems for quantities like the polymer free energy have been proved for a few exactly solvable models. This enhanced level of solvability is due to connections to algebraic structures such as symmetric functions and combinatorial structures such as the Robinson-Schensted-Knuth correspondence. These probabilistic advances have already sparked interesting new directions within the realm of algebraic combinatorics.



The purpose of this workshop was to bring together leading experts and new researchers in random polymers and algebraic combinatorics to help build further bridges between these areas. One of its goals was to try to bridge the language gap between the probabilistic and algebraic communities so as to further develop their rich interaction. In addition to the scheduled talks, informal discussion as well as evening questions sessions, helped to achieve this goal.

Organizers

Ivan Corwin (Columbia University, IHP, CMI)
Nikos Zygouras (University of Warwick)

Speakers

Guillaume Barraquand (Université Paris Diderot)
Dan Betea (Université Pierre et Marie Curie)
Alexey Bufetov (Higher School of Economics)
Reda Chhaibi (Universität Zürich)
Sunil Chhita (Universität Bonn)
Francis Comets (Université Paris Diderot)
Sylvie Corteel (Université Paris Diderot)
Philippe Di Francesco (University of Illinois)
Elnur Emrah (University of Wisconsin, Madison)
Vadim Gorin (Massachusetts Institute of Technology)
Chris Janjigian (University of Wisconsin, Madison)
Kurt Johansson (KTH Stockholm)
Rinat Kedem (University of Illinois)
Anatol N. Kirillov (RIMS University of Kyoto)
Christian Korff (University of Glasgow)
Jeffrey Kuan (Harvard University)
Thomas Lam (University of Michigan)
James Martin (University of Oxford)
Konstantin Matveev (Harvard University)
Vu-Lan Nguyen (Université Paris Diderot)
Mihai Nica (Courant Institute)
Sergey Oblezin (University of Nottingham)
Janosch Ortmann (University of Toronto)
Yuchen Pei (University of Warwick)
Leonid Petrov (University of Virginia)
Alexander Povolotsky (Higher School of Economics)
Dan Romik (University of California, Davis)
Tomohiro Sasamoto (Tokyo Institute of Technology)
Hao Shen (University of Warwick)
Yi Sun (Massachusetts Institute of Technology)

IUT THEORY OF SHINICHI MOCHIZUKI

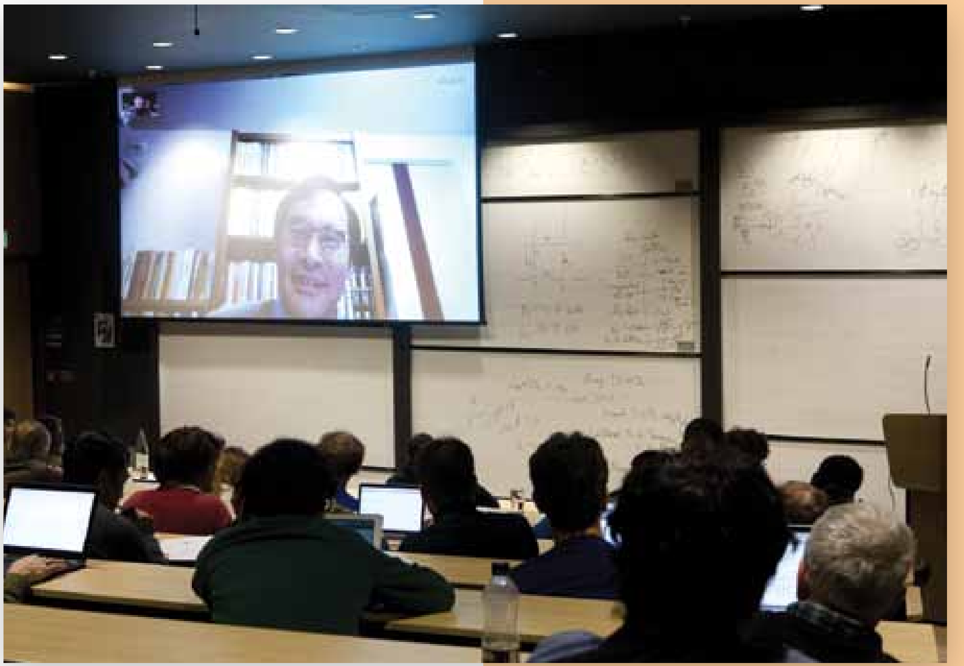
December 7 – 11, 2015

The work of Shinichi Mochizuki on inter-universal Teichmüller theory (arithmetic deformation theory) and its application to famous conjectures in diophantine geometry, including the *abc* conjecture, became publicly available in August 2012. Mochizuki's theory, developed over 20 years, introduces a vast collection of novel ideas, methods, and objects. It operates with full Galois and fundamental groups in a way that no other classical or non-classical theories do. Its objects, such as Frobenioids and theatres, extend scheme theoretical objects outside conventional arithmetic geometry so that arithmetic deformation tools become available. Aspects of the theory have the potential to open new fundamental areas of mathematics.

The aim of this workshop, the first international workshop on the IUT theory of Shinichi Mochizuki, was to present and analyze key principles, concepts, objects and proofs of the theory, to study its relations with existing theories in different areas, to help to increase the number of experts in the theory, and to stimulate its further applications. The purpose was not to evaluate the correctness of the proof, but rather to introduce some of the key concepts involved.

The first three days of the workshop featured lectures on Mochizuki's pre-IUT papers that he later used in IUT. These included talks on a proof of geometric Szpiro inequality inspired by Bogomolov (Zhang), Belyi maps (Kühn), mono-anabelian transport (Hoshi), absolute anabelian geometry (Tan, Stix, Kühne), Frobenioids (Ben-Bassat, Czerniawska), semi-graph anabelioids (Szamuely, Lepage), and Hodge-Arakelov theory (Yamashita). Two lectures by Kedlaya on the étale theta function explained how Mochizuki intended to use Frobenioids in a proof of *abc* and clarified a central concept in his method. Lectures by Mok, Hoshi, and Yamashita on the fourth and fifth days of the workshop covered Mochizuki's four IUT papers.

In addition to a full schedule of lectures, two, two-hour question-and-answer sessions with Shinichi Mochizuki, conducted via Skype, were held on the second and fifth days.



Organizers

Ivan Fesenko (University of Nottingham)
Minhyong Kim (University of Oxford)
Kobi Kremnitzer (University of Oxford)

Speakers

Oren Ben-Bassat (University of Haifa)
Weronika Czerniawska (University of Nottingham)
Ivan Fesenko (University of Nottingham)
Yuichiro Hoshi (RIMS Kyoto University)
Ariyan Javanpeykar (Universität Mainz)
Kiran Kedlaya (University of California, San Diego)
Ulf Kühn (Universität Hamburg)
Lars Kühne (MPIM Bonn)
Emmanuel Lepage (Université Paris 6)
Chung Pang Mok (Purdue University)
Jakob Stix (Universität Frankfurt)
Tamás Szamuely (ARIM Budapest)
Fucheng Tan (Jiao Tong University)
Go Yamashita (RIMS Kyoto University)
Shou-Wu Zhang (Princeton University)

Photo courtesy of Wadham College

PROMYS Europe

UNIVERSITY OF OXFORD

July 11 – August 22, 2015

by Henry Cohn, Microsoft Research

The summer of 2015 marked the beginning of PROMYS Europe, a residential summer program in Oxford for mathematically ambitious secondary school students, which Vicky Neale, Glenn Stevens, Nicholas Woodhouse, and I organize. Modelled after the Program in Mathematics for Young Scientists (PROMYS) at Boston University, PROMYS Europe is a partnership of PROMYS; the Clay Mathematics Institute; the Mathematical Institute at the University of Oxford; and Wadham College, University of Oxford. In this summer program, carefully selected students from around Europe gather at Wadham College for six weeks of intensive mathematical activities.

The goal of PROMYS Europe is neither acceleration nor contest preparation, but rather for each student to have a deep experience of discovering mathematics. We don't teach them mathematics, but rather welcome them to a community that will support them in doing mathematics. Each morning, the students receive a number theory problem set to guide them on their journey. The problems push them to make computations, gather data, identify patterns, formulate conjectures, and prove theorems. The problem sets are intentionally flexible and open-ended, while pointing the students in the direction of some of the highlights of 19th century mathematics, such as quadratic reciprocity.

Glenn Stevens and I also teach a morning number theory class, which intentionally stays several days behind the problem sets. By the time we reach a topic in class, the students will already have worked out the mathematics for themselves. Our role is not to teach number theory, but rather to help the students systematize and reflect on their discoveries.

For many of the students, PROMYS Europe is the first time they have ever been pushed to their limits intellectually. The problem sets are difficult, perhaps unreasonably difficult, and students cannot expect to solve all the problems. Instead, mathematics at PROMYS Europe is much like research mathematics: further insights are always possible, and there's no notion of being "done" with a topic. This can be an unsettling experience for the students, but it's a valuable shift in perspective.

In 2015, PROMYS Europe started small, with twelve first-year students, four returning students (former participants in PROMYS and the CMI-PROMYS Alliance), and five university students as counsellors. Over time we expect the program to grow, and we intend to build a community that extends beyond the bounds of any single summer and brings together participants at many different career stages, ranging from high school students to senior faculty. Everyone in the PROMYS Europe community is actively engaged in mathematics themselves; for example, the counsellors have their own seminar, which in 2015 was on elliptic curves.

Returning students are a key part of building this community. They extend their understanding of number theory, but they also need further mathematical experiences, and Vicky Neale's course plays this role. In 2015 she taught a graph theory course, which took the students from an elementary beginning to deep concepts such as Szemerédi regularity; this coming summer she will teach group theory. In addition to their target audience of returning students, these courses are also open to first-years looking for an additional challenge.

Returning students also take on mini-research projects in small groups. In 2015, they

Photos courtesy of Wadham College

explored a problem on modular representation theory proposed by Laurent Berger and Sandra Rozensztajn and a problem on graph coloring proposed by David Conlon.

As a break from the formal courses, PROMYS Europe has a program of guest lectures by well-known mathematicians, such as Andrew Wiles and Ben Green. These visits give students the opportunity to interact with a wide variety of mathematicians.

PROMYS Europe is still early in its development, but I'm excited by the start we have made in 2015, and I'm eager to see how the program grows and develops over time. This quote from a PROMYS Europe student illustrates the perspective we wish to encourage:

"PROMYS Europe is something different from how I had done mathematics before. Previously I had always learnt strategies and techniques and ideas and applied these to solving problems in order to get good marks in a test. PROMYS however is all about exploration and learning through experimentation, building the tools to ask and then tackle progressively more difficult and interesting questions. This makes the work deeply satisfying, because the results one discovers not only feel but truly are deep and meaningful mathematics."

LMS/CMI Research Schools

Following the success of 2014's program, CMI again partnered with the London Mathematical Society to deliver four week-long research schools at various locations in the United Kingdom. In 2015, the schools attracted a very strong pool of applicants from around the world, with a total of 288 participants from 25 countries attending.

STATISTICAL PROPERTIES OF DYNAMICAL SYSTEMS

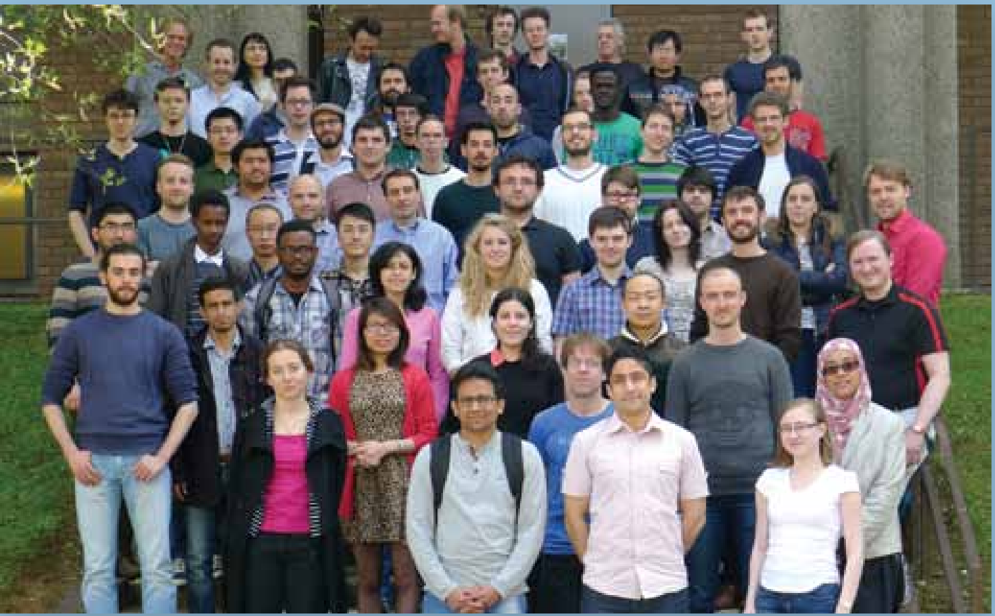
April 13 – 17, 2015 | University of Loughborough

Dynamical systems is a modern mathematical field concerned with studying phenomena that evolve over time. In dynamical systems, the time-evolution of the process is modelled by the iterates of a map (or a flow). Most systems of interest are chaotic, i.e. their orbits are sensitive dependent to initial conditions. Consequently, the long-term behaviour of such systems cannot be predicted by following orbits. Hence, a statistical approach is the best candidate to tackle such unpredictable systems.

This research school provided PhD students with knowledge and training on the basic tools needed to study statistical problems of dynamical systems. Three lecture courses, supported by tutorial sessions, covered:

- *Fast-slow skew product systems and convergence to stochastic differential equations* by Ian Melbourne
- *Strong statistical properties of hyperbolic and partially hyperbolic dynamical systems* by Carlangelo Liverani
- *Open systems: fundamental questions and emerging perspectives* by Mark Demers

During the first two days, lectures focused on general background and tools needed, while more advanced topics, related to current advances in the area, were presented later in the week. The school also hosted an LMS 150th Anniversary competition, at which four students from among 14 submissions were selected to present their work. Following the competition, guest lecturer Mark Pollicott spoke on dynamical quantities and their estimation.



Organizer

Wael Bahsoun (University of Loughborough)

Lecturers

Mark Demers (Fairfield University)
Carlangelo Liverani (Università di Roma Tor Vergata)
Ian Melbourne (University of Warwick)

Guest Lecturer

Mark Pollicott (University of Warwick)

Photo by Elena Heaviside

REGULARITY AND ANALYTIC METHODS IN COMBINATORICS

July 1 – 5, 2015 | University of Warwick

Organizers
Peter Keevash (University of Oxford)
Daniel Kráľ (University of Warwick)
Oleg Pikhurko (University of Warwick)

Lecturers
Christian Borgs (Microsoft)
Henry Cohn (Microsoft)
David Conlon (University of Oxford)
Asaf Shapira (Tel Aviv University)

Guest Lecturers
Noga Alon (Tel Aviv University)
Ben Green (University of Oxford)

The Regularity Methods of Szemerédi gives a way to approximate a large combinatorial object by one of a bounded size. The method has many applications inside and outside mathematics, with those in number theory and in theoretical computer science being most often cited. It also forms one of the foundations of the (recently emerged) theory of combinatorial limits, which led to substantial progress on many classical problems in extremal combinatorics.

The research school consisted of three lecture courses, each covering recent exciting developments in one of three interlinked discrete mathematics topics on the interface with computer science:

- *Regularity of methods* by David Conlon
- *Limits of combinatorial structures* by Christian Borgs and Henry Cohn
- *Property testing* by Asaf Shapira

The course lectures were complemented by more general talks by Noga Alon, *Easily testable graph properties*, Ben Green, *Arithmetic regularity lemmas*, and Christian Borgs, *From graph limits to non-parametric graph estimation*.

DEVELOPMENTS IN MODERN PROBABILITY

July 5 – 10, 2015 | University of Oxford

Organizers
Dmitry Belyaev (University of Oxford)
Alison Etheridge (University of Oxford)
Christina Goldschmidt (University of Oxford)
Ben Hambly (University of Oxford)

Lecturers
Nathanaël Berestycki (University of Cambridge)
Paul Bourgade (Courant Institute)
Ivan Corwin (Columbia University, IHP, CMI)

Guest Lecturer
Steve Evans (UC Berkeley)

This research school was designed to tie in with the annual international conference in probability, *Stochastic Processes and their Applications Conference (SPA2015)*, which took place the following week. The school provided PhD students and early career researchers with background on some of the important topics in modern probability which would feature prominently in the plenary talks at *SPA2015*.

Three five-hour lecture courses, each supported by tutorial sessions, covered some of the most exciting areas of current research in probability.

- *Two-dimensional Liouville quantum gravity and the Gaussian free field* by Nathanaël Berestycki
- *Random matrices and PDEs* by Paul Bourgade
- *Integrable probability and the KPZ universality classes* by Ivan Corwin

In addition to the course lectures, the students were offered a talk on *The fundamental theorem of arithmetic for metric measure spaces* by guest lecturer Steve Evans.

Photo by Tim Dokchitser

Organizers

Tim Dokchitser (University of Bristol)
Vladimir Dokchitser (University of Warwick)

Lecturers

Tim Dokchister (University of Bristol)
Vladimir Dokchister (University of Warwick)
Andrew Granville (University of Montreal and University College London)
Jennifer Park (McGill University)
Alexei Skorobogatov (Imperial College London)
Michael Stoll (Jacobs University Bremen)
Trevor Wooley (University of Bristol)

Guest Lecturer

John Coates (University of Cambridge)

DIOPHANTINE EQUATIONS

September 14 – 20, 2015 | **Baskerville Hall, Hay-on-Wye**

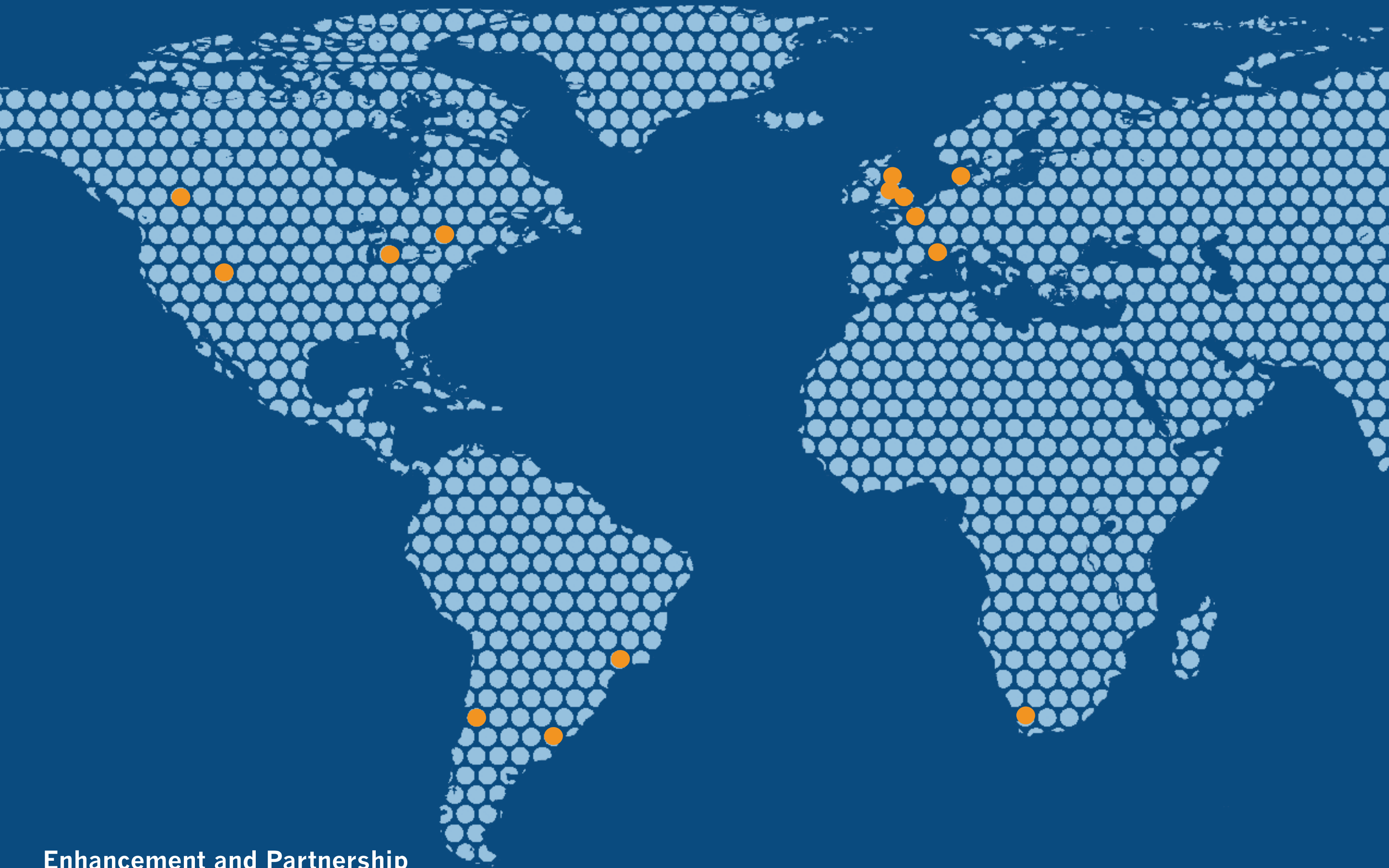
The aim of this research school was to present an overview of all the existing methods for studying diophantine equations, at a level accessible to a first-year graduate student. This is a vast topic, covering major parts of both algebraic and analytic number theory, but this is what made the research school useful. Many people working in algebraic number theory have little or no idea of what is happening in analytic number theory, algebraic geometry or the logic theory related to Hillbert's tenth problem, and the same appears to be true of those working in these other areas. Such an overview of the various techniques and recent advances proved very helpful to number theorists.

The school consisted of six mini-courses, with each lecturer setting at least one problem for each of the lectures:

- *Elliptic curves* by Tim Dokchitser and Vladimir Dokchitser
- *ABC conjecture* by Andrew Granville
- *Curves and Jacobians* by Michael Stoll
- *Higher-dimensional varieties* by Alexei Skorobogatov
- *Circle method* by Trevor Wooley
- *Integral points* by Jennifer Park

The afternoons were spent solving the problems and discussing them. By the end of the week, the participants has a good overview of the tools available in the different branches of number theory for working with diophantine equations and, through the problem sessions, gained some experience of using them.

The school also included a talk by guest lecturer John Coates, who spoke about the history of Diophantine equations, the congruent number problem, and its connections to the topics of the research school.



Enhancement and Partnership

CMI's Enhancement and Partnership Program aims to add value to activities that have already been planned, particularly by increasing international participation. In accordance with CMI's mission and its status as an operating foundation, its funding is utilized to enhance mathematical activities organized by, or planned in partnership with, other organizations. In 2015, CMI partnered in 16 initiatives in nine countries, often by funding a distinguished international speaker or supporting participants from outside the host country.

January 19 – 23 | **AIMS-Stellenbosch University Number Theory Conference**
AIMS, Cape Town, South Africa

February 9 – 20 | **X Americas Conference on Differential Equations and Nonlinear Analysis**
University of Buenos Aires, Argentina

March 9 | **SET for Britain** | London, UK

June 1 – 5 | **Non-Archimedean Geometry and its Applications** | University of Michigan, Ann Arbor, MI

June 15 – July 11 | **PIMS Summer School in Probability** | McGill University, Montreal, Canada

June 15 – 26 | **International Conference in Number Theory and Physics** | IMPA, Rio de Janeiro, Brazil

June 22 – 26 | **Arithmetic Geometry, Representation Theory and Applications**
CIRM, Luminy, France

July 7 – 10 | **Future Directions in Model Theory and Analytic Functions** | University of Manchester, UK

July 12 – 31 | **Algebraic Geometry Summer Research Institute** | University of Utah, Salt Lake City, UT

July 20 – 24 | **Advances in Geometric Analysis**
University of Warwick, UK

July 27 – August 1 | **XVIII International Congress on Mathematical Physics (ICMP 2015)** | Santiago, Chile

August 3 – 12 | **Center for Quantum Geometry on Moduli Spaces Summer School** | University of Aarhus, Denmark

September 23 – 25 | **Algebraic Geometry**
IHES, Bures-sur-Yvette, France

October 26 – 30 | **Moduli Spaces in Geometry**
CIRM, Luminy, France

November 22 – 27 | **BIRS International Math Outreach Workshop** | BIRS, Banff, Canada

December 9 – 10 | **Ada Lovelace Symposium**
University of Oxford, UK

Publications

Selected Articles by Research Fellows

Ivan Corwin

Stochastic six-vertex model, with Alexei Borodin and Vadim Gorin, to appear in *Duke Mathematics Journal*. arXiv: 1407.6729

Stochastic higher spin vertex models on the line, with Leonid Petrov, to appear in *Communications in Mathematical Physics*. arXiv: 1502.07374

Semyon Dyatlov

Spectral gaps, additive energy, and a fractal uncertainty principle, with Joshua Zahl. arXiv: 1504.06589

Improved fractal Weyl bounds for hyperbolic manifolds, with an appendix by David Borthwick and Tobias Weich. arXiv: 1512.00836

June Huh

Hodge theory for combinatorial geometries, with Karim Adiprasito and Eric Katz. arXiv: 1511.02888

A tropical approach to the strongly positive Hodge conjecture, with Farhad Babaee. arXiv: 1502.00299

James Maynard

Primes represented by incomplete norm forms, submitted. arXiv: 1507.05080

Chains of large gaps between primes, with Kevin Ford and Terence Tao, submitted. arXiv: 1511.04468

John Pardon

Contact homology and virtual fundamental cycles, submitted. arXiv: 1508.03873

Existence of Lefschetz fibrations on Stein and Weinstein domains, with Emmanuel Giroux, submitted. arXiv: 1411.6176

Aaron Pixton

Gromov-Witten/Pairs correspondence for the quintic 3-fold, with Rahul Pandharipande. To appear in *Journal of the American Mathematical Society*. arXiv: 1206.5490

Double ramification cycles on the moduli spaces of curves, with Felix Janda, Rahul Pandharipande and Dmitri Zvonkine. arXiv: 1602.04705

Peter Scholze

On the generic part of the cohomology of compact unitary Shimura varieties, with Ana Caraiani. http://www.math.uni-bonn.de/people/scholze/Torsion-vanishing.pdf

Projectivity of the Witt vector affine Grassmannian, with Bhargav Bhatt. http://www.math.uni-bonn.de/people/scholze/WittGrass.pdf

Jack Thorne

Elliptic curves over Q_∞ are modular, submitted. arXiv: 1505.04769

Torsion Galois representations over CM fields and Hecke algebras in the derived category, with James Newton, submitted. arXiv: 1511.04913

Miguel Walsh

Bounded rational points on curves, *International Mathematics Research Notes*, 2015, no. 14, 5644-5658.

The algebraicity of ill-distributed sets, *Geometric and Functional Analysis*, 24 (2014), 959-967.

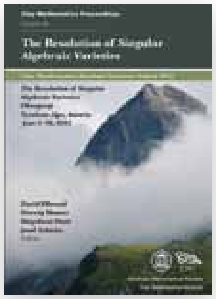
Alex Wright

The boundary of an affine invariant submanifold, with Maryam Mirzakhani, submitted. arXiv: 1508.01446

A smooth mixing flow on a surface with non-degenerate fixed points, with Jon Chaika, submitted. arXiv: 1501.02881

Books

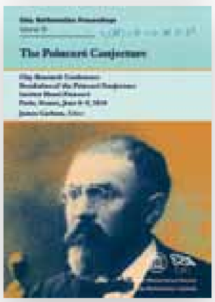
The Resolution of Singular Algebraic Varieties



Editors: David Ellwood, Herwig Hauser, Shigefumi Mori and Josef Schicho. CMI/AMS, 2014, 340 pp., softcover, ISBN: 0-8218-8982-4. List price: $101. AMS Members: $80.80. Order Code: CMIP/20.

Resolution of singularities has long been considered a difficult to access area of mathematics. The more systematic and simpler proofs that have appeared in the last few years in zero characteristic now give us a much better understanding of singularities. They reveal the aesthetics of both the logical structure of the proof and the various methods used in it. This volume is intended for readers who are not yet experts but always wondered about the intricacies of resolution. As such, it provides a gentle and quite comprehensive introduction to this amazing field. The book may tempt the reader to enter more deeply into a topic where many mysteries—especially the positive characteristic case—await discovery.

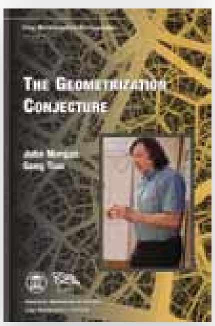
The Poincaré Conjecture



Editor: James Carlson. CMI/AMS, 2014, 178 pp., softcover, ISBN: 0-8218-9865-5. List price: $69. AMS Members: $55.20. Order Code: CMIP/19.

The conference to celebrate the resolution of the Poincaré conjecture, one of CMI's seven Millennium Prize Problems, was held at the Institut Henri Poincaré in Paris. Several leading mathematicians gave lectures providing an overview of the conjecture—its history, its influence on the development of mathematics, and its proof. This volume contains papers based on the lectures at that conference. Taken together, they form an extraordinary record of the work that went into the solution of one of the great problems of mathematics.

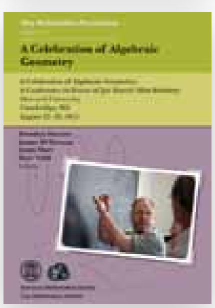
The Geometrization Conjecture



Authors: John Morgan and Gang Tian. CMI/AMS, 2014, 291 pp., hardcover, ISBN: 0-8218-5201-9. List price: $81. AMS Members: $64.80. Order Code: CMIM/5.

This book gives a complete proof of the geometrizaton conjecture, which describes all compact 3-manifolds in terms of geometric pieces, i.e., 3-manifolds with locally homogeneous metrics of finite volume. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. In the course of proving the geometrization conjecture, the authors provide an overview of the main results about Ricci flows with surgery on 3-dimension manifolds, introducing the reader to difficult material. The book also includes an elementary introduction to Gromov-Hausdorff limits and to the basics of the theory of Alexandrov spaces. In addition, a complete picture of the local structure of Alexandrov surfaces is developed.

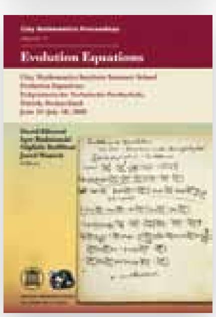
A Celebration of Algebraic Geometry



Editors: Brendan Hassett, James McKernan, Jason Starr and Ravi Vakil. CMI/AMS, 2013, 599 pp., softcover, ISBN: 0-8218-8983-4. List Price: $149. AMS Members: $119.20. Order Code: CMIP/18.

This volume resulted from the conference held in honor of Joe Harris' 60th birthday. Harris is famous around the world for his lively textbooks and enthusiastic teaching, as well as for his seminal research contributions. The articles are written in this spirit: clear, original, engaging, enlivened by examples, and accessible to young mathematicians. The articles focus on the moduli space of curves and more general varieties, commutative algebra, invariant theory, enumerative geometry both classical and modern, rationally connected and Fano varieties, Hodge theory and abelian varieties, and Calabi-Yau and hyperkähler manifolds. Taken together, they present a comprehensive view of the long frontier of current knowledge in algebraic geometry.

Evolution Equations



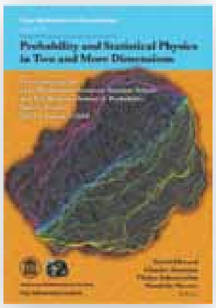
Editors: David Ellwood, Igor Rodnianski, Gigliola Staffilani and Jared Wunsch. CMI/AMS, 2013, 572 pp., softcover, ISBN: 0-8218-6861-6. List Price: $149. AMS Members: $119.20. Order Code: CMIP/17.

This volume is a collection of notes from lectures given at the 2008 Clay Mathematics Institute Summer School, held in Zurich, Switzerland. The lectures were designed for graduate students and mathematicians within five years of their PhD and the main focus of the program was on recent progress in the theory of evolution equations. Such equations lie at the heart of many areas of mathematical physics and arise not only in situations with a manifest time evolution (such as nonlinear wave and Schrödinger equations) but also in the high energy or semi-classical limits of elliptic problems.

Topics in Noncommutative Geometry

Editor: Guillermo Cortiñas. CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16.

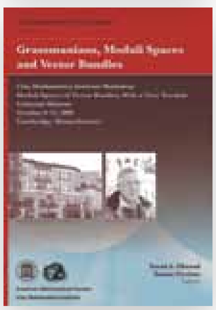
This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.



Probability and Statistical Physics in Two and More Dimensions

Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner. CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15.

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, “Probability and Statistical Physics in Two and More Dimensions” and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.



Grassmannians, Moduli Spaces and Vector Bundles

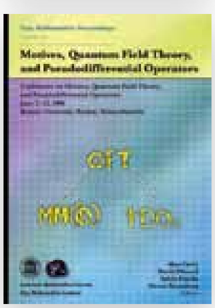
Editors: David A. Ellwood, Emma Previato. CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $58. AMS Members: $46.40. Order Code: CMIP/14.

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

On Certain *L*-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu. CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $136. AMS Members: $108.80. Order Code: CMIP/13.

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their L-functions, and both local and global theory are addressed.



Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg. CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $94. AMS Members: $75.20. Order Code: CMIP/12.

This volume contains articles related to the conference “Motives, Quantum Field Theory, and Pseudodifferential Operators” held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

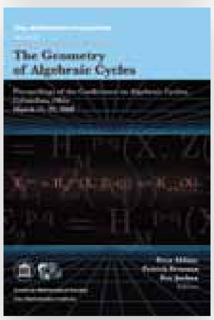
Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa. CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $136. AMS Members: $108.80. Order Code: CMIP/11.

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Kleinbock, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz. CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $104. AMS Members: $83.20. Order Code: CMIP/10.

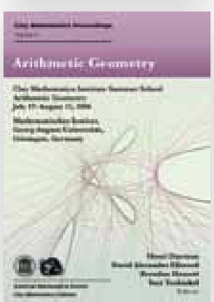
This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.



The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua. CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $55. AMS Members: $44. Order Code: CMIP/9.

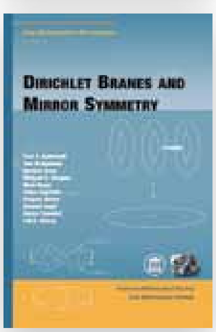
The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.



Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel. CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $125. AMS Members: $100. Order Code: CMIP/8.

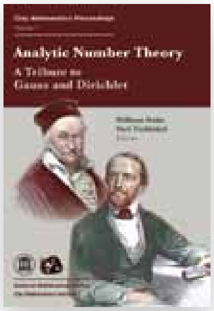
This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.



Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross. CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $115. AMS Members: $92. Order Code: CMIM/4.

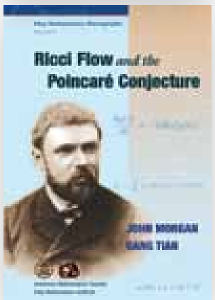
The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.



Analytic Number Theory: A Tribute to Gauss and Dirichlet

Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $53. AMS Members: $42.40. Order Code: CMIP/7.

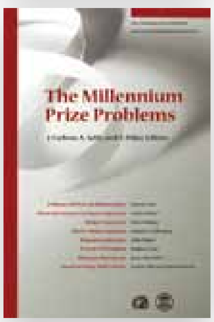
This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture

Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $75. AMS Members: $60. Order Code: CMIM/3.

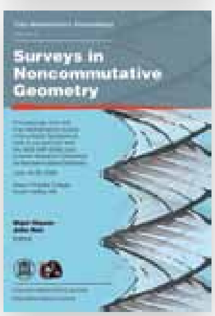
This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.



The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $32. AMS Members: $25.60. Order Code: MPRIZE.

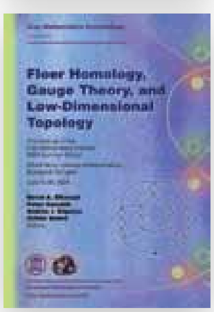
This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.



Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $53. AMS Members: $42.40. Order Code: CMIP/6.

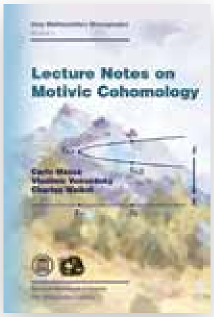
In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.



Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $70. AMS Members: $56. Order Code: CMIP/5.

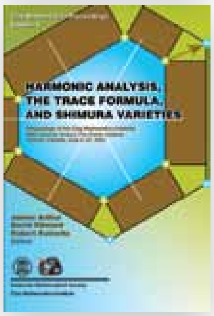
This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $50. AMS Members: $40. Order Code: CMIM/2.S.

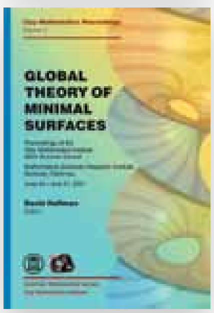
This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.



Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $138. AMS Members: $110.40. Order Code: CMIP/4.

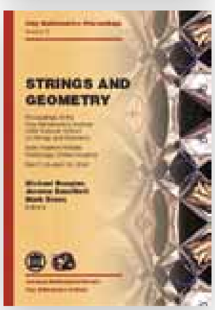
The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.



Global Theory of Minimal Surfaces

Editor: David Hoffman. CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $138. AMS Members: $110.40. Order Code: CMIP/2

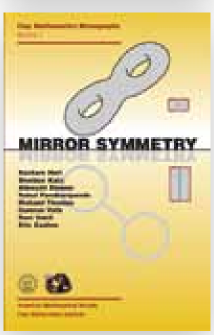
This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and thedouble-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.



Strings and Geometry

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $80. AMS Members: $64. Order Code: CMIP/3.

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.



Mirror Symmetry

Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $144. AMS Members: $115.20. Order Code: CMIM/1

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $91. ASM Members: $72.80. Order Code: CMIP/1.

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

To order print copies of these books, please visit www.ams.org/bookstore. PDF versions are posted on CMI's Online Library six months after publication and can be found at www.claymath.org/node/262.

Digital Library

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Ada Lovelace Papers

Often called the first computer programmer, Ada Lovelace is celebrated for her pioneering work on programming Charles Babbage's Analytical Engine. These papers offer a rounded picture of the development of Lovelace's mathematical and scientific interests and include both sides of the extensive correspondence between Ada and the mathematician Augustus DeMorgan. CMI is very grateful to Ada's descendent, the Earl of Lytton, for his family's permission to undertake this project. High resolution images will be available through the Bodleian Library, University of Oxford.

www.claymath.org/ada-lovelaces-mathematical-papers

Quillen Notebooks

Daniel Quillen obtained his PhD under the supervision of Raoul Bott at Harvard in 1961. He worked at MIT before moving to the University of Oxford in 1984. During his long mathematical career, Quillen kept a set of detailed notes which give a day-to-day record of his mathematical research.

www.claymath.org/publications/quillen-notebooks

Euclid's Elements

The manuscript MS D'Orville 301 contains the thirteen books of Euclid's Elements, copied by Stephen the Clerk for Arethas of Patras in Constantinople in 888 AD. It is kept in the Bodleian Library at the University of Oxford where high resolution copies of the manuscript are available for study.

www.claymath.org/euclids-elements

Riemann's 1859 Manuscript

Bernhard Riemann's paper, Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse (On the number of primes less than a given quantity), was first published in the Monatsberichte der Berliner Akademie, in November 1859. Just six manuscript pages in length, it introduced radically new ideas to the study of prime numbers.

www.claymath.org/publications/riemanns-1859-manuscript

Klein Protokolle

The “Klein Protokolle,” comprising 8600 pages in 29 volumes, is a detailed handwritten registry of seminar lectures given by Felix Klein, his colleagues and students, and distinguished visitors in Göttingen for the years 1872-1912.

www.claymath.org/publications/klein-protokolle

James Arthur Archive

James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur’s professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author’s complete published work in an easily accessible set of searchable PDFs.

www.claymath.org/publications/collected-works-james-g-arthur

High resolution copies of the manuscript are available for study at the Bodleian Library, University of Oxford.

Notes of Talks at the I. M. Gelfand Seminar

The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin’s notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education.

www.claymath.org/publications/notes-talks-imgelfand-seminar

Nominations, Proposals and Applications

Research Fellowship Nominations

Nominations for Clay Research Fellows are considered once a year. The primary selection criteria for the Fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader. Selection decisions are made by the Scientific Advisory Board based on the nominating materials: letter of nomination, names and contact information for two other references, Curriculum Vitae, publication list for the nominee.

Address all nominations to Nick Woodhouse at president@claymath.org, copied to Naomi Kraker at admin@claymath.org.

Workshops at the Mathematical Institute

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty people, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. An application submitted three months before the workshop is sufficient. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

Enhancement and Partnership

The Clay Mathematics Institute invites proposals under its Enhancement and Partnership Program. The aim is to enhance activities that are already planned, particularly by funding international participation. The program is broadly defined, but subject to the general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board.

Nominations for Senior Scholars will be considered within the scope of the Enhancement and Partnership Program. The aim of the Senior Scholar program is to foster mathematical research and the exchange of ideas by providing support for senior mathematicians who will play a leading role as "senior scientist" in a topical program at an institute or university. Senior Scholars will be in residence throughout the program and are expected to interact extensively with the other participants.

For more information, visit www.claymath.org/programs/enhancement-and-partnership-program. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

Annual Deadlines

- Research Fellows nominations: November 16
- Workshop proposals: March 1, June 1, September 1, December 1
- Enhancement and Partnership proposals, including Senior Scholars nominations: March 1, June 1, September 1, December 1

Nominations and proposals may also be mailed to:

Clay Mathematics Institute
Office of the President
Andrew Wiles Building
Radcliffe Observatory Quarter
Woodstock Road
Oxford OX2 6GG
United Kingdom

2016 Institute Calendar

Jan 1 – Dec 31	PRIMES	MIT, Cambridge, MA
Jan 4 – 8	AIMS Conference on Quadratic Differentials	AIMS, Cape Town, South Africa
Jan 25 – 29	Analysis, PDEs and Geometry	Princeton University, NJ
Spring 2016	Senior Scholar Tobias Colding, Differential Geometry	MSRI, Berkeley, CA
March 7	SET for Britain 2016	London, UK
Mar – July	Constructive Approximation and Harmonic Analysis	CRM, Barcelona, Spain
Mar 20 – 24	Explicit p-Adic Methods in Number Theory Workshop (Sage Days 71)	University of Oxford, UK
Mar 21 – 24	British Mathematical Colloquium	University of Bristol, UK
Mar 21 – 25	Kähler Geometry, Einstein Metrics and Generalizations	MSRI, Berkeley, CA
April 5 – 8	British Applied Mathematics Colloquium	University of Oxford, UK
April 10 – 16	European Girls Mathematical Olympiad 2016	Bușteni, Romania
May 2 – 6	Geometric Flows in Riemannian and Complex Geometry	MSRI, Berkeley, CA
May 23 – June 6	Geometric Group Theory and Low-Dimensional Topology	ICTP, Trieste, Italy
June 13 – 16	*L*-functions and Arithmetic	Harvard University, Cambridge, MA
June 13 – 17	Dynamics, Geometry and Number Theory	IHP, Paris, France
June 13 – 17	Leuca 2016	Patù, Italy
June 20 – 24	LMS-CMI Research School: Developments in Contact and Symplectic Topology	University of Glasgow, UK
June 27 – July 1	EMS/EWM Summer School: Geometric and Physical Aspects of Trudinger-Moser Type	Institute Mittag-Leffler, Djursholm, Sweden
June 27 – July 2	String-Math 2016	College de France, Paris, France
July 4 – 8	LMS-CMI Research School: Modern Topics in Nonlinear PDE and Geometric Analysis	University of Reading, UK

July 4 – Aug 13	PROMYS	Boston University, MA
July 10 – 20	Summer School in Rational Homotopy Theory and its Interactions	Rabat, Morocco
July 10 – Aug 20	PROMYS Europe	University of Oxford, UK
July 17 – 20	EMS/EWM Survey Lectures 2016	TU Berlin, Germany
July 18 – 29	IHES Summer School on Non-linear Waves	IHES, Bures-sur-Yvette, France
Aug 15 – Dec 16	Senior Scholar Karen Vogtmann, *Geometric Group Theory*	MSRI, Berkeley, CA
Sept 1 – April 30, 2017	Senior Scholar Dmitry Orlov, *Homological Mirror Symmetry*	IAS, Princeton, NJ
Sept 5 – 16	Hitchin70	Aarhus Univ., Denmark; Univ. of Oxford, UK; ICMAT, Spain
Sept 5 – Dec 16	Large Cardinals and Strong Logics	CRM, Barcelona, Spain
Sept 12 – 23	Quantum Integrable Systems, Conformal Field Theories and Stochastic Processes	Institute d'Etudes Scientifiques de Cargèse, Corsica
Sept 25 – Oct 2	Women in Numbers Europe 2	Sirince, Turkey
Sept 28	Clay Research Conference	University of Oxford, UK
Sept 26 – 30	Algebraic Geometry: Old and New	University of Oxford, UK
Sept 26 – 30	Geometric Representation Theory and Beyond	University of Oxford, UK
Sept 26 – 30	Mean Curvature Flow	University of Oxford, UK
Sept 26 – 30	Recent Developments on Elliptic Curves	University of Oxford, UK
Fall 2016	Thematic Program on Combinatorial Algebraic Geometry	Fields Institute, Toronto, Canada
Oct 1 – Dec 15	Senior Scholar Pham Huu Tiep, *Local Representation Theory and Simple Groups*	EPFL Lausanne, Switzerland
Dec 5 – 9	Deformation Theory, Completed Cohomology, Leopoldt Conjecture and K-theory	CIRM, Luminy, France
Dec 5 – 9	Generalised Geometry & Noncommutative Algebra Workshop	University of Oxford, UK
Dec 6 – 9	Amenability, Coarse Embeddability and Fixed Point Properties Workshop	MSRI, Berkeley, CA

CMI

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CLAY MATHEMATICS INSTITUTE

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PRESIDENT'S OFFICE

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