CLAY MATHEMATICS INSTITUTE

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ANNUAL REPORT

2014

CLAY MATHEMATICS INSTITUTE

contents

LETTER FROM THE PRESIDENT

Each year, the CMI appoints two or three Clay Research Fellows. All are recent PhDs, and most are selected as they complete their theses. Their fellowships provide a generous stipend, research funds, and the freedom to carry on research for up to five years anywhere in the world and without the distraction of teaching and administrative duties. There are 31 alumni of this program, and eight fellows in post, with two more, John Pardon and James Maynard, joining them in July.

The extraordinary distinction and achievements of this group were evident at the International Congress of Mathematicians in Seoul in the summer of 2014: thirteen were invited to speak, including two of the current fellows. Four were invited to give plenary talks and three of the four Fields Medals awarded went to former Clay Fellows, to Artur Avilla, Manjul Bhargava, and Maryam Mirzakhani. It is hard to think of a clearer demonstration of the huge positive impact that Landon Clay's foundation has had on the international mathematical community.

While the Clay Research Fellowships and the Millennium Prize Problems define for many the public image of CMI, it also has a quieter but significant influence through its 'Enhancement and Partnership Program'. This builds on the excellent work of those organising mathematical meetings throughout the world, by providing the funds to bring in distinguished international speakers whose long-distance travel costs would be beyond local budgets and to widen participation of young researchers. In 2014, CMI supported some 42 mathematical events throughout the world through partnerships with other organisations under the program. The map on pp 26–27 shows an impressive geographic spread, which will extend further in 2015 and 2016 with events in Africa, north and south.

At the beginning of October, CMI held its second Research Conference in the Andrew Wiles Building at Oxford, along with four workshops. It was an exciting celebration of mathematics, with spectacular talks by Jonathan Pila, Scott Sheffield, Paul Seidel and Ben Green. Michael Rapoport gave an excellent presentation on the work of Peter Scholze, before Landon Clay presented him with one of the 2014 Clay Research awards. Rapoport's talk is written up in this report (pp. 11–13). The second award, to Maryam Mirzakhani, will be presented at the 2015 conference.

Sincerely,

Nick Woodhouse

N. M. J. Woodhouse

Annual Meeting

Clay Research Conference

NICK WOODHOUSE

The 2014 Clay Research Conference was held in Oxford on Wednesday, 1 October 2014. There were four one-hour lectures, by Jonathan Pila, Scott Sheffield, Paul Seidel, and Ben Green. Michael Rapoport then spoke about the work of Peter Scholze (see page 11) before Landon Clay presented one of the 2014 Clay Research Awards to Scholze. A second award had been made to Maryam Mirzakhani, but she was unable to attend. Her award will be presented at the 2015 conference.

The Schanuel Paradigm

Jonathan Pila spoke about various themes surrounding Schanuel's conjecture—its roots in transcendental number theory, its functional analogues, and its connections with diophantine problems.

The number theory story begins with Hermite's proof (1873) of the transcendence of e and its extension to the Lindemann–Weierstrass theorem, that if a finite set of algebraic numbers is linearly independent over the rationals, then their exponentials are algebraically independent over the rationals.

Similar questions arise for logarithms, in this case arising from the Gelfond–Schneider solution (1934) of Hilbert's seventh problem: if $a \neq 0,1$ is algebraic and b is algebraic and irrational, then a^b is transcendental. But the established analogues of the Lindemann–Weierstrass theorem, with exponentials replaced by logarithms, are not as strong as in the exponential case.

By taking the Gelfond–Schneider result in a slightly different direction, Gelfond conjectured that if $a \neq 0, 1$ and β are algebraic, and β has degree d over the rationals, then the $\beta, \beta^2, \dots, \beta^{d-1}$ powers of α are algebraically independent over the rationals. He proved this in the case $d=3$.

Schanuel's 1966 conjecture takes this further: given a set of n complex numbers, linearly independent over the rationals, the transcendence degree of the n numbers and their exponentials is at least n. Schanuel made an analogous conjecture about functions, which was proved by Ax in 1971. Roughly it states that the intersection of the graph of

$$\exp : \mathbb{C}^n \to (\mathbb{C}^\times)^n, \quad \exp(z_1, \dots, z_n) = (e^{z_1}, \dots, e^{z_n})$$

with an algebraic variety can only exceed the expected dimension in very special circumstances.

The conjecture has been refined and extended in various ways, not least in Boris Zilber's *uniform Schanuel conjecture* (2002). The truth of this would follow from the original conjecture together with a further conjecture, Zilber's 'CIT'—a challenging conjecture in diophantine geometry going far beyond the conjectures of Mordell–Lang and Manin–Mumford, which were proved in the 1980s.

In its most general form, due to Pink, CIT includes the André–Oort conjecture. Pila was given a Clay Research Award in 2011 for his partial resolution of the latter. In this approach, certain analogues of Ax's 1971 theorem play a key role. The lecture explored these connections and recent further progress by Pila himself and others

Photos by Mott Carter

Chinese Dragons and Mating Trees

Scott Sheffield has made many fundamental contributions in probability theory, in most recent years through his study of random conformally invariant objects. His lecture focused on some of these, in particular describing how random growth models can become tractable in surprising ways when they are run on a background that is itself random. This he illustrated with recent results on Diffusion Limited Aggregation (DLA). In the standard picture, in which growth occurs in a Euclidean background, a huge amount is known through simulations in a variety of specific applications (mineral deposits, Hele-Shaw flow, electrodeposition, lichen growth, lightning paths, coral growth, and so forth), but there are virtually no rigorous mathematical results.

On a random surface, however, the process can be understood in some detail when the two parameters that control the surface and the growth of the process are related in a special way. In particular, a certain scaling dimension emerges naturally. The 'Chinese dragons' of the title are suggested by the images of growth in this more general context.

Sheffield's lecture began by introducing his 'cast of characters'. First, for background, the familiar fractals from complex dynamics—Julia sets and so forth. Then random trees constructed from Brownian motion; random surfaces and the random area measures found by regularization in Liouville quantum gravity (LQG); random paths and Schramm Loewner evolution (SLE); and finally random growth, the Eden model and DLA.

The persona are linked in a variety of ways through their conformal properties. So the conformal welding of two LQG random surfaces is a fractal curve generated by SLE, and two conformally mated random trees give a space-filling path on the sphere.

In its final act, the lecture drew together all the elements of the play, in the remarkable story of random growth on random surfaces.

Steenrod Squares and Symplectic Fixed Points

Paul Seidel is known for his pioneering work on the application of advanced algebraic methods in symplectic topology, and in particular for his European Mathematical Society monograph *Fukaya categories and Picard–Lefschetz theory.* This was cited in the award of the Oswald Veblen Prize in 2010, along with several other fundamental contributions.

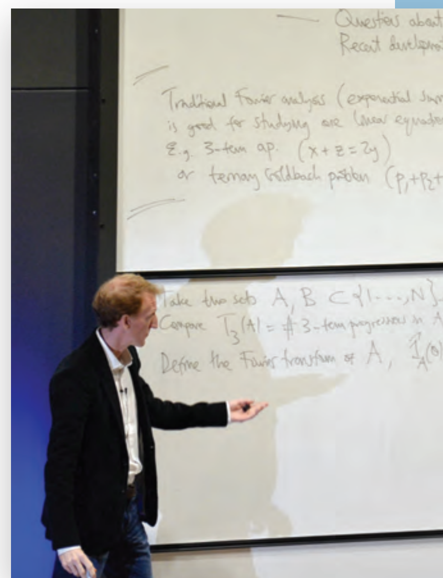
In his lecture, Seidel explained a geometry-algebra dictionary which takes ideas from classical algebraic topology, turns them into tools for exploring dynamical problems in symplectic geometry, and may eventually allow one to export the resulting insights into homological algebra. The classical background was equivariant cohomology, specialized to involutions, and the main application was recent work on fixed point Floer cohomology.

The scene was set by recalling the historical motivation for studying diffeomorphisms φ of a symplectic manifold, namely Poincaré return maps. Fixed points of such diffeomorphisms have received much attention. The lecture focused on periodic points of index two, which are fixed points of φ^2. On the algebraic side, the basic linear algebra model is the theory of involutions acting on vector spaces (or chain complexes) over a field of characteristic two. The properties of the involution can be encoded into cohomology groups, which involve an auxiliary formal variable u. This, as well as the Tate version (defined by introducing a formal inverse u^{-1}), are the key algebraic objects. Given a manifold with an involution, one can apply this construction to the singular cochain complex, and that yields equivariant cohomology (as well as its Tate version, which appears in the classical localization theorem).

The new results are built on an analogous construction in Floer cohomology (half-infinite dimensional Morse theory). The starting point is the action functional on a twisted loop space associated to a symplectic diffeomorphism. Its critical points are the fixed points of the diffeomorphism, and one imitates Morse's variational approach to define a chain complex generated by them. If the diffeomorphism is a square φ^2, the loop space comes with a natural involution (half-rotation, corresponding to the natural involution on periodic points of index two). The main result is that the equivariant version of the Floer cohomology of φ^2 agrees with the Floer cohomology of φ up to u-torsion, a kind of localization result. The relation is established via a natural product (the equivariant pair-of-pants product).

Under certain additional assumptions, it is possible to construct, in a non-canonical way, a topological "space" whose cohomology is Floer cohomology. In particular, one can then define cohomology operations (Steenrod squares) on Floer cohomology. Recent work of Hendricks provides a conjectural relation between these operations, localization theorems, and the previously mentioned pair-of-pants product.

Higher Order Fourier Analysis and Applications



Ben Green is well known for his contributions in number theory and combinatorics, and in particular for the Green–Tao theorem, that for all n, there are infinitely arithmetic progressions of length n in the primes.

His lecture on *Higher Order Fourier Analysis* explored an extension of the traditional application of Fourier analysis in number theory to problems involving systems of linear equations, in particular to questions about four-term arithmetic progressions and to the study of large gaps between primes.

The standard theory is a very effective tool in problems that can be formulated as single linear equations in the integers, such as the ternary Goldbach conjecture ($p_1 + p_2 + p_3 - N = 0$). A key example that sets the scene for the higher order theory is the problem of comparing the number of arithmetic progressions of length three in two sets of integers A, B. With the Fourier transforms defined by summing $\exp(2\pi i n\theta)$ over the sets, it can be shown that if the transforms of A and B are close pointwise in θ, then the numbers of three-term arithmetic progressions in A and B are also close.

The analogous statement for four-term progressions does not hold, essentially because the condition that (x_1, x_2, x_3, x_4) should be in arithmetic progression involves two linear conditions. In fact there are examples of pairs of sets for which the Fourier transforms are close but the numbers of arithmetic progressions of length four are not. A new approach is needed.

That is found by adapting a concept from ergodic theory: *s-step nilsequences* are defined as maps $\varphi : \mathbb{Z} \to \mathbb{C}$ of the form $\varphi(n) = F(g^n)$, where $F : G \to \mathbb{C}$, with G a Lie group of nilpotency class s and F automorphic with respect to a lattice in G. These are the appropriate higher order characters for systems of s linear equations. In particular, 2-step nilsequences provide a solution to the four-term problem, although the proof is very much longer and more complicated than in the three-term case.

In two papers, one with Tao [1] and the other with Tao and Ziegler, [2] Green used nilsequences to derive an asymptotic for the number of solutions in the primes of a 'non-degenerate' system of linear equations. He sketched the example of counting the number of 100-term progressions of primes less than X with common difference equal to the product of a prime with 100! (a number that is 'basically a prime'). The count is asymptotic to $cX^2(\log X)^{-101}$.

Such results unlock the 'large gaps' problem—the problem of determining the behaviour for large X of the function $G(X)$ which denotes the largest gap between consecutive primes less than or equal to X. In 1938, Rankin showed that

$$G(X) \geq \frac{(\log X \,.\, \log\log X \,.\, \log\log\log\log X)}{(\log\log\log X)^2}$$

In 2014 Green, with Ford, Konyagin, and Tao, provide that G(X) exceeds the product of the expression on the right with a function that goes to infinity as $X \to \infty$. [3] The proof involves a sieving argument using 'basically prime' progressions. Green ended his lecture by giving an outline.

[1] B.J. Green and T.C. Tao, *Linear equations in primes*, Annals of Math. 171 (2010), 1753-1850.

[2] B.J. Green, T.C. Tao and T. Ziegler, *An inverse theorem for the Gowers $U^{s+1}[N]$-norm*, Annals of Math. 176 (2012), 1231-1372.

[3] Kevin Ford, Ben Green, Sergei Konyagin, Terence Tao. *Large gaps between consecutive prime numbers*. arXiv:1408.4505 (2014). James Maynard simultaneously obtained the same result by a different method.

Clay Research Conference Workshops

ADVANCES IN PROBABILITY: INTEGRABILITY, UNIVERSALITY AND BEYOND

September 28 – October 2, 2014

Since its discovery over two hundred years ago, the Gaussian distribution has come to represent one of mathematics greatest societal contributions—a robust theory explaining and analyzing much of the randomness inherent in the physical world. However, not all systems are well described by Gaussian theory. For example, classical extreme value statistics or Poisson statistics better capture the randomness and severity of events ranging from natural disasters to emergency room visits.

Recently, significant research efforts have been focused on understanding systems which are not well described in terms of any of the classically developed statistical universality classes. The failure of these systems to conform with classical descriptions is generally due to the non-linear relationship between natural observables and underlying sources of random inputs and noise. The integrability (or exact solvability) of a few key models for these systems allows for detailed descriptions of new scaling limits and statistics, while universality results prove that these limiting behaviors are robust with respect to changing some of the underlying details of the models.

This workshop brought together experts at the forefront of recent advances in the probabilistic study of complex random systems to probe the interplay between newly developed methods of integrability and universality in relation to these systems, focusing on:

- Random interface growth, particle systems, stochastic PDEs, and the Kardar–Parisi–Zhang equation and universality class
- Random matrix theory
- Two-dimensional equilibrium statistical mechanics such as the Ising model, percolation, quantum Liouville gravity and its relation to the Schramm Loewner evolution
- Logarithmically correlated processes such as Gaussian free field, and branching diffusion processes

There were quite active discussions among participants during talks and breaks. A number of participants gave feedback to the organizers that the workshop was inspiring in so far as it showed the strength and potential of young probabilists (many of the speakers are relatively early in their career development). Additionally, Scott Sheffield, as the probability representative at the Clay Research Conference, gave a beautiful talk which received accolades from many audience members.

Organizers

Ivan Corwin (Columbia University, IHP and CMI)
Martin Hairer (University of Warwick)

Speakers

Gérard Ben Arous (New York University)
Nathanael Berestycki (University of Cambridge)
Alexei Borodin (Massachusetts Institute of Technology)
Amir Dembo (Stanford University)
Christina Goldschmidt (University of Oxford)
Geoffrey Grimmett (University of Cambridge)
Alice Guionnet (Massachusetts Institute of Technology)
Grégory Miermont (ENS Lyon)
Jason Miller (Massachusetts Institute of Technology)
Ashkan Nikeghbali (Universität Zürich)
Neil O'Connell (University of Warwick)
Yuval Peres (Microsoft Research)
Jeremy Quastel (University of Toronto)
Fabio Toninelli (University Lyon 1)
Craig Tracy (University of California, Davis)
Vincent Vargas (ENS Paris)
Ofer Zeitouni (Weizmann Institute of Science, New York University)

ANALYTIC NUMBER THEORY

September 29 – October 3, 2014

This workshop covered all aspects of analytic number theory. There have been a number of spectacular advances in recent years, on gaps between primes, mean-values of L-functions, Vinogradov's mean-value theorem, rational points on algebraic varieties, and additive combinatorics, for example.

The aim of the workshop was to bring together researchers in these various fields to learn about the latest developments, facilitate interaction between different areas, promote collaborations, and to suggest future directions.

There was a full programme of 18 invited 60-minute talks and four contributed 30-minute talks. Amongst the highlights were (i) Trevor Wooley's talk describing the latest applications of his novel "efficient congruencing" technique; (ii) James Maynard speaking on his solution of Erdős' prize problem on large gaps between primes; (iii) Harald Helfgott describing the full solution of the ternary Goldbach problem; and (iv) Maksym Radziwill's account of his joint work with Kaisa Matomaki, on sign changes of the Liouville function.

It would have been inconceivable when the workshop was first planned that we could have such results to hear about. Such dramatic progress, on many disparate fronts, was reflected in a general air of excitement noticeable throughout the workshop.

Organizers

Ben Green (University of Oxford)
Roger Heath-Brown (University of Oxford)

Speakers

Christoph Aistleitner (Kobe University)
Jonathan Bober (University of Bristol)
Julia Brandes (University of Goettingen)
Tim Browning (University of Bristol)
Vorrapan Chandee (Burapha University)
Chantal David (Concordia University)
John Friedlander (University of Toronto)
Leo Goldmakher (Williams College)
Andrew Granville (University of Montreal)
Gergely Harcos (Renyi Institute)
Adam Harper (University of Cambridge)
Roger Heath-Brown (University of Oxford)
Harald Helfgott (École Normale Supérieure)
Emmanuel Kowalski (ETH Zürich)
Dimitris Koukoulopoulos (University of Montreal)
Stephen Lester (Tel Aviv University)
Manfred Madritsch (University of Lorraine)
James Maynard (University of Montreal and University of Oxford)
Lillian Pierce (Hausdorff Centre)
Maksym Radziwill (Institute for Advanced Study)
Fernando Shao (Stanford University)
Kannan Soundararajan (Stanford University)
Jesse Thorner (Emory University)
Trevor Wooley (University of Bristol)

September 29 – October 3, 2014

Schanuel's conjecture governs the transcendence properties of the exponential function. It remains open except in some very special cases. In a differential field setting it is a theorem of Ax (1971). Analogues for other special functions, in particular for uniformising maps of Shimura varieties, are central objects of study in parts of Diophantine geometry, differential algebra, differential Galois theory, model theory, and complex geometry.

The aim of the workshop was to reflect this diversity of perspectives around the shared common theme. It gathered participants from all the above mentioned disciplines with an interest in such questions to share methods, problems, results, and conjectures. As well as results directly within the "Ax–Schanuel" framework, the workshop included contributions devoted to the model-theory of exponentiation more generally, to ideas about "differentiating" numbers, to Diophantine applications of functional transcendence results within the framework of the Zilber–Pink conjecture but also going outside that framework, to the theory of the model-theoretic "pseudo"-exponential and "pseudo"-modular functions, to Ax–Schanuel formulations in positive characteristic, and to topological questions seeking to refine the "Ax–Schanuel" statements.

It was thus a diverse programme giving all participants exposure to a wide variety of ideas around the central Ax–Schanuel formulation.

Organizers

Jonathan Pila (University of Oxford)
Alex Wilkie (University of Manchester)

Speakers

Daniel Bertrand (Institut de Mathematiques de Jussieu)
Alexandru Buium (University of New Mexico)
Laura Capuano (Scuola Normale Superiore di Pisa)
Christopher Daw (IHES)
Philipp Habegger (TU Darmstadt)
Adam Harris (University of Oxford)
Jonathan Kirby (University of East Anglia)
Bruno Klingler (Institut de Mathematiques de Jussieu)
Piotr Kowalski (Wroclaw University)
Angus Macintyre (Queen Mary University London)
David Masser (Mathematisches Institut Basel)
Ngaiming Mok (Hong Kong University)
Martin Orr (University College London)
Anand Pillay (University of Notre Dame)
Harry Schmidt (University of Basel)
Michael Singer (North Carolina State University)
Emmanuel Ullmo (Université Paris Sud)
Sai Kee Yeung (Purdue University)
Umberto Zannier (Scuola Normale Superiore di Pisa)
Boris Zilber (University of Oxford)

September 29 – October 3, 2014

The origins of symplectic topology lie in classical dynamics and the search for periodic orbits of Hamiltonian systems. It is now understood to arise naturally in algebraic geometry, in low-dimensional topology, in representation theory and in string theory. Following seminal ideas of Gromov and Floer from the 1980s, several of the most powerful tools in symplectic topology revolve around invariants counting pseudoholomorphic curves. An important theme in recent years has been that holomorphic curve invariants are not independent, but are bound together and governed by very rich algebraic structures, making connections to integrable systems and to the theory of A-infinity and L-infinity algebras. On the one hand, the algebraic structures make the invariants more powerful; on the other, these algebraic structures often underscore the connections to other fields, where similar structures arise with different origins.

This workshop brought together experts in various parts of holomorphic curve theory to expose the latest developments in: (i) Floer cohomology theory, the Fukaya category, and aspects of homological mirror symmetry and (ii) contact and Stein topology, and their connections to low-dimensional topology and to dynamics.

Recent areas of progress include flexibility results in Stein topology, formulations and proofs of homological mirror symmetry, renewed ideas about flux and categorical dynamics, existence theorems for quasi-morphisms, sharp constraints on symplectic or Lagrangian embeddings, and relations to knot theory and finite type invariants. Whereas much recent activity has been directed towards foundational issues, the workshop focused more on applications of the theory, particularly those where progress has been and is being made, and highlighted some of the new questions that progress raises.

The workshop provided a very stimulating and friendly environment for both experts in the area and for newcomers to discuss recent trends and progress in symplectic topology. This helped foster new and strengthen existing collaborations. The participants were particularly engaged in conversations between lectures and during coffee and lunch breaks.

Organizers

Dominic Joyce (University of Oxford)
Alexander Ritter (University of Oxford)
Ivan Smith (University of Cambridge)

Speakers

Mohammed Abouzaid (Columbia University)
Denis Auroux (University of California, Berkeley)
Paul Biran (ETH Zürich)
Vincent Colin (University of Nantes)
Tobias Ekholm (University Uppsala)
Yasha Eliashberg (Stanford University)
Kenji Fukaya (Simons Center, Stony Brook)
Helmut Hofer (Institute for Advanced Study)
Michael Hutchings (University of California, Berkeley)
Dusa McDuff (Columbia University)
Mark McLean (State University of New York, Stony Brook)
Emmy Murphy (Massachusetts Institute of Technology)
Tim Perutz (University of Texas, Austin)
Leonid Polterovich (Tel Aviv University)
Paul Seidel (Massachusetts Institute of Technology)
Nick Sheridan (Princeton University)

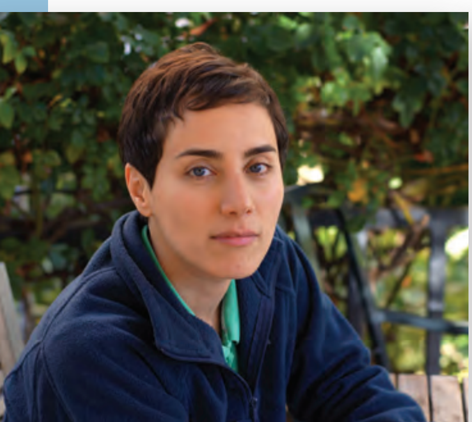
Recognizing Achievement

Clay Research Awards

The Clay Millennium Prize Problems have given the Clay Mathematics Institute a high public profile. Less well known outside of the mathematical world, but widely appreciated within it, are the annual Clay Research Awards. These celebrate the outstanding achievements of the world's most gifted mathematicians.

In 2014 a Clay Research Award was presented to Peter Scholze for his outstanding work in arithmetic algebraic geometry, particularly in the development and applications of the theory of perfectoid space.

Maryam Mirzakhani was awarded a 2014 Clay Research Award for her many and significant contributions to geometry and ergodic theory, in particular to the proof of an analogue of Ratner's theorem on unipotent flows for moduli of flat surfaces. Professor Mirzakhani will be presented with her award at the 2015 Clay Research Conference.



CLAY RESEARCH AWARDEES

2014	Maryam Mirzakhani Peter Scholze
2013	Rahul Pandharipande
2012	Jeremy Kahn and Vladimir Markovic
2011	Yves Benoist and Jean-François Quint Jonathan Pila
2009	Ian Agol, Danny Calegari and David Gabai Jean-Loup Waldspurger
2008	Cliff Taubes Claire Voisin
2007	Alex Eskin Christopher Hacon and James McKernan Michael Harris and Richard Taylor
2005	Manjul Bhargava Nils Dencker
2004	Ben Green Gérard Laumon and Ngô Bảo Châu
2003	Richard Hamilton Terence Tao
2002	Manindra Agrawal Oded Schramm
2001	Stanislav Smirnov Edward Witten
2000	Alain Connes Laurent Lafforgue
1999	Andrew Wiles

Highlights of Peter Scholze's Contributions

MICHAEL RAPOPORT

Peter Scholze (Universität Bonn) was given the 2014 Clay Research Award for "his many and significant contributions to arithmetic algebraic geometry, particularly in the development and applications of the theory of perfectoid spaces". Here is a list of highlights of Scholze's contributions to date.

• *A new proof of the Local Langlands Correspondence* (LLC): Scholze's main contribution to this circle of ideas is the determination of the *semi-simple* sheaf of nearby-cycles in certain geometric situations. The hypotheses of Scholze's formula are satisfied in the case of Shimura varieties studied by Kottwitz, Harris and Taylor, and which are used as the basic geometric input of the proof of Harris/Taylor and Henniart of the local Langlands correspondence for the general linear group of a local field of characteristic zero. Based on this geometric theorem, Scholze gave a completely different proof of LLC that does not use the cumbersome reduction to characteristic p and avoids complicated ad hoc methods that lie behind the *numerical Langlands correspondence* of Henniart.

We state Scholze's theorem in the case when the local field is $\mathbb{Q}_p$. Let τ be an element of the Weil group $W_{\mathbb{Q}_p}$ projecting to the rth power of the Frobenius element. Let $h \in C_c^\infty(\mathrm{GL}_n(\mathbb{Z}_p))$ have values in $\mathbb{Q}$. Scholze defines a function $\beta \mapsto \varphi_{\tau,h}(\beta)$ on $\mathrm{GL}_n(\mathbb{Q}_{p^r})$ by associating to β the alternating trace of $\tau \times h^\vee$ on the ℓ-adic sheaf of nearby cycles of a certain moduli space of p-divisible groups that depends on β. What is essential here is the fact that the function $\varphi_{\tau,h}$ is defined in a geometric way. It lies in $C_c^\infty(\mathrm{GL}_n(\mathbb{Q}_{p^r}))$ and takes values in $\mathbb{Q}$ independent of ℓ.

Theorem 1. *(i) For any irreducible smooth representation π of $\mathrm{GL}_n(\mathbb{Q}_p)$, there exists a unique n-dimensional representation $\mathrm{rec}(\pi)$ of $W_{\mathbb{Q}_p}$ such that for all τ and h as above,*

$$\mathrm{trace}(f_{\tau,h}|\pi) = \mathrm{trace}(\tau|\mathrm{rec}(\pi)) \cdot \mathrm{trace}(h|\pi).$$

Here $f_{\tau,h} \in C_c^\infty(\mathrm{GL}_n(\mathbb{Q}_p))$ is any function matching $\varphi_{\tau,h}$ in the sense of local harmonic analysis.

(ii) The map $\pi \mapsto \mathrm{rec}(\pi)$ induces a bijection between the set of isomorphism classes of supercuspidal irreducible smooth representations of $\mathrm{GL}_n(F)$ and the set of isomorphism classes of irreducible n-dimensional representations of W_F.

(iii) If π is supercuspidal, then, up to Tate twist, $\mathrm{rec}(\pi)$ is in Langlands correspondence with π.

Here (i) asserts that the geometrically defined identity appearing in it sets up a map between irreducible smooth representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and n-dimensional representations of the Weil group $W_{\mathbb{Q}_p}$; (ii) asserts that this sets up a bijection between the two sets in question; and (iii) asserts that, for supercuspidal representations, this bijection can be characterized à la Langlands through L-functions and ε-factors.

• *The theory of perfectoid spaces:* This is a general method to reduce problems of algebraic geometry over fields of mixed characteristic, like $\mathbb{Q}_p$, to fields of positive characteristic, like $\mathbb{F}_p((t))$, which are often easier to solve, mainly due to the power of the Frobenius map in characteristic p. One application of this theory is a general form of Faltings' *almost purity theorem*. Rather than explaining this method, we will give here only a sampling of the many applications of this theory. Detailed accounts of this theory have been given by Scholze himself (in his original paper in Pub. math. IHES, in his paper for the *Current developments in Mathematics* series, and his ICM 2014 paper, and his video talk at the ICM in Seoul; also an exposition by J.-M. Fontaine in the Bourbaki seminar is devoted to this topic, as well as a short note by B. Bhatt in the *What is . . .?* series of the *Notices of the AMS*).

• *A proof of the Weight Monodromy Conjecture (WMC) in many cases:* As a first application of perfectoid spaces, Scholze proved the WMC in many cases. The conjecture explains, through the monodromy operator, the lack of Frobenius purity in the cohomology of smooth projective algebraic varieties over fields of mixed characteristic with bad reduction modulo p. This conjecture, due to Deligne in 1970, is *the* major open problem in the etale cohomology of algebraic varieties, and Scholze's theorem is the first significant advance in over 30 years. His theorem is as follows.

Theorem 2. *Let X be a geometrically connected projective smooth variety over a finite extension of $\mathbb{Q}_p$ which is a set-theoretic complete intersection in projective space. Then the WMC holds for X.*

Scholze's proof uses perfectoid spaces to reduce the assertion to the equi-characteristic analogue which is due to Deligne.

• *p-adic Hodge theory for rigid-analytic spaces:* This is a second application of perfectoid spaces, and generalizes and streamlines the p-adic Hodge theory for schemes, due to Fontaine, Messing, Faltings, Kato and Tsuji, and Beilinson and Niziol. Tate asked more than 40 years ago whether such a theory exists. The next theorem (which incorporates improvements due to Gabber and Conrad on Scholze's original theorem) gives a confirmation of one of Tate's conjectures; it is the rigid-analytic analogue of theorems of Poincaré, Hodge and deRham for complex-analytic manifolds.

Theorem 3. *Let C be an algebraically closed complete p-adic field, and let X be a proper smooth rigid-analytic space over C. Let*

$$h^{ij} = \dim H^j(X, \Omega^i_X), \quad h^n_{\mathrm{dR}} = \dim H^n_{\mathrm{dR}}(X), \text{ and } h^n_{\mathrm{et}} = \dim H^n_{\mathrm{et}}(X, \mathbb{Q}_\ell).$$

Then

$$h^n_{\mathrm{et}} = h^n_{\mathrm{dR}} = \sum_{i+j=n} h^{ij}.$$

• *Moduli of p-divisible groups:* This is a third application of perfectoid spaces. Scholze and J. Weinstein develop a theory of universal coverings of p-divisible groups, with a number of striking applications. Here is a sample application of this theory. It is the analogue of Riemann's classification of abelian varieties over $\mathbb{C}$ by their first singular homology, together with the Hodge filtration.

Theorem 4. *Let C be an algebraically closed complete p-adic field, and O_C its ring of integers. There is an equivalence of categories*

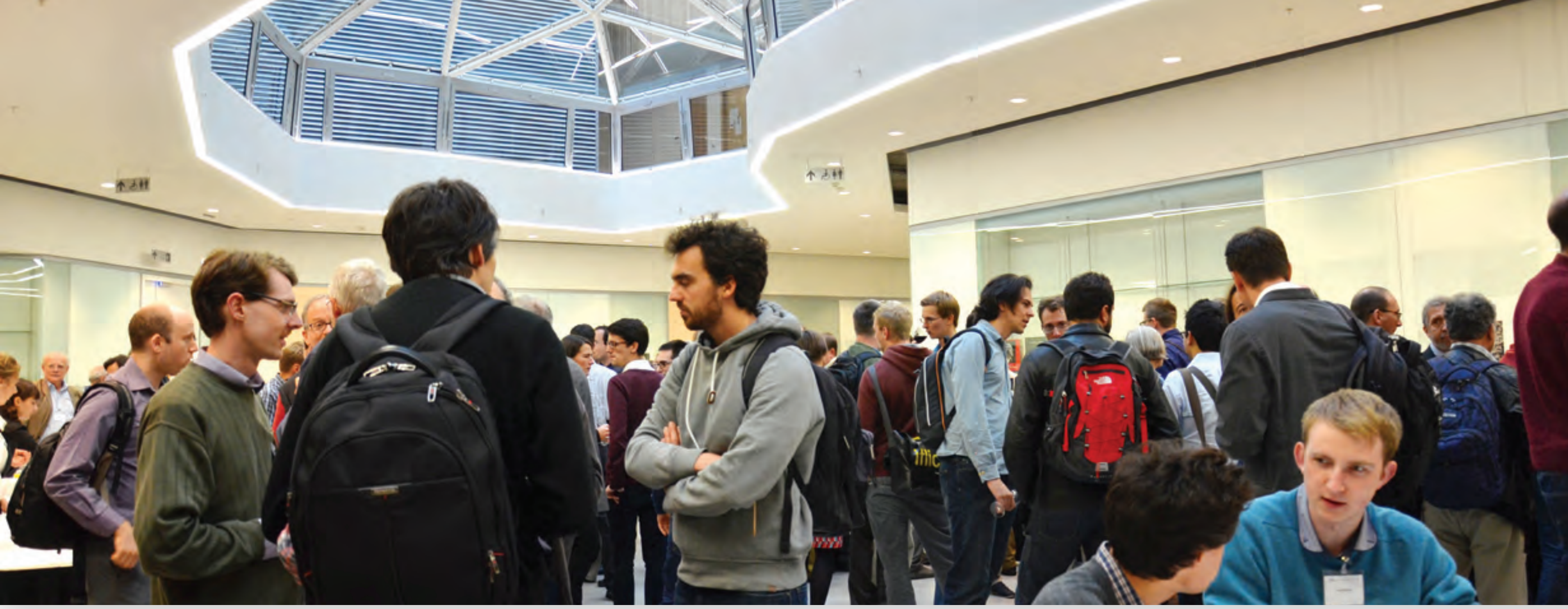
$$\{\text{ p-divisible groups over } O_C\} \xrightarrow{\sim} \{\text{ pairs } (\Lambda, W), \text{ where } \Lambda \text{ is a finite free } \mathbb{Z}_p\text{-module}, \text{ and } W \subset \Lambda \otimes C \text{ is a } C\text{-subvectorspace}\}.$$

• *Torsion in the cohomology of symmetric spaces* : This is a fourth application of perfectoid spaces. Scholze shows that one can associate a Galois representation to any system of Hecke eigenvalues appearing in the p-power torsion cohomology of GL_n over a totally real or CM field, with matching Frobenius eigenvalues. This confirms conjectures of Grunewald, Ash and others that were open for 40 years. Here is a more precise version.

Theorem 5. *Let G be the restriction of scalars of GL_n from a totally real or CM field F down to $\mathbb{Q}$. Let $K \subset G(\mathbb{A}_f)$ be an open compact subgroup, with corresponding locally symmetric space Y_K. Then for any system of Hecke eigenvalues ψ appearing in $H^i(Y_K, \bar{\mathbb{F}}_p)$, there exists a continuous semisimple Galois representation*

$$\rho_\psi : \mathrm{Gal}_F \to \mathrm{GL}_n(\bar{\mathbb{F}}_p)$$

characterized by the property that for all but finitely many 'ramified' places v of F, the characteristic polynomial of $\rho_\psi(\mathrm{Frob}_v)$ is described by the Hecke eigenvalues ψ.



The proof proceeds by realizing ψ as a *boundary contribution* to the cohomology of a Shimura variety (a device invented by Harris/Lan/Taylor/Thorne in the context of *rational cohomology*), and uses perfectoid spaces obtained in the limit as the *p*-component of the level structure shrinks to zero.

Here is a cohomological vanishing theorem that results from this theory, and which proves a conjecture of Calegari and Emerton.

Theorem 6. *Let* $\mathrm{Sh}_K = \mathrm{Sh}_{K^pK_p}$ *be a Shimura variety, assumed compact for convenience. Let*

$$H^i(K^pK_p, \mathbb{F}_p) = H^i(\mathrm{Sh}_{K^pK_p}, \mathbb{F}_p) \quad \text{and} \quad H^i(K^p, \mathbb{F}_p) = \varinjlim_{K_p} H^i(K^pK_p, \mathbb{F}_p)$$

Then

$$H^i(K^p, \mathbb{F}_p) = (0), \qquad i > \dim \mathrm{Sh}_K.$$

It is in a weak sense an analogue of the theorem of Borel, Casselman, Garland, Prasad, Wallach, Zuckerman that states that $H^i(K^pK_p, \mathbb{Q})$ is *banal* for $i > 2\dim \mathrm{Sh}_K - \mathrm{rank}_{\mathbb{R}}(G)$.

• *The pro-etale site* : Grothendieck had explained 50 years ago that the étale cohomology of non-torsion sheaves is essentially trivial and that, in order to obtain interesting cohomology groups in characteristic zero, one had to first take the cohomology of torsion sheaves and then pass to the limit. Here Grothendieck was following Weil, who treated the case of abelian varieties through their Tate modules which arise by considering the ℓ^n-torsion points and passing to the limit. Scholze introduces a new topology on the category of schemes which has the virtue that sheaves like $\mathbb{Z}_\ell$ and $\mathbb{Q}_\ell$ behave in just the way we expect from classical topology and, in joint work with B. Bhatt also introduces a fundamental group of schemes which is reasonable even for highly singular schemes. To make the latter statement precise, call a topological group *G* a *Noohi group* if *G* is complete, and admits a basis of open neighborhoods of the unit element given by open subgroups (examples of Noohi groups are locally pro-finite groups and discrete groups).

Theorem 7. *Let X be a connected scheme whose underlying topological space is locally noetherian. For any geometric point x of X, there is a Noohi group* $\pi_{\mathrm{proet}}(X, x)$ *such that local systems in* $\mathbb{Q}_\ell$*-vector spaces on X are equivalent to continuous representations of* $\pi_{\mathrm{proet}}(X, x)$ *on finite-dimensional* $\mathbb{Q}_\ell$*-vector spaces.*

This enumeration is not complete. It leaves out many further results and concepts that Scholze introduced in the last four years. His ICM 2014 paper gives an excellent overview of these; of course, that overview does not contain those results that Scholze has obtained in the last few months; to get an idea of some of these, in particular of his theory of *diamonds*, one may consult the video of his Berkeley course in the fall of 2014 that is available on the homepage of the MSRI.

Profile

INTERVIEW WITH **IVAN CORWIN**, CLAY RESEARCH FELLOW

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I always remember wanting to do mathematics. One salient memory is of when I was about five staying with my older brother at my grandparents. During a car ride he bet me $10 that I couldn't find three positive numbers such that the sum of the cubes of the first two equaled the cube of the third. I spent a few years hand calculating cubes and trying to find a solution to his problem. Unfortunately this is impossible (it's the simplest case of Fermat's last theorem). But, in retrospect, this was probably the best spent $10 of my life since it gave me my first glimpse into the beautiful structure of numbers.

Could you talk about your mathematical education? What experiences and people were especially influential?

I attended public school near Poughkeepsie, NY. I was able to take math classes with the students a year or two ahead of me, and in my senior year of high school I took two math classes at the local community college. My siblings and parents were supportive of my interest in math and introduced me to some more advanced topics when I was young. I also enjoyed borrowing the limited number of popular math books available in our local and school libraries. In high school, I participated in some math competitions and went to a math camp one summer. All in all, though, when I arrived as an undergrad at Harvard, I was grossly underprepared for the famed Math 55. It took a while to catch up!

When I was about five, my brother bet me $10 that I couldn't find three positive numbers such that the sum of the cubes of the first two equaled the cube of the third. … This was probably the best spent $10 of my life.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

As an undergraduate, I participated in the SMALL REU at Williams College. Frank Morgan led my group and devoted most of his summer towards guiding four of us into our first serious research experience. He has done this for decades with countless "Geometry Group" members benefiting. In graduate school my advisor, Gerard Ben Arous, as well as Percy Deift helped direct me towards a very fertile area of probability research. Alexei Borodin, my principle collaborator during my post-doctoral studies, was incredibly thoughtful, not only helping to shape my taste in mathematics but also serving as an all-around role model. I have also enjoyed the support of many senior colleagues, such as Craig Tracy and Horng-Tzer Yau, and collaborators, such as Jeremy Quastel.

From your own experience at high school, are there any aspects of mathematics education that you would like to see changed?

I think that more than ever, people need to know certain aspects of math as adults. Unfortunately, I don't think that these aspects are the focus of high school math classes. I'm certainly biased, but I think that there should be much more emphasis on probability, statistics, algorithms, logic, and relevant aspects of data processing. So many more jobs rely on these skills today than ever before.

What attracted you to the particular problems you have studied?

Some people are attracted to the pure beauty of mathematics while others value its application. I am attracted to a middle-ground. My research attempts to build bridges between algebraic structures and problems of some physical interest.

Can you describe your research in accessible terms?

An overarching aim of my research is to study how systems with various types of microscopic rules and randomness can manifest universal and precisely quantifiable behaviors as their size and time grow large. The archetypical example is the ubiquitous occurrence of the bell-curve (Gaussian distribution) in the context of random walks. I study more complex systems than random walks, such as models for interface growth, traffic flow, mass transport, turbulence and shock-fronts. Within these contexts I try to determine what universal behaviors arise by finding special examples of these type of systems which have enhanced algebraic structure admitting exact analysis.

What research problems and areas are you likely to explore in the future?

I would like to build stronger links between probability and quantum integrable systems, using the latter to discover new probabilistic phenomena. I would also like to develop new methods to prove the universality of these phenomena.

How has the Clay Fellowship made a difference for you?

The Clay Fellowship afforded me the opportunity to travel extensively to build up many collaborations and move into areas of research a little distant from my graduate training. I have also enjoyed the opportunity to organize two workshops at their new facilities in Oxford.

What advice would you give lay persons who would like to know more about mathematics? What should they read? How should they proceed?

Some of the most beautiful mathematics is very old and very accessible. Most libraries stock a few popular math books and I suggest checking these out. I think that people should understand that math is often motivated by other fields, such as physics. I really enjoyed reading some of the popular work of Feynman or Hawking. Even though these are more physics oriented, I think that they demonstrate how math is fundamental to our understanding of everything. At a more elementary level, Flatland is an eye opening journey through geometry. I recently visited the Museum of Mathematics in New York. Places like this are great since they showcase the beauty and use of mathematics in a very exciting manner.

How do you think mathematics benefits culture and society?

Mathematics has always been a lens through which to understand and appreciate the world. Now, more than ever, it is an engine for the development of large parts of our culture and society. Computers, communications, medicine, politics, finance and much more, all rely upon advances in mathematics, and require some amount of math to properly understand and control. Most of my high school friends who used to dislike math are now working in jobs which rely upon it in one way or another. I think it’s important for us to realize this and help train people at a younger age.

I think it is also important to be honest about the fact that practitioners of mathematics have been partially responsible for certain societal ills, such as those which led to the recession in the past decade. Mathematics does not have built-in ethics, so it is upon each of us to take responsibility for how we use it.

Please tell us about things you enjoy when not doing mathematics.

I really enjoy cooking and experimenting with new recipes. Recently, my fiancée and I have gotten into fermentation, making things like Kimchi, Sauerkraut, and apple cider.

My research attempts to build bridges between algebraic structures and problems of some physical interest.

Program Overview

Summary of 2014 Research Activities

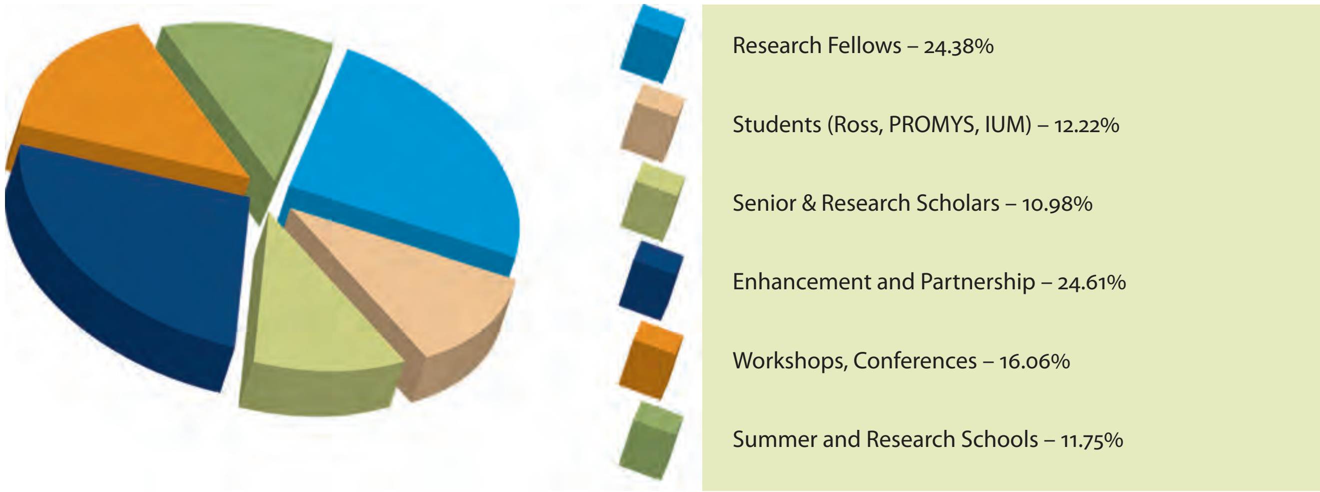
The activities of CMI researchers and research programs are sketched on the following pages. Researchers and programs are selected by the Scientific Advisory Board (see inside front cover).

PROGRAM ALLOCATION

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2014

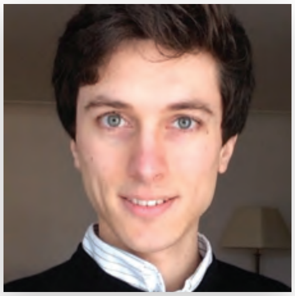
26	Research Fellows, Research Awardees, Senior Scholars, Research Scholars
123	CMI Workshops
317	Summer and Research Schools
60	PROMYS/Ross/PRIMES Faculty and Participants
>600	Participants attending Conferences and Joint Programs
80	Independent University of Moscow

RESEARCH EXPENSES FOR FISCAL YEAR 2014



Clay Research Fellows

June Huh received his PhD in 2014 from the University of Michigan under the supervision of Mircea Mustaţă. He applies algebraic geometry and singularity theory to problems in combinatorics and other areas. His recent interests include singularities of projective hypersurfaces, positivity of Chern classes of Schubert varieties, and connections between realizability problems in algebraic geometry and combinatorial geometry. June was appointed as a Clay Research Fellow for a term of five years beginning 1 July 2014.



Miguel Walsh was born in Buenos Aires, Argentina. He received his "Licenciatura" degree in 2010 from Universidad de Buenos Aires and his PhD from the same institution in 2012, under the supervision of Román Sasyk. During this period he held a CONICET doctoral fellowship. He is currently based at the University of Oxford. His research so far has focused on inverse problems in arithmetic combinatorics, the limiting behaviour of ergodic averages and the estimation of rational points on curves. Miguel was appointed as a Clay Research Fellow for a term of four years beginning 1 July 2014.

Alex Wright received his PhD in 2014 from the University of Chicago under the supervision of Alex Eskin. His recent work concerns dynamics on moduli spaces and special families of algebraic curves that arise in this context. His interests include dynamics, geometry, and especially ergodic theory on homogenous spaces and Teichmüller theory. Alex received his BMath from the University of Waterloo in 2008. He was appointed as a Clay Research Fellow for a term of five years beginning 1 July 2014.

June, Miguel and Alex joined CMI's group of research fellows: Tim Austin (New York University), Ivan Corwin (Columbia University and IHP), Semyon Dyatlov (MIT), Aaron Pixton (Harvard University), Peter Scholze (University of Bonn) and Jack Thorne (Harvard University).

RESEARCH SCHOLARS

Roman Travkin
2012-2015
Harvard University

Kiran Kedlaya
2013-2014
University of California at San Diego

SENIOR SCHOLARS

Lars Hesselholt (MSRI)
March – April 2014
Algebraic Topology

Ehud Hrushovski (MSRI)
March – April 2014
Model Theory, Arithmetic Geometry and Number Theory

L. Mahadevan (PCMI)
June 29 – July 19, 2014
Materials

Felix Otto (PCMI)
June 29 – July 19, 2014
Materials

Yves Benoist (INI)
June 9 – July 4, 2014
Interactions between Dynamics of Group Actions and Number Theory

Alex Eskin (INI)
June 9 – July 4, 2014
Interactions between Dynamics of Group Actions and Number Theory

Pierre Colmez (MSRI)
August – October 2014
New Geometric Methods in Number Theory and Automorphic Forms

Simon Brendle (Fields Institute)
August – November 2014
Thematic Program on Variational Problems in Physics, Economics and Geometry

Nader Masmoudi (Fields Institute)
August – December 2014
Thematic Program on Variational Problems in Physics, Economics and Geometry

Joseph Bernstein (MSRI)
September 2014
Geometric Representation Theory

Ngô Bảo Châu (MSRI)
September – October 2014
Geometric Representation Theory

Rick Durret (MBI, Ohio State University)
September 2014 – June 2015
Cancer and its Environment

Vladimir Voevodsky (University of Oxford)
September – December 2014
Homotopy Type Theory and its Applications

CMI Workshops

CMI conducts a program of workshops at the Mathematical Institute in Oxford, UK which bring together a small set of researchers quickly, outside the usual grant and application cycle, when this is likely to result in significant progress.

GEOMETRY AND FLUIDS

April 7-11, 2014

The application of ideas from the theory of complex manifolds to fluids mechanics has revealed important connections between complex structures and the dynamics of vortices in many different fluid flows. Large-scale atmospheric flows, optimal transport and complex geometry have each provided a framework for studying (Monge–Ampère) partial differential equations, their transformation properties, and solutions. Recently, new connections have been established between these seemingly disparate areas, as well as between coherent vortices in incompressible Navier–Stokes flows and almost-complex structures. The application of geometry to fluid mechanics has opened up promising new perspectives on some enduring problems and facilitates a unification of otherwise ostensibly disparate topics, including singular behaviour conservation laws and the PDEs describing vortex dynamics.

The interplay between hyper-Kähler geometry and the Monge–Ampère equation also has a long cherished history in string theory, a subject far-removed from fluid dynamics. Recently, several new tools have been developed, such as generalized geometry, flux compactifications and string theory dualities for understanding the structure of these equations and their solutions.

This workshop brought together experts from complex manifolds and string theory with those from fluid mechanics to study the interplay between geometry, optimal transport and their applications. It focused on three topics: elliptic Monge–Ampère equations, hyper-Kähler geometry and the classification of incompressible flows in two dimensions (four dimensions in terms of the phase space of the fluid); optimal transport on manifolds and the mathematics of quasi-equilibrium atmosphere/ocean flows; and general complex structures and the local balance between vorticity and rate of strain in 3d turbulent flows (six dimensions in terms of the phase space of the fluid).

The workshop led to new collaborations, and several important new issues have emerged, which are the subject of ongoing research. In particular, the question as to whether a given Calabi–Yau metric can be associated with a solution of the Navier–Stokes equations is being investigated. Further, some exact solutions of the incompressible Navier–Stokes equations in three dimensions, such as Burgers' vortex, have been shown to be related to solutions of the incompressible equations in two dimensions using Hamiltonian symmetry reduction. Finally, the Poisson equation for the pressure and the condition for incompressibility have been shown to be encoded in an anti-self-dual holomorphic three-form: this construction may facilitate the application of twistor theory.

Organizers

Jock McOrist (University of Surrey)
Ian Roulstone (University of Surrey)
Martin Wolf (University of Surrey)

Speakers

Bertrand Banos (Université de Bretagne-Sud)
Yann Brennier (École Polytechnique)
Bin Cheng (University of Surrey)
Mike Cullen (Met Office)
Xenia de la Ossa (University of Oxford)
Sylvain Delahaies (University of Surrey)
Philippe Delanoë (Université de Nice Sophia-Antipolis)
Mikhail Feldman (University of Wisconsin)
Wilfrid Gangbo (Georgia Institute of Technology)
John Gibbon (Imperial College London)
Marco Gualtieri (University of Toronto)
Bob Kerr (University of Warwick)
Jock McOrist (University of Surrey)
John Norbury (University of Oxford)
Ian Roulstone (University of Surrey)
Vladimir Rubtsov (University of Angers)
Edriss Titi (Weizmann Institute of Science)
Chuong Tran (University of St. Andrews)
Cesare Tronci (University of Surrey)
David Waldram (Imperial College London)
Martin Wolf (University of Surrey)

EXTREMAL AND PROBABILISTIC COMBINATORICS

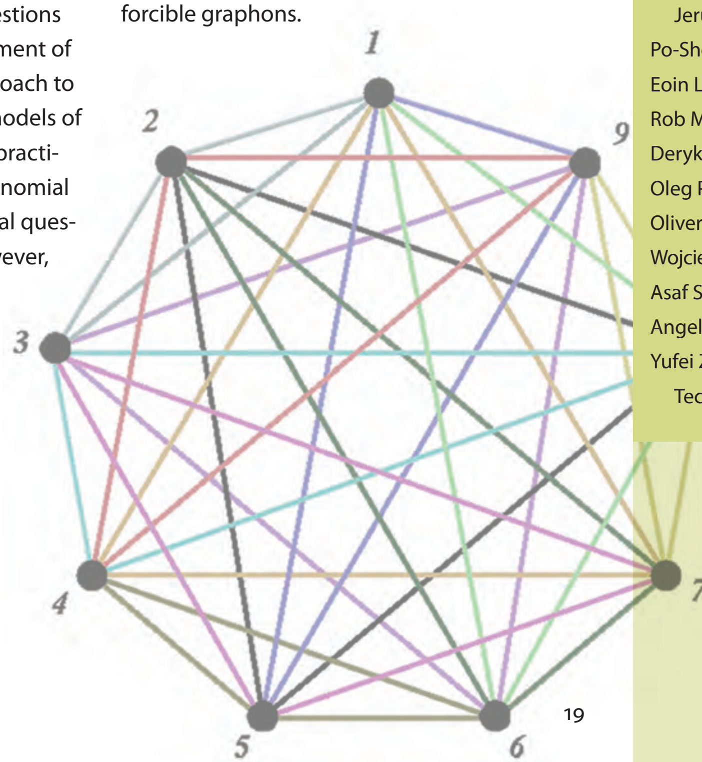
June 2-6, 2014

Extremal combinatorics and probabilistic combinatorics are two of the most central branches of modern combinatorics. Extremal combinatorics deals with problems of determining or estimating the maximum or minimum possible cardinality of a collection of finite objects satisfying certain requirements. Such problems are often related to other areas including computer science, logic, number theory and geometry. Probabilistic combinatorics can be described informally as a hybrid between combinatorics and probability, where the main object of study is probability distributions on discrete structures, a notable example being the theory of random graphs.

Recent years have seen some truly spectacular developments in both subjects, whose impact is likely to go far beyond their immediate context. They include: the transference principle and its applications to extremal problems on random structures; recent breakthroughs on classical questions in graph Ramsey theory; the development of flag algebras as a computational approach to extremal questions; analysis of new models of random objects, partly motivated by practical applications and settings; the polynomial method and its applications to classical questions in combinatorial geometry. However, these are only a few representative examples of a healthy and burgeoning field.

This workshop focused on the interactions between various areas of extremal and probabilistic combinatorics. Topics included extremal problems for graphs and set systems, Ramsey theory, combinatorial number theory, combinatorial geometry, random graphs, probabilistic methods and graph limits. Researchers in the field from around the world, as well as several of the brightest and most promising researchers from the younger generation, were brought together to facilitate the exchange of the latest ideas and developments.

Some highlights of the conference included Peter Keevash's talk describing his recent breakthrough on constructing combinatorial designs of all orders; Jacob Fox's talk on the combinatorics of permutations; and Daniel Král's talk on the construction of exotic finitely forcible graphons.



Organizers

David Conlon (University of Oxford)
Michael Krivelevich (Tel Aviv University)
Alex Scott (University of Oxford)
Benny Sudakov (ETH Zürich)

Speakers

Peter Allen (London School of Economics)
Julia Böttcher (London School of Economics)
David Ellis (Queen Mary, University of London)
Jacob Fox (Massachusetts Institute of Technology)
Dan Hefetz (University of Birmingham)
Jeff Kahn (Rutgers University)
Gil Kalai (Hebrew University of Jerusalem)
Peter Keevash (University of Oxford)
Daniel Kráľ (University of Warwick)
Daniela Kuhn (University of Birmingham)
Choongbum Lee (Massachusetts Institute of Technology)
Anita Liebenau (University of Warwick)
Nathan Lineal (Hebrew University of Jerusalem)
Po-Shen Loh (Carnegie Mellon University)
Eoin Long (University of Oxford)
Rob Morris (IMPA)
Deryk Osthus (University of Birmingham)
Oleg Pikhurko (University of Warwick)
Oliver Riordan (University of Oxford)
Wojciech Samotij (University of Cambridge)
Asaf Shapira (Tel Aviv University)
Angelika Steger (ETH Zürich)
Yufei Zhao (Massachusetts Institute of Technology)

Photo by Eduardo de Córdoba

CMI Summer School

PERIODS AND MOTIVES:
FEYNMAN AMPLITUDES IN THE 21ST CENTURY

June 30 – July 25, 2014
Kurusch Ebrahimi-Fard

Quantum field theory (QFT) is one of the most successful theories in physics. Perturbative QFT led to theoretical predictions that are in breathtaking agreement with experimental data. It is based on power series expansions of probability amplitudes for physical processes. The coefficients of these power series expansions are given by integrals over a large number of variables. They are known as Feynman amplitudes, and can be represented pictorially by the so-called Feynman diagrams. They are named after their inventor, the theoretical physicist and Nobel laureate Richard P. Feynman.

The predictive power of perturbative QFT hinges on finding efficient ways of calculating Feynman amplitudes. In this respect, the past two decades have seen enormous progress. Nowadays, advanced techniques from algebra, geometry and combinatorics play an important role, which has turned the subject of calculating Feynman amplitudes into a fruitful playground for mathematicians and physicists alike. The works of Spencer Bloch, Francis Brown, Alain Connes, Hélène Esnault, Alexander Goncharov, Dirk Kreimer, Matilde Marcolli, and others, uncovered exciting links between the Feynman amplitudes on the one hand, and the theory of motives of algebraic varieties and their periods on the other hand, which generated highly successful research collaborations between mathematicians and physicists, with tremendous impact in this area.

The mathematical concepts of periods and motives are pivotal in these developments. Periods are just numbers that are computed as integrals of algebraic differential forms over topological cycles on algebraic varieties. The term originated from the period of a periodic elliptic function, which can be computed as an elliptic integral. The theory of motives was introduced by Grothendieck as a "universal cohomology theory" to explain the common features of different cohomology theories. One may describe motives as an intermediate step between algebraic varieties and their linear invariants (cohomology). The motives associated to smooth projective varieties are called "pure", while the motives associated to quasi-projective and singular varieties are called "mixed" (they consist of different pure pieces). Although a complete and satisfactory theory of mixed motives is not yet available, impressive results and tools have already been developed. For instance the category of mixed Tate motives (the simplest mixed motives) is well understood.

The Institute de Ciencias Matemáticas (ICMAT – Institute of Mathematical Sciences) in Madrid, Spain, offered the perfect setting for the 2014 Clay Mathematics Institute Summer School. The school was aimed at training a new generation of graduate students and young researchers in the state of the art techniques behind these deep and important results.

The summer school consisted of three weeks of lecture courses supplemented by exercise and problem sessions, as well as a student seminar:

- *The mathematics of Feynman amplitudes* by Spencer Bloch
- *Motivic multiple zeta values* by José I. Burgos Gil
- *An introductory course on ℓ-adic Galois representations of function fields* by Hélène Esnault, Lars Kindler and Kay Rülling
- *Feynman integrals, periods and motives* by Matilde Marcolli

These courses provided a pedagogical introduction to advanced tools and modern techniques necessary to play an active role in current research.

Spencer Bloch developed a rigorous background on the mathematics of Feynman amplitudes. Although guided by physics, the lectures were unabashedly mathematical in character. Starting with the integrals of interest in physics, Bloch showed, in concrete examples, how these integrals arise as periods of geometric motives. Then, using the machinery of Hodge structures and motives, he showed how to express Feynman amplitudes in terms of special functions like polylogarithms and elliptic polylogarithms. The series of lectures ended with a discussion on thresholds in physics that correspond to the concept of monodromy in mathematics.

José Burgos sought to explain the results of F. Brown, A. Goncharov and T. Terasoma, covering multiple zeta values (MZVs) and their combinatorial properties. The set of MZVs is a collection of numbers with a very rich combinatorial structure that appear frequently in the computation of Feynman amplitudes. There are conjectures predicting the dimension over d_n of the space of MZVs of a given weight. We have only been able to prove that the expected dimensions are an upper bound of the real dimensions, and the proof of such results uses deep properties of motives. To explain the main ingredients of such proofs, Burgos discussed the notions of mixed Hodge structures, polylogarithms and MZVs as periods, mixed Tate motives, and the Tannakian formalism for defining the fundamental group of a category.

The main objective of the course given by Hélène Esnault, Lars Kindler and Kay Rülling, was to understand a recent result by Pierre Deligne, that states that there are only finitely many irreducible lisse Q_ℓ-sheaves up to twist on an open variety X, with suitably bounded ramification at infinity. The course was divided into two parts, an introductory part, where the basics of ramification theory were presented, and an advanced part devoted to the proof of Deligne's main theorem. The topics treated in the introductory part included infinite Galois theory, ramification groups, the Swan conductor, the Swan representation, ℓ-adic sheaves and the formula of Grothendieck-Ogg-Shararevich, as well as a discussion on the difficulties one encounters, when studying ramification in dimensions bigger that one.

The course by Matilde Marcolli covered different aspects of Feynman integrals in momentum and configuration spaces, and some closely related mathematical generalizations. The course entered a fruitful and exciting interaction with the courses by Bloch and Burgos. This was, in fact, very efficient in providing motivations together with theoretical framework for the students. The list of topics of Marcolli's course included algebraic renormalization and the Connes-Kreimer Hopf algebra of Feynman graphs, algebro-geometric Feynman rules arising from geometry of singular varieties and varieties over "the field of one element".

The courses of the fourth week of the school were aiming at a higher level. In five mini-courses, leading experts presented topics of great future promise, and outlined some open problems under current investigation. Each of the five mini-courses focused on topics that are at the forefront of current research:

- *Motives, motivic Galois groups and periods* by Joseph Ayoub
- *Multiple modular values* by Francis Brown
- *Bootstraps for scattering amplitudes in N=4 super Yang–Mills theory* by James Drummond
- *Feynman integrals, scattering Amplitudes and the Hopf algebra of multiple polylogarithms* by Claude Duhr
- *ℓ-adic Galois representations of function fields over finite fields* by Hélène Esnault

The lectures took students deep into the current research, and got them acquainted with problems right at the research frontier of the interface between modern mathematics and physics.

The backgrounds, levels and origins of students were rather mixed. Therefore it turned out to be of great advantage that all students were lodged at the same student residence on campus of Universidad Autónoma de Madrid, near the venue. It took only a few days into the lectures before the students started to interact. In fact, it was rather exciting (and surprising) to see how quickly the audience started to engage with the lectures. In this respect, it was of some advantage that a senior participant (himself a researcher from Paris) was participating. His insistence on understanding details, motivations and arguments encouraged younger participants to take an active role in the lectures. An important part in the school was the student seminars. Several participants presented research projects (all of them related to the topics of the school) in one hour talks. This broadened the scope of the school and created research discussions between the students. In addition, several students started projects.

Organizers

José Ignacio Burgos Gil (ICMAT-CSIC)
Kurusch Ebrahimi-Fard (ICMAT-CSIC)
David Ellwood (Harvard University)
Dominique Manchon (CNRS)
Juano Rué Perna (Freie Universität Berlin)
Nick Woodhouse (CMI)

Lecturers

Joseph Ayoub (Universität Zürich)
Spencer Bloch (University of Chicago)
Francis Brown (CNRS, IHES)
José I. Burgos Gil (CSIC, Madrid)
James Drummond (CNRS)
Claude Duhr (Durham University)
Hélène Esnault (Freie Universität Berlin)
Lars Kindler (Freie Universität Berlin)
Matilde Marcolli (California Institute of Technology)
Kay Rülling (Freie Universität Berlin)

Photo by Chrystal Cherniwchan

LMS/CMI Research Schools

In 2014 CMI entered into a partnership with the London Mathematical Society to deliver four week-long research schools at various locations in the UK. Following a call in Spring 2013, a number of excellent proposals were received, from which the following were selected.

The schools were successful in attracting a very strong pool of applicants from around the world, with a total of 173 participants from 24 countries across the four schools.

AUTOMORPHIC FORMS AND RELATED TOPICS

June 30 – July 5, 2014
University of Bristol, UK

Automorphic forms are present in almost every area of modern number theory. In recent decades there has been a starburst of activity and progress in this broad area, leading to many new directions, applications and links with other areas within mathematics and mathematical physics.

This research school brought together an international group of PhD students and early-career researchers to provide training on some topics that are having great impact on current research in automorphic forms. It aimed to facilitate new connections between topics of the school and topics of the participants' current research programs, as well as new connections between researchers.

The school was comprised of three two-day intensive mini-courses, each team-taught by a pair of experts. :

Organizers
Jennifer Beineke (Western New England College)
Jonathan Bober (University of Bristol)
Lynne Walling (University of Bristol)

Lecturers
Jim Cogdell (Ohio State University)
John Cremona (University of Warwick)
Tim Dokchitser (University of Bristol)
Bill Duke (UCLA)
Solomon Friedberg (Boston College)
Paul Jenkins (Brigham Young University)

Colloquium Speaker
Sarah Zerbes (University College London)

- *Explicit methods for modular forms and L-functions* by John Cremona and Tim Dokchister
- *The legacy of Ramanujan* (focusing mainly on weakly holomorphic modular forms) by William Duke and Paul Jenkins
- *The Langlands program* by James Cogdell and Solomon Friedberg

A particularly successful aspect of the school was Jim Cogdell's lectures on the final day, which brought together topics from all three mini-courses. A colloquium by Sarah Zerbes on *Euler systems and the Birch–Swinnerton-Dyer conjecture* was well received by the students, instructors and TAs.

AN INVITATION TO GEOMETRY AND TOPOLOGY VIA G2

Organizers
Mark Haskins (Imperial College London)
Jason Lotay (University College London)
Simon Salamon (Kings College London)

Lecturers
Robert Bryant (Duke University)
Jason Lotay (University College London)
Johanes Nordström (University of Bath)

Guest Lecturers
Nigel Hitchin (University of Oxford)
Bobby Acharya (Kings College London)
Mark Haskins (Imperial College London)

July 7 – 11, 2014
Imperial College London, UK

Research in G_2 geometry plays a crucial role in modern geometry and topology and provides a focused area where a wide variety of mathematical techniques are implemented which are of use to graduates and post-doctoral researchers working on a range of topics.

The aim of this research school was to give a thorough introduction to G_2 geometry, starting from fundamental material and progressing through to recent breakthroughs and current research. The school also aimed to introduce participants to topics of broader interest in algebra (e.g. representation theory), analysis (e.g. elliptic regularity), geometry (e.g. holonomy) and topology (e.g. characteristic classes). It also indicated some connections beyond mathematics to contemporary theoretical physics.

To cover the fundamental material, the main body of the research school consisted of three lecture courses, each supported by tutorials:

- *An introduction to special holonomy* by Robert Bryant
- *Calibrated submanifolds* by Jason Lotay
- *Compact G_2 manifolds* by Johannes Nordström

Three one-hour lectures by Bobby Acharya, Mark Haskins and Nigel Hitchin bridged the gap between the material in the main lectures and current research.

ALGEBRAIC LIE THEORY AND REPRESENTATION THEORY

Organizers
Gwyn Bellamy (University of Glasgow)
Simon Goodwin (University of Birmingham)

Lecturers
Iain Gordon (University of Edinburgh)
Andrew Mathas (University of Sydney)
Catharina Stroppel (University of Bonn)

Guest Lecturer
Peter Tingley (Loyola University Chicago)

August 25 – 29, 2014
University of Glasgow, UK

The theory of Lie groups and Lie algebras is a central theme in mathematics, which has found striking applications in diverse areas of mathematics and physics. In the 1950s the "analytic theory" was extended to arbitrary algebraically closed fields and introduced the area of mathematics now known as algebraic Lie theory. This led to the introduction of many important new algebraic structures, which play a fundamental role in in Lie theory and in other areas of mathematics and physics. Recently, there have been many exciting and groundbreaking advances, which provide new insights to this field, often involving a novel mixture of algebraic, combinatorial and geometric techniques.

The research school offered three lecture courses, each of which covered areas of major current research interest and gave the participants an overview of where future research is likely to be focused:

- *Rational Cherednik algebras* by Iain Gordon
- *Quiver Hecke algebras* by Andrew Mathas
- *Categorification in Lie theory* by Catharina Stroppel

The topics were selected because they are dynamic research areas with major recent international interest in which there is definite scope for further breakthroughs. In addition, these areas are enriched by their interaction with each other. Each lecture course consisted of five lectures, complemented by tutorial problem sessions.

A guest lecture by Peter Tingley on *Mirkovic–Vilonen Polytopes* complemented the lecture courses and was well received by the participants.

BOUNDED GAPS BETWEEN PRIMES

September 22 – 26
University of Oxford, UK

In a spectacular breakthrough, Yitang Zhang proved that there are infinitely many pairs of primes differing by at most 70 million. Due to further advances of Maynard and Tao, and the collaborative Polymath Project, 70 million has been reduced to a few hundred. This course introduced attendees to the mathematics surrounding these developments.

The school consisted of four lecture courses, each supplemented by tutorial sessions:

- *Introduction to prime number theory;* ζ- and L-functions, the prime number theorem by Andrew Granville
- *The Bombieri–Vinogradov theorem about distribution of primes in progressions; Introduction to sieve theory* by Kannan Soundararajan
- *The methods of Goldston, Pintz and Yildirim and Maynard–Tao* by James Maynard
- *Inputs from algebraic geometry* by Emmanuel Kowalski

Guest lecturer Terence Tao spoke on his recent joint work on large gaps between primes and Yitang Zhang spoke on his original breakthrough on bounded gaps between primes. A discussion session on the notion of *Polymath* (massively collaborative mathematics) was chaired by Ben Green, with panelists Emmanuel Kowalski, James Maynard and Terence Tao.

Organizers
Ben Green (University of Oxford)
Roger Heath-Brown (University of Oxford)

Lecturers
Andrew Granville (University of Montreal)
Kannan Soundararajan (Stanford University)
Emmanuel Kowalski (ETH Zürich)
James Maynard (University of Oxford)

Guest Lecturers
Terence Tao (University of California, Los Angeles)
Yitang Zhang (University of New Hampshire)

Enhancement and Partnership

2013 activities
2014 activities

CMI's Enhancement and Partnership Program, introduced in 2012, aims to add value to activities that have already been planned, particularly by increasing international participation. In accordance with CMI's mission and its status as an operating foundation, its funding is utilized to enhance mathematical activities organized by, or planned in partnership with, other organizations. In 2014, CMI partnered in 42 initiatives in 18 countries, often by funding a distinguished international speaker or supporting participants from outside the host country.

January – July | **Central Configurations, Periodic Orbits and Beyond in Celestial Mechanics** | CRM, Bellaterra, Spain

January 8 – March 15 | **Oscar Garcia Prada visit to University of Oxford** | Oxford, UK

January 20 – 24 | **Representation Theory Days in Patagonia** | Punta Arenas, Chile

February 17 – 21 | **Perfectoid Spaces and their Applications** | MSRI, Berkeley, CA

March 17 | **SET for Britain 2014** | London, UK

April 7 – 10 | **British Mathematical Colloquium** | Queen Mary University of London, UK

April 12 – 15 | **Applications of Automorphic Forms in Number Theory and Combinatorics** | Louisiana State University, Baton Rouge, LA

April 20 – 24 | **Women in Numbers** | Banff International Research Station, Canada

May 12 – 16 | **Rational and Integral Points on Higher-dimensional Varieties** | American Institute of Mathematics, Palo Alto, CA

May 19 – 23 | **Representation of Reductive Groups** | MIT, Cambridge, MA

May 26 – 30 | **From Macdonald Processes to Hecke Algebras and Quantum Integrable Systems** | Institut Henri Poincaré, Paris, France

June 2 – 7 | **Partial Differential Equations and Geometric Measure Theory** | Cetraro, Cosenza, Italy

June 2 – 14 | **Algebraic Geometry and Number Theory** | Galatasaray University, Istanbul, Turkey

June 2 – 27 | **Summer School in Probability** University of British Columbia, Vancouver, BC, Canada

June 16 – 20 | **12th Workshop on Interactions between Dynamical Systems and Partial Differential Equations (JISD 2014)** | Universitat Politècnica de Catalunya, Barcelona, Spain

June 16 – 27 | **Summer School on Quiver Hecke Algebras and Conference on Geometric Representation** | Institut d'Etudes Scientifiques de Cargese, Corsica

June 23 – 27 | **Algebra, Geometry and Physics** | IHES, Paris, France

June 23 – 27 | **Apollonian Circle Packings Summer School** | Institute Mittag-Leffler, Stockholm, Sweden

June 23 – 27 | **Manifolds, K-theory and Related Topics** | Inter University Centre Dubrovnik, Croatia

June 23 – 27 | **Conference in Honor of Richard Stanley** | MIT, Cambridge, MA

June 23 – 27 | **Strings 2014** | Princeton University, NJ

June 27 | **Non-Western Mathematics** | University of Oxford, UK

June 29 – July 3 | **International Conference on Formal Power Series and Algebraic Combinatorics** | DePaul University, Chicago, IL

June 30 – July 4 | **Groups, Numbers and Dynamics Workshop** | Isaac Newton Institute, Cambridge, UK

June 30 – July 11 | **Summer Graduate School on Algebraic Topology** | Centro de Investigación en Matemáticas, Guanajuato, Mexico

July 5 – 8 | **Symmetries and Correspondences in Number Theory, Geometry, Algebra and Quantum Computing** | University of Oxford, UK

July 7 – 11 | **Building Bridges: 2nd EU/US Workshop on Automorphic Forms and Related Topics** | University of Bristol, UK

July 14 – 18 | **30th International Colloquium on Group Theoretical Methods in Physics** | University of Ghent, Belgium

July 14 – 18 | **Analytic Number Theory and its Applications** | Perrotis College, Thessaloniki, Greece

July 27 – August 1 | **37th Conference on Stochastic Processes and their Applications** | Universidad de Buenos Aires, Argentina

August 17 – 22 | **UKMT Summer School for Girls** | Balliol College, University of Oxford, UK

August 23 – 26 | **Holomorphic Dynamics in One and Several Variables** | Gyeong-ju, Korea

September 1 – 5 | **Algebraic Varieties: Bundles, Topology, Physics (VBAC 2014)** | Freie Universität Berlin, Germany

September 19 – 21 | **Harmonic Analysis and Partial Differential Equations** | University of Chicago, IL

September 24 – 26 | **New Geometric Structures in Scattering Amplitudes** | University of Oxford, UK

October 26 | **Michael Sipser Colloquium** | MIT, Cambridge, MA

November 7 – 10 | **Homotopy Type Theory** | University of Oxford, UK

November 17 – 21 | **Categorical Structures in Harmonic Analysis** | MSRI, Berkeley, CA

December 1 – 5 | **Automorphic Forms, Shimura Varieties, Galois Representations and L-functions** | MSRI, Berkeley, CA

December 1 – 5 | **Multiple Zeta Values, Modular Forms and Elliptic Motives** | ICMAT, Madrid, Spain

December 11 – 20 | **Foundations of Computational Mathematics** | Universidad de la Republica del Uruguay, Montevideo, Uruguay

December 15 – 19 | **Analysis, Spectra, and Number Theory** | Princeton University and Institute for Advanced Study, Princeton, NJ

Publications

Selected Articles by Research Fellows

Tim Austin

Scenery entropy as an invariant of RWRS processes, submitted. arXiv: 1405.1468.

Partial difference equations over compact Abelian groups, I: modules of solutions, submitted. arXiv: 1305.7269

Partial difference equations over compact Abelian groups, II: step-polynomial solutions, submitted. arXiv: 1309.3577

Ivan Corwin

Macdonald processes, with Alexei Borodin. *Probability Theory and Related Fields*, 158 (2014), 225-400.

Tropical combinatorics and Whittaker functions, with Neil O'Connell, Timo Seppäläinen, Nikos Zygouras. *Duke Mathematical Journal*, 163 (2014), 513-563.

Semyon Dyatlov

Spectral gaps for normally hyperbolic trapping, submitted. arXiv: 1403:6401.

Pollicott–Ruelle resonances for open systems, with Colin Guillarmou, submitted. arXiv: 1410:5516.

June Huh

Minor numbers of projective hypersurfaces with isolated singularities. *Duke Mathematical Journal*, 163 (2014), 1525-1548.

Positivity of Chern classes of Schubert cells and varieties, to appear in *Journal of Algebraic Geometry*. arXiv: 1302.5852.

Aaron Pixton

The Chern character of the Verlinde bundle over the moduli space of stable curves, with Alina Marian, Dragos Oprea, Rahul Pandharipande, Dimitri Zvonkine, to appear in *Journal für die Reine und Angewante Mathematik*. arXiv: 1311.3028.

Relations on $\overline{M}_{g,n}$ via 3-spin structures, with Rahul Pandharipande and Dimitri Zvonkine, to appear in *Journal of the American Mathematical Society*. arXiv: 1303:1043.

Peter Scholze

Perfectoid spaces and their applications, *Proceedings of the ICM 2014*.

The pro-étale topology for schemes, with Bhargav Bhatt. arXiv: 1309.1198.

Jack Thorne

Raising the level for GL_n, *Forum of Mathematics, Σ*, 2 (2014), e16.

E_6 and the arithmetic of a family of non-hyperelliptic curves of genus 3, to appear in *Forum of Mathematics, Pi*. www.dpmms.cam.ac.uk/~jat58/.

Miguel Walsh

The algebraicity of ill-distributed sets. *Geometric and Functional Analysis*, 24 (2014), 959-96.

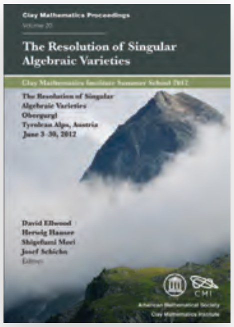
Bounded rational points on curves, to appear in *International Mathematics Research Notices*. arXiv: 1308.0574.

Alex Wright

Classification of higher rank orbit closures in $H^{odd}(4)$, with David Aulicino and Duc-Mahn Nguyen, to appear in *Journal of the European Mathematical Society*. arXiv: 1308.5879.

Hodge-Teichmüller planes and finiteness results for Teichmüller curves, with Carlos Matheus, to appear in *Duke Mathematical Journal*. arXiv: 1308.0832.

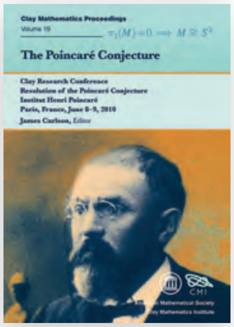
Books



The Resolution of Singular Algebraic Varieties

Editors: David Ellwood, Herwig Hauser, Shigefumi Mori and Josef Schicho. CMI/AMS, 2014, 340 pp., softcover, ISBN: 0-8218-8982-4. List price: $101. AMS Members: $80.80. Order Code: CMIP/20.

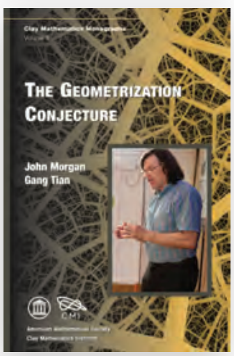
Resolution of Singularities has long been considered as being a difficult to access area of mathematics. The more systematic and simpler proofs that have appeared in the last few years in zero characteristic now give us a much better understanding of singularities. They reveal the aesthetics of both the logical structure of the proof and the various methods used in it. This volume is intended for readers who are not yet experts but always wondered about the intricacies of resolution. As such, it provides a gentle and quite comprehensive introduction to this amazing field. The book may tempt the reader to enter more deeply into a topic where many mysteries—especially the positive characteristic case—await to be discovered.



The Poincaré Conjecture

Editor: James Carlson. CMI/AMS, 2014, 178 pp., softcover, ISBN: 0-8218-9865-5. List price: $69. AMS Members: $55.20. Order Code: CMIP/19.

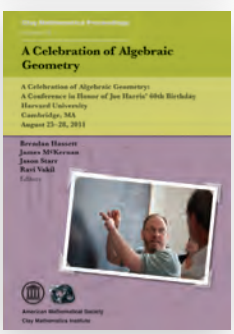
The conference to celebrate the resolution of the Poincaré conjecture, which is one of CMI's seven Millennium Prize Problems, was held at the Institut Henri Poincaré in Paris. Several leading mathematicians gave lectures providing an overview of the conjecture—its history, its influence on the development of mathematics, and its proof. This volume contains papers based on the lectures at that conference. Taken together, they form an extraordinary record of the work that went into the solution of one of the great problems of mathematics.



The Geometrization Conjecture

Authors: John Morgan and Gang Tian. CMI/AMS, 2014, 291 pp., hardcover, ISBN: 0-8218-5201-9. List price: $81. AMS Members: $64.80. Order Code: CMIM/5.

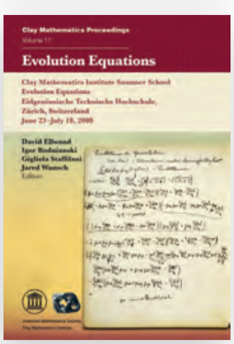
This book gives a complete proof of the geometrizaton conjecture, which describes all compact 3-manifolds in terms of geometric pieces, i.e., 3-manifolds with locally homogeneous metrics of finite volume. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. In the course of proving the geometrization conjecture, the authors provide an overview of the main results about Ricci flows with surgery on 3-dimension manifolds, introducing the reader to difficult material. The book also includes an elementary introduction to Gromov-Hausdorff limits and to the basics of the theory of Alexandrov spaces. In addition, a complete picture of the local structure of Alexandrov surfaces is developed.



A Celebration of Algebraic Geometry

Editors: Brendan Hassett, James McKernan, Jason Starr and Ravi Vakil. CMI/AMS, 2013, 599 pp., softcover, ISBN: 0-8218-8983-4. List Price: $149. AMS Members: $119.20. Order Code: CMIP/18.

This volume resulted from the conference held in honor of Joe Harris' 60th birthday. Harris is famous around the world for his lively textbooks and enthusiastic teaching, as well as for his seminal research contributions. The articles are written in this spirit: clear, original, engaging, enlivened by examples, and accessible to young mathematicians. The articles focus on the moduli space of curves and more general varieties, commutative algebra, invariant theory, enumerative geometry both classical and modern, rationally connected and Fano varieties, Hodge theory and abelian varieties, and Calabi-Yau and hyperkähler manifolds. Taken together, they present a comprehensive view of the long frontier of current knowledge in algebraic geometry.



Evolution Equations

Editors: David Ellwood, Igor Rodnianski, Gigliola Staffilani and Jared Wunsch. CMI/AMS, 2013, 572 pp., softcover, ISBN: 0-8218-6861-6. List Price: $149. AMS Members: $119.20. Order Code: CMIP/17.

This volume is a collection of notes from lectures given at the 2008 Clay Mathematics Institute Summer School, held in Zurich, Switzerland. The lectures were designed for graduate students and mathematicians within five years of their PhD and the main focus of the program was on recent progress in the theory of evolution equations. Such equations lie at the heart of many areas of mathematical physics and arise not only in situations with a manifest time evolution (such as nonlinear wave and Schrödinger equations) but also in the high energy or semi-classical limits of elliptic problems.

Topics in Noncommutative Geometry

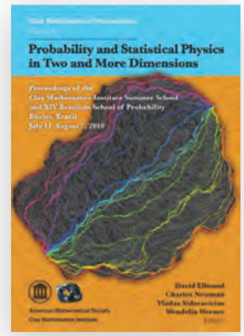
Editor: Guillermo Cortiñas. CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16.

This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.

Probability and Statistical Physics in Two and More Dimensions

Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner. CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15.

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, "Probability and Statistical Physics in Two and More Dimensions" and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.



Grassmannians, Moduli Spaces and Vector Bundles

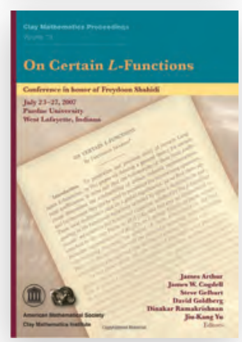
Editors: David A. Ellwood, Emma Previato. CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $58. AMS Members: $46.40. Order Code: CMIP/14.

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

On Certain *L*-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu. CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $136. AMS Members: $108.80. Order Code: CMIP/13.

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their *L*-functions, and both local and global theory are addressed.



Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg. CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $94. AMS Members: $75.20. Order Code: CMIP/12.

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

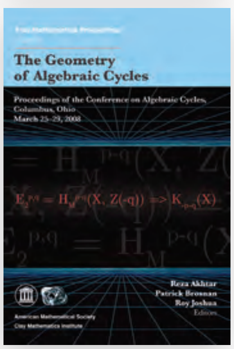
Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa. CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $136. AMS Members: $108.80. Order Code: CMIP/11.

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Kleinbock, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz . CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $104. AMS Members: $83.20. Order Code: CMIP/10.

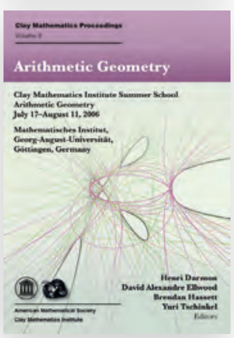
This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.



The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua. CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $55. AMS Members: $44. Order Code: CMIP/9.

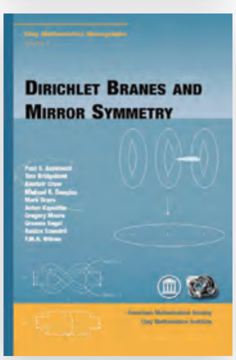
The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.



Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel. CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $125. AMS Members: $100. Order Code: CMIP/8.

This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

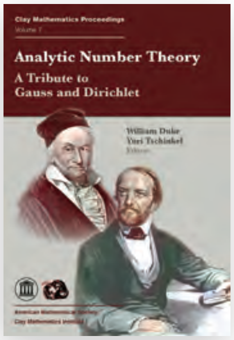


Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross. CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $115. AMS Members: $92. Order Code: CMIM/4.

The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.

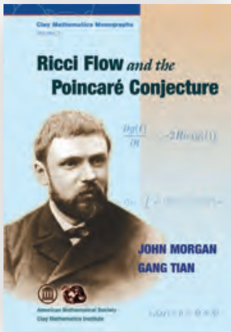
Analytic Number Theory: A Tribute to Gauss and Dirichlet



Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $53. AMS Members: $42.40. Order Code: CMIP/7.

This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.

Ricci Flow and the Poincaré Conjecture



Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $75. AMS Members: $60. Order Code: CMIM/3.

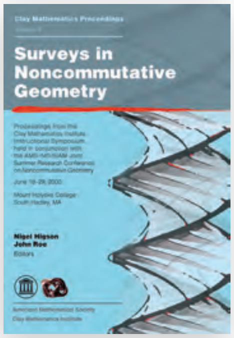
This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $32. AMS Members: $25.60. Order Code: MPRIZE.

This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

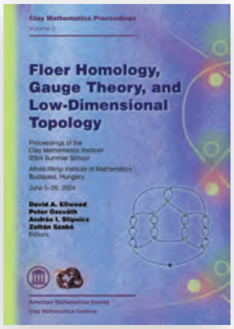
Surveys in Noncommutative Geometry



Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $53. AMS Members: $42.40. Order Code: CMIP/6.

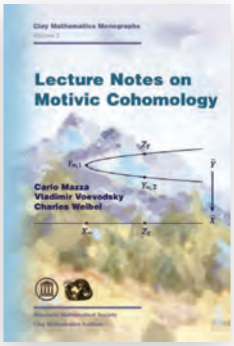
In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

Floer Homology, Gauge Theory, and Low-Dimensional Topology



Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $70. AMS Members: $56. Order Code: CMIP/5.

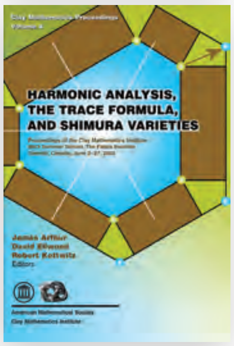
This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $50. AMS Members: $40. Order Code: CMIM/2.S.

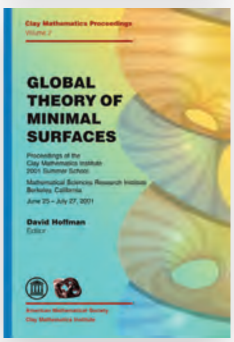
This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.



Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $138. AMS Members: $110.40. Order Code: CMIP/4.

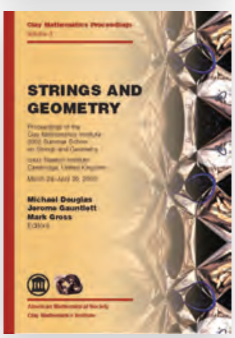
The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.



Global Theory of Minimal Surfaces

Editor: David Hoffman. CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $138. AMS Members: $110.40. Order Code: CMIP/2

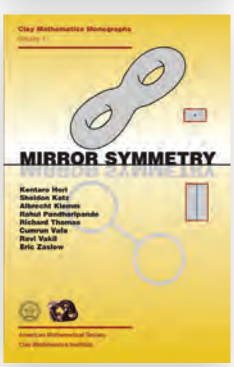
This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and thedouble-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.



Strings and Geometry

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $80. AMS Members: $64. Order Code: CMIP/3.

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.



Mirror Symmetry

Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $144. AMS Members: $115.20. Order Code: CMIM/1

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $91. ASM Members: $72.80. Order Code: CMIP/1.

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

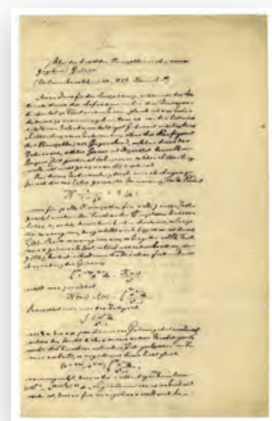
To order print copies of these books, please visit www.ams.org/bookstore. PDF versions are posted on CMI's Online Library six months after publication and can be found at www.claymath.org/node/262.

Digital Library

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Quillen Notebooks

Daniel Quillen obtained his PhD under the supervision of Raoul Bott at Harvard in 1961. He worked at MIT before moving to the University of Oxford in 1984. During his long mathematical career, Quillen kept a set of detailed notes which give a day-to-day record of his mathematical research.
www.claymath.org/publications/quillen-notebooks



Euclid's Elements, Constantinople, 888 AD (Greek). MS at the Bodleian Library

The oldest extant manuscript and printed editions of Euclid's Elements, in Greek (888 AD) and Latin (1482 AD), respectively. High resolution copies of the manuscript are available for study at the Bodleian Library, Oxford University. **www.claymath.org/euclids-elements**

Riemann's 1859 Manuscript

The manuscript in which Riemann formulated his famous conjecture about the zeroes of the zeta function.
www.claymath.org/publications/riemanns-1859-manuscript

Klein Protokolle

The "Klein Protokolle," comprising 8600 pages in 29 volumes, record the activity of Felix Klein's seminar in Göttingen for the years 1872-1912. **www.claymath.org/publications/klein-protokolle**

James Arthur Archive

James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author's complete published work in an easily accessible set of searchable PDFs. **www.claymath.org/publications/collected-works-james-g-arthur**

Notes of Talks at the I. M. Gelfand Seminar

The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin's notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education. **www.claymath.org/publications/notes-talks-imgelfand-seminar**

Nominations, Proposals and Applications

RESEARCH FELLOWSHIP NOMINATIONS

Nominations for Clay Research Fellows are considered once a year. The primary selection criteria for the Fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader. Selection decisions are made by the Scientific Advisory Board based on the nominating materials: letter of nomination, names and contact information for two other references, Curriculum Vitae, publication list for the nominee.

Address all nominations to Nick Woodhouse at president@claymath.org, copied to Naomi Kraker at admin@claymath.org.

WORKSHOPS AT THE MATHEMATICAL INSTITUTE

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty people, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. An application submitted three months before the workshop is sufficient. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

ENHANCEMENT AND PARTNERSHIP

The Clay Mathematics Institute invites proposals under its Enhancement and Partnership Program. The aim is to enhance activities that are already planned, particularly by funding international participation. The program is broadly defined, but subject to the general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board.

Nominations for Senior Scholars will be considered within the scope of the Enhancement and Partnership program. The aim of the Senior Scholar program is to foster mathematical research and the exchange of ideas by providing support for senior mathematicians who will play a leading role as "senior scientist" in a topical program at an institute or university. Senior Scholars will be in residence throughout the program and are expected to interact extensively with the other participants.

For more information, visit www.claymath.org/programs/enhancement-and-partnership-program. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

ANNUAL DEADLINES

- Research Fellows nominations: November 16
- Workshop proposals: March 1, June 1, September 1, December 1
- Enhancement and Partnership proposals, including Senior Scholars nominations: March 1, June 1, September 1, December 1

Nominations and proposals may also be mailed to:

Clay Mathematics Institute
Office of the President
Andrew Wiles Building
Radcliffe Observatory Quarter
Woodstock Road
Oxford OX2 6GG
United Kingdom

2015 Institute Calendar

Jan 1 – Dec 31	PRIMES	MIT, Cambridge, MA
Jan – June	Senior Scholar Artur Avila, *Periodic and Ergodic Spectral Problems*	Isaac Newton Institute, Cambridge, UK
Jan 19 – 23	AIMS-Stellenbosch University Number Theory Conference	AIMS, Cape Town, South Africa
Jan – July	Senior Scholar Wendelin Werner, *Random Geometry*	Isaac Newton Institute, Cambridge, UK
Feb 9 – 20	X Americas Conference on Differential Equations and Nonlinear Analysis	Buenos Aires, Argentina
Spring 2015	Senior Scholar Marc Burger, *Dynamics on Moduli Spaces of Geometric Structures*	MSRI, Berkeley, CA
Spring 2015	Senior Scholar Elon Lindenstrauss, *Geometric and Arithmetic Aspects of Homogeneous Dynamics*	MSRI, Berkeley, CA
Spring 2015	Senior Scholar Jeff Lagarias, *Phase Transitions and Emergent Properties*	ICERM, Providence, RI
March 9	SET for Britain	London, UK
April 13 – 17	Asymptotic Invariants of Groups Workshop	University of Oxford, UK
April 13 – 17	LMS-CMI Research School: Statistical Properties of Dynamical Systems	Loughborough University, UK
May 25 – 29	Random Polymers and Algebraic Combinatorics Workshop	University of Oxford, UK
June 1 – 5	Non-Archimedean Geometry and its Applications	University of Michigan, Ann Arbor, MI
June 8 – July 4	PIMS Summer School in Probability	McGill University, Montreal, Canada
June 15 – July 24	Ross Program	Ohio State University, Columbus, OH
June 15 – 16	International Conference in Number Theory and Physics	IMPA, Rio de Janeiro, Brazil
June 22 – 26	Arithmetic Geometry, Representation Theory and Applications	CIRM, Luminy, France
June 28 – July 18	Senior Scholars Ngô Bảo Châu and Andrei Okounkov, *Geometry of Moduli Spaces and Representation Theory*	PCMI, Park City, UT

June 28 – Aug 8	PROMYS	Boston University, MA
July 1 – 5	LMS-CMI Research School: Regularity and Analytic Methods in Combinatorics	University of Warwick, UK
July 6 – 10	LMS-CMI Research School: Developments in Modern Probability	University of Oxford, UK
July 7 – 10	Future Directions in Model Theory and Analytic Functions	University of Manchester, UK
July 12 – 31	Algebraic Geometry Summer Research Institute	University of Utah, Salt Lake City, UT
July 13 – Aug 21	PROMYS Europe	University of Oxford, UK
July 27 – Aug 1	XVIII International Congress on Mathematical Physics (ICMP 2015)	Santiago, Chile
Aug 3 – 12	Centre for Quantum Geometry on Moduli Spaces Masterclass	Aarhus University, Denmark
Aug – Dec	Senior Scholars Martin Hairer and Pierre Raphael, *New Challenges in PDE*	MSRI, Berkeley, CA
Sept 14 – 20	LMS-CMI Research School: Diophantine Equations	Baskerville Hall, Hay-on-Wye, UK
Sept 23 – 25	Algebraic Geometry	IHES, Bures-sur-Yvette, France
Sept 28 – Oct 2	Clay Research Conference and Workshops	University of Oxford, UK
Sept 28 – Oct 2	Geometry and Dynamics on Moduli Spaces	University of Oxford. UK
Sept 28 – Oct 2	Motives and Automorphic Forms	University of Oxford, UK
Sept 28 – Oct 2	Algebraic Topology: Manifolds unlocking higher structures	University of Oxford, UK
Sept 28 – Oct 2	Water Waves and Related Fluid Models	University of Oxford, UK
Sept 30	Clay Research Conference	University of Oxford, UK
Oct 26 – 30	Moduli Spaces in Geometry	CIRM, Luminy, France
Nov 22 – 27	BIRS International Math Outreach Workshop	BIRS, Banff, Canada
Dec 7 – 11	IUT Theory of Shinichi Mochizuki Workshop	University of Oxford, UK
Dec 9 – 10	Ada Lovelace Symposium	University of Oxford, UK

mission

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.
The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

CLAY MATHEMATICS INSTITUTE

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