



James Carlson

Dear Friends of Mathematics,

I would like to single out four activities of the Clay Mathematics Institute this past year that are of special interest. The first was a small workshop, organized on short notice, on the topic of resolution of singularities of algebraic varieties in characteristic $p > 0$. The characteristic zero case was solved by Heisuke Hironaka in a monumental paper, “Resolution of singularities of an algebraic variety over a field of characteristic zero I, II,” *Annals of Mathematics* 1964 (pp 109-203 and 205-326). Without resolution of singularities, modern algebraic geometry would be a far different subject. Since Hironaka’s paper, resolution of singularities in characteristic zero has been better and better understood by a series of authors including Bierstone and Millman (1997), Encinas and Villamayor (1998) Encinas and Hauser (2002), Cutkosky (2004), Włodarczyk (2005), and Kollár (2007). A complete proof, suitable for an advanced graduate course, can now be given in a tenth of the space of the original. Such is the progress of mathematics!

In characteristic $p > 0$, much less is known. The case of curves is elementary (normalize) while the case of surfaces, due to Abhyankar (1956) is difficult. Abhyankar also proved resolution for threefolds in characteristic $p > 5$. Cutkosky recently gave a short proof of this result; Cossart and Piltant have given a proof valid in all characteristics.

The major area of progress in the general case has been the work of de Jong (1996) and Abramovich-de Jong (1998), who proved a weaker result: a singular variety is the image under a dominant finite map of

smooth variety. This is sufficient for many, but not all applications. For instance, it is still not known whether the dimension of the space of holomorphic q -forms is a birational invariant in characteristic p . In recent years there has been renewed progress on the problem by Hironaka, Villamayor and his collaborators, Włodarczyk, Kawanoue-Matsuki, Teissier, and others. A workshop at the Clay Institute brought many of those involved together for four days in September to discuss recent developments. Participants were Dan Abramovich, Dale Cutkosky, Herwig Hauser, Heisuke Hironaka, János Kollár, Tie Luo, James McKernan, Orlando Villamayor, and Jaroslaw Włodarczyk. A superset of this group met later at RIMS in Kyoto at a workshop organized by Shigefumi Mori.

Second was the CMI workshop organized by Rahul Pandharipande and Daves Maulik. Workshops are intended to foster communication and hence the creation of new mathematical knowledge. This one had a quick payoff: the solution of the Yau-Zaslow conjecture for rational curves on $K3$ surfaces (see “Noether-Lefschetz theory and the Yau-Zaslow conjecture,” A. Klemm, D. Maulik, R. Pandharipande, and E. Scheidegger; arXiv:0807.2477).

Third was the CMI workshop “Stringy Reflections on the LHC.” This meeting, organized by Cumrun Vafa, brought together leading string theorists and particle phenomenologists to discuss the potentially observable data that could emerge from the Large Hadron Collider in Geneva after it begins operation in 2009.

Fourth was the Clay Research Summer School, held in Zürich, Switzerland on the subject of Evolution Equations. The month-long school was organized by David Ellwood, Igor Rodnianski, Gigliola Staffilani, and Jared Wunsch. It was the first such school in analysis per se. Especially noteworthy was the large number of participants, one hundred seventy-eight, and the fact that thirty-eight came with their own funding. As with all Clay Summer Schools, a volume with written versions of the courses and the topical lectures will appear in the CMI-AMS proceeding series.

Sincerely,

James A. Carlson