Poisson kernel. Given f on $\mathbb{T}$ extend as a harmonic fn to D.

$f = e^{in\theta}$ $\quad u = z^n \quad n \geq 0$

$e^{-in\theta}$ $\quad u = \bar{z}^n \quad n \geq 0$

$$\sum_{n \geq 0} z^n \int e^{-in\phi} f(\phi) \frac{d\phi}{2\pi}$$

$\zeta = e^{i\phi}$
$$\sum_{n \geq 0} z^n \zeta^{-n} + \sum_{n \geq 1} \bar{z}^n \zeta^n$$

$$= \frac{1}{1 - z\zeta^{-1}} + \frac{\bar{z}\zeta}{1 - \bar{z}\zeta} = \frac{1 - \bar{z}\zeta + \bar{z}\zeta - |z|^2}{|1 - z\zeta^{-1}|^2}$$

$$f(z) = \sum_{n \geq 0} a_n z^n \quad \text{anal in } D$$

$$\operatorname{Im} f(\zeta) = \frac{f(\zeta) - \overline{f(\zeta)}}{2i} \qquad |\zeta| = 1.$$

$$2i \operatorname{Im} f(\zeta) = \sum_{n \geq 1} a_n \zeta^n + a_0 - \bar{a}_0 - \sum_{n \leq -1} \bar{a}_{-n} \zeta^{-n}$$

$$f(z) = \sum_{n \geq 0} a_n z^n \qquad \text{anal in } D$$

$$2 \operatorname{Re} f(z) = \sum_{n \geq 0} a_n z^n + \bar{a}_n \bar{z}^{+n}$$

$$2 \operatorname{Re} f(e^{i\phi}) = \sum_{n \geq 0} a_n e^{in\phi} + \bar{a}_n e^{-in\phi}$$

$$\int e^{-in\phi} \overbrace{2 \operatorname{Re} f(e^{i\phi})}^{\rho(e^{i\phi})} \frac{d\phi}{2\pi} = \begin{cases} a_n & n \geq 1 \\ a_0 + \bar{a}_0 & n = 0 \\ \bar{a}_{-n} & n \leq -1 \end{cases}$$

$$f(z) = a_0 + \sum_{n \geq 1} z^n \int e^{-in\phi} \rho(e^{i\phi}) \frac{d\phi}{2\pi}$$

$$f(z) = \sum_{n \geq 0} a_n z^n \qquad \text{analytic in } D$$

$$2i \,\mathrm{Im}\, f(z) = [illegible]\left(f(z) - \overline{f(z)}\right)$$

$$= \sum_{n \geq 0} a_n z^n - \overline{a}_n \overline{z}^n$$

$\xi = e^{i\phi}$

[illegible]

$$\int \xi^{-n} \left(2i \,\mathrm{Im}\, f(\xi)\right) \frac{d\phi}{2\pi} = \begin{cases} a_n & n \geq 1 \\ a_0 - \overline{a}_0 & n = 0. \end{cases}$$

[illegible]

$$f(z) = a_0 + \sum_{n \geq 1} \int e^{-in\phi} \left(2i \,\mathrm{Im}\, f(e^{i\phi})\right) \frac{d\phi}{2\pi}$$

[illegible]

$$\int \left(2i \,\mathrm{Im}\, f(e^{i\phi})\right) \frac{d\phi}{2\pi} = a_0 - \overline{a}_0$$

$$\frac{a_0 - \overline{a}_0}{2}$$

$$f(z) = \sum_{n\geq 0} a_n z^n \qquad \text{analytic in } D$$

$$J = [illegible]\, f(z) - \overline{f(z)} = \sum_{n\geq 0} a_n z^n - \bar{a}_n \bar{z}^n$$

$$\int e^{-in\theta} J(e^{i\theta}) \frac{d\theta}{2\pi} = a_n \qquad n \geq 1$$

$$= a_0 - \bar{a}_0 \qquad n = 0$$

~~$\sum_{n\geq 1} a_n z^n = \overline{\ldots}$~~

$$f(z) = \sum_{n\geq 0} a_n z^n$$

$$\operatorname{Im} f(z) = \sum_{n\geq 0} \frac{a_n z^n - \bar{a}_n z^{-n}}{2i}$$

$$\int e^{-in\theta} \operatorname{Im} f(e^{i\theta}) \frac{d\theta}{2\pi} = \begin{cases} \dfrac{a_0 - \bar{a}_0}{2i} & n = 0 \\ \dfrac{a_n}{2i} & n \geq 1 \end{cases}$$

~~$\int \sum_{n\geq 1} z^n e^{-in\theta}\, 2i \operatorname{Im} f(e^{i\theta}) \frac{d\theta}{2\pi} = \overline{\ldots}$~~

~~$a_0 + \frac{1}{2}$~~

$$f(z) = \sum a_n z^n$$

$$f(z) - \overline{f(z)} = \sum_{n\geq 0} a_n z^n - [illegible]\, \bar{a}_n \bar{z}^n$$

$$\int e^{-in\theta}\, [illegible]\, 2i \operatorname{Im} f(e^{i\theta}) \frac{d\theta}{2\pi} = \begin{cases} a_0 - \bar{a}_0 & n = 0 \\ a_n & n \geq 1 \end{cases}$$

$$f(z) = \sum_{n \geq 0} a_n z^n \quad \text{anal in } D$$

$$f(z) - \overline{f(z)} = \sum_{n \geq 0} a_n z^n - \bar{a}_n \bar{z}^n$$

$$f(e^{i\theta}) - \overline{f(e^{i\theta})} = \sum_{n \geq 0} a_n e^{in\theta} - \bar{a}_n e^{-in\theta}$$

$$\int e^{-in\theta} \left(f(e^{i\theta}) - \overline{f(e^{i\theta})} \right) \frac{d\theta}{2\pi} = \begin{cases} a_0 - \bar{a}_0 & n = 0 \\ a_n & n \geq 1 \end{cases}$$

[illegible] $f(0) = a_0$

[illegible]

$$\int \left(\sum_{n=1}^{\infty} z^n e^{-in\theta} + \frac{1}{2} \right) \left(f(e^{i\theta}) - \overline{f(e^{i\theta})} \right) \frac{d\theta}{2\pi}$$

$$= \frac{1}{2}(a_0 - \bar{a}_0) + \sum_{n=1}^{\infty} z^n a_n = f(z) - \frac{a_0 + \bar{a}_0}{2}$$

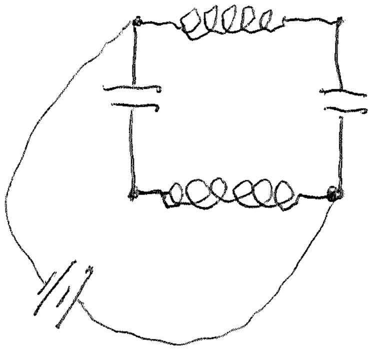
$$\frac{1}{2} + \frac{z e^{-i\theta}}{1 - z e^{-i\theta}} = \frac{1 - z e^{-i\theta} + 2 z e^{-i\theta}}{2(1 - z e^{-i\theta})} = \frac{1}{2} \frac{1 + z e^{-i\theta}}{1 - z e^{-i\theta}}$$

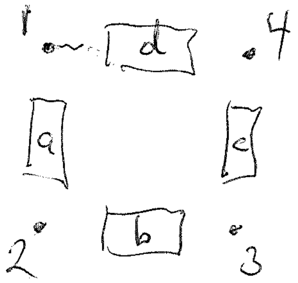
$$f(z) = \sum_{n \geq 0} a_n z^n \quad \text{anal in } D$$

[illegible] $$f(z) - \overline{f(z)} = \sum_{n \geq 0} a_n z^n - \bar{a}_n \bar{z}^n$$

$$\int e^{-in\theta} \left(f(z) - \overline{f(z)} \right) \frac{d\theta}{2\pi} = \begin{cases} a_n & n > 0 \\ a_0 - \bar{a}_0 & n = 0 \end{cases}$$

Do resistance case first





connected
4 vertices
4 edges
1 loop.

~~applied voltage across~~
1-port with $\{1,3\}$ external
$\{2,4\}$ internal

$$\frac{1}{\frac{1}{a+b}+\frac{1}{c+d}}$$

LC 1-port has [redacted] an impedance fn. $Z(s)$ with certain properties [redacted] that you need to understand properly.

Start at the LC side. Γ connected

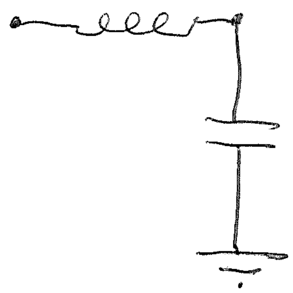
$$V = C^0(\Gamma)/\mathbb{R} \rightarrowtail C^1(\Gamma) \twoheadrightarrow H^1(\Gamma)$$
$$\downarrow$$
$$V/W$$

example: [redacted]

$W = C^0(\Gamma_{int})$ [redacted]

Then $V/W = C^0(\Gamma_{ext})/\mathbb{R}$

e.g.



This seems correct.

Return to ~~...~~ LC circuits + partial unitaries.

~~...~~ Go over steps carefully

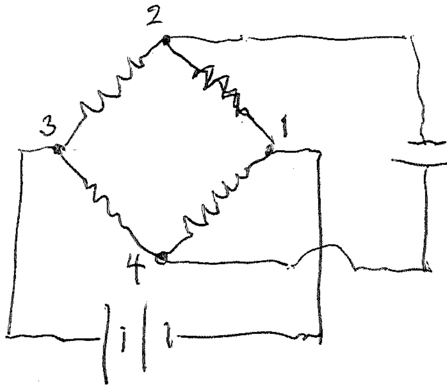
direct current, resistance > 0 values for edges

a set of external vertices containing *? ?

~~...~~ data: external

bridge: How are you going to handle

~~...~~ independent batteries?



Could this square be related to scattering?

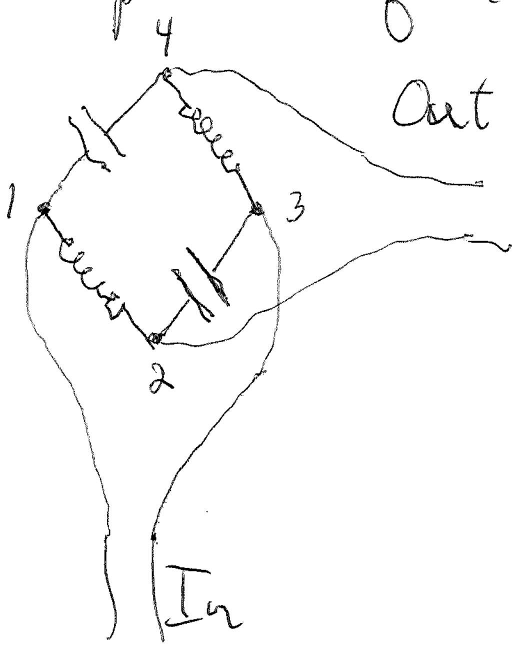
It looks like you want to drop the ground.

$$0 \to C^{1}/\mathbb{C} \xrightarrow{\delta} C^{1} \to H^{1} \to 0$$

(3 above $C^1/\mathbb{C}$, 4 above C^1, 1 above H^1)

$$C^{1}/\mathbb{C} \xrightarrow{\begin{pmatrix} V_1 - V_3 \\ V_2 - V_4 \end{pmatrix}} \mathbb{C}^2$$

Problem: Find the impedance of a "bridge"



You want to go over the calculation of the quadratic form $s\|\xi_+\|^2 + s^{-1}\|\xi_-\|^2$ on $H_+ \oplus H_-$ when restricted to $V \subset H_+ \oplus H_-$ and afterward when pushed down to V/W.

Let the components of the inclusion $V \subset H$ be $j_\pm : V \longrightarrow H_\pm$. Use the inner $\|\xi\|^2 = \|\xi_+\|^2 + \|\xi_-\|^2$ on H to make V a Euclidean subspace of H, and to make V/W a Euclidean quotient space. The variable quadratic form induced on V from $s\|\xi_+\|^2 + s^{-1}\|\xi_-\|^2$ is

$$Q_s(v) = s\|j_+ v\|^2 + s^{-1}\|j_- v\|^2 = v^*\left(s j_+^* j_+ + s^{-1} j_-^* j_-\right) v$$

Put $\rho = j_+^* j_+$; it's a self adjoint operator sat $0 \le \rho \le 1$ since $1 - \rho = 1 - j_+^* j_+ = j_-^* j_- \ge 0$.

$$1_V = \begin{pmatrix} j_+^* & j_-^* \end{pmatrix} \begin{pmatrix} j_+ \\ j_- \end{pmatrix} \Rightarrow \begin{pmatrix} j_+ \\ j_- \end{pmatrix} \begin{pmatrix} j_+^* & j_-^* \end{pmatrix}$$ is the self-adj operator projection in $\hat{H}$

with image $\begin{pmatrix} j_+ \\ j_- \end{pmatrix} V$.

You can split V into eigenspaces for the operator $\rho = j_+^* j_+$: $V = \bigoplus_{0 \le \lambda \le 1} V_\lambda$, $\rho(\xi_\lambda) = \lambda \xi_\lambda$ for $\xi \in V_\lambda$.

?

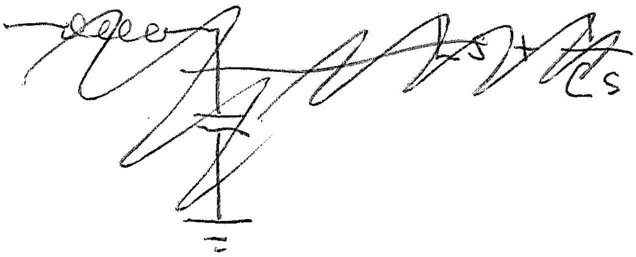
Try your Grassmannian version. Given $V \subset \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$ $\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, let $F = \begin{cases} +1 & \text{on } V \\ -1 & \text{on } V^\perp \end{cases}$: Suppose $V = \begin{pmatrix} 1 \\ T \end{pmatrix} H_+$ $V^\perp = \begin{pmatrix} -T^* \\ 1 \end{pmatrix} H_-$. Then $F\begin{pmatrix} 1 & -T^* \\ T & 1 \end{pmatrix} = \begin{pmatrix} 1 & -T^* \\ T & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$F(1+X) = (1+X)\varepsilon, \quad F = (1+X)\varepsilon(1+X)^{-1} = \frac{1+X}{1-X}\varepsilon$$

$V \hookrightarrow \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$ $\qquad s\rho_+ + s^{-1}\rho_- \qquad \rho_+ + \rho_- = 1_V$

$$\rho_+ = \sum_{0\le\lambda\le 1} \lambda \pi_\lambda, \quad \sum_{0\le\lambda\le 1} \pi_\lambda = 1_V, \quad \rho_- = \sum_{0\le\lambda\le 1} (1-\lambda)\pi_\lambda$$

$\sum_\lambda \left(s\lambda + s^{-1}(1-\lambda)\right)\pi_\lambda$. now you need to link $0 \le \lambda \le 1$ to frequency $0 \le \omega \le \infty$. ~~Bute $\frac{1}{s-i\omega} + \frac{1}{s+i\omega} = \frac{2s}{s^2+\omega^2}$?~~



$$\frac{1}{\lambda} - 1 = \frac{1-\lambda}{\lambda} = \omega^2 \qquad \lambda = \frac{1}{1+\omega^2}$$

transformation $\quad s\lambda + s^{-1}(1-\lambda) = \frac{s^2\lambda + (1-\lambda)}{s}$

$$s\frac{1}{1+\omega^2} + s^{-1}\frac{\omega^2}{1+\omega^2} = \frac{1}{s}\,\frac{s^2+\omega^2}{1+\omega^2}$$

Get $\sum_{0\le\omega\le\infty} \frac{s + s^{-1}\omega^2}{1+\omega^2}\pi_\omega$ $\qquad$ You notice $s=1$ gives 1 on V.

~~There seems to be ...~~

The quadratic form Q_s on $\begin{pmatrix} H_+ \\ H_- \end{pmatrix}$ does not yield an operator unless there is already given a quadratic form on H. What structure is there on $C^1 = C^1_L \oplus C^1_C$

energy $\quad \sum \frac{V_L^2}{Ls} \qquad \sum V_C^2 Cs$

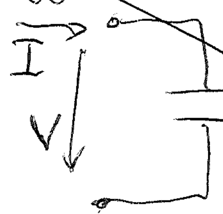
What you've learned. LC circuit described by [illegible] the real vector space $H \simeq C^1 = H_+ \oplus H_-$ equipped with the family of quadratic forms

$$Q_s\begin{pmatrix}\xi_+\\ \xi_-\end{pmatrix} = s\|\xi_+\|^2 + s^{-1}\|\xi_-\|^2$$

$\xi_+ = \{V_\sigma,\ \sigma \text{ C-type}\}$ $\quad \|\xi_+\|^2 = \sum C_\sigma V_\sigma^2$?

$\xi_- = \{V_\sigma,\ \sigma \text{ L-type}\}$ $\quad \|\xi_-\|^2 = \sum \frac{1}{L_\sigma} V_\sigma^2$

Energy calculation

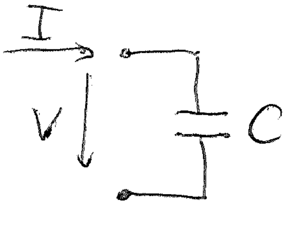


$$C V = \dot{I}$$

$$\underbrace{C V I}_{\text{power}} = I\dot{I} = \partial_t\left(\frac{I^2}{2}\right)$$

Energy in the capacitor at time t is

$$\int_{-\infty}^{t} V I\, dt = \int_{-\infty}^{t} \partial_t\left(\frac{I^2}{C2}\right) dt = \frac{I^2}{2C}$$



charge of capacitor $\quad Q = CV$

$$I = C\,\partial_t V$$

power $\quad VI = C V \partial_t V = \partial_t\left(\frac{1}{2} C V^2\right)$

Energy stored in capacitor at time t is $\quad \int_{-\infty}^{t} VI\,dt = \left[\frac{1}{2}CV^2\right]_{-\infty}^{t} = \frac{1}{2}CV(t)^2$

Try for some progress on LC circuits

Γ connected graph, whose edges are either L or C type

$$V = C^0(\Gamma_0)/\mathbb{C} \xhookrightarrow{\delta} C(\Gamma_1)$$

$$\downarrow$$

$$V/W$$

e.g. $W = C(\Gamma_{0,int})$

$V/W = C(\Gamma_{0,ext})/\mathbb{C}$

Here you have set Γ_0 of vertices partitioned ~~divided~~ into $\Gamma_{0,ext}$ and $\Gamma_{0,int}$. You want the internal voltages to be free

---

It seems that you should work over $\mathbb{R}$ when ~~considering~~ defining an "abstract" LC circuit. Such a thing is equivalent to subquotient of a polarized real ~~inner product~~ Euclidean space

$$V \hookrightarrow H_+ \oplus H_- = H$$

$$\downarrow$$

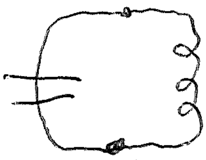
$$V/W$$

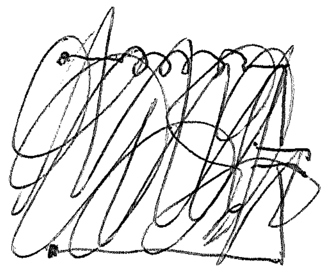
On H you have the quadratic form

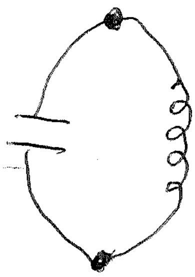
$$s\|\xi_+\|^2 + s^{-1}\|\xi_-\|^2 = Q_s$$

which induces a family of quadratic forms on V/W depending rational on s. Your first task will be to understand this well. Need to understand dynamics: s is the "frequency" parameter occuring in the L.T. You are studying a module over the (real) time translation group, The L.T. turns this into a module over functions of s now you have a new idea – to work with the DE's hopefully to understand variational principles – to handle circuit constraints.

where to start? where to begin? a specific example!

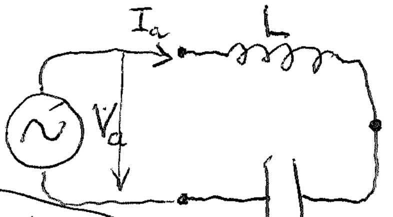




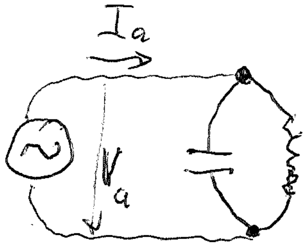


You need a way to handle the constraints. What is a solution? Prove a solution exists and is unique.

Aim: To obtain a real functions of time picture of some simple LC circuits. Try



$$\boxed{\frac{V_a}{I_a} = Ls + \frac{1}{Cs}}$$

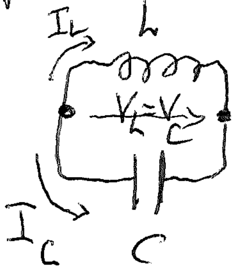


$$\frac{C^{-1}s}{s^2+\frac{1}{CL}}$$

$$\parallel$$

$$\frac{V_a}{I_a} = \frac{C^{-1}s}{\frac{1}{LC} + s^2}$$

But simpler is the closed circuit: $\boxed{\frac{V_a}{I_a} = \frac{C^{-1}s}{s^2+\omega^2}}$ $\omega = \frac{1}{\sqrt{LC}}$



What exactly is the system of equations you want to solve? In terms of the 4 variables V_L, V_C, I_L, I_C you have

$$V_L = L\dot{I}_L \qquad I_C = C\dot{V}_C$$

circuit equations $\quad V_L = V_C, \quad I_L + I_C = 0$

4 equations in 4 variables

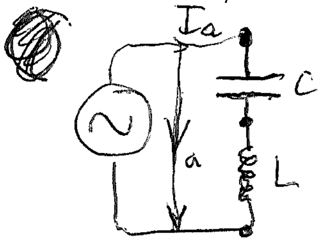
$$V_L = L\dot{I}_L \qquad -I_L = C\dot{V}_L = CL\ddot{I}_L$$

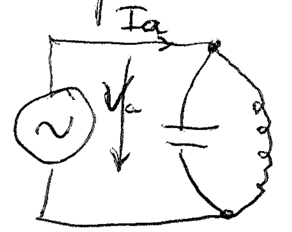
$$(CL\,\partial_t^2 + 1)I_L = 0$$

$$CLs^2 + 1 = 0 \qquad s = \pm i\sqrt{\frac{1}{LC}}$$

What comes next should be one of the two cases above with a "forcing" term – current or voltage source applied to the series or parallel LC circuit.

Aim: real function of time picture for an LC circuit, simple examples with applied voltage or current. This is a 1-port. Two simple cases





series first. variables are $V_C, I_C, V_L, I_L, V_a, I_a$

$$I_a = I_C = I_L \ , \quad V_a = V_C + V_L$$

$$I_C = C\dot{V}_C \ , \quad \dot{V}_L = L\dot{I}_L \qquad ?$$

Take L.T. $\quad I_a = I_C = I_L, \quad V_a = V_C + V_L$

$$I_a = Cs V_C \qquad V_L = Ls I_a \qquad V_a = \left(\frac{1}{Cs} + Ls\right) I_a$$

parallel example. $\quad V_a = V_C = V_L \ , \quad I_a = I_C + I_L$

$$I_C = C\dot{V}_C \ , \quad V_L = L\dot{I}_L$$

Take L.T. $\quad I_C = Cs V_a \ , \quad I_L = \frac{1}{Ls} V_a \Rightarrow I_a = \left(Cs + \frac{1}{Ls}\right) V_a$

Go back to energy - something seems "strange" if you are using real functions of time. Start where?

Start with resistance a network. direct ~~current~~ voltage sources (batteries). Review the existance + uniqueness of the linear equations

Consider a resistance network, a graph where each edge σ has a resistance of $R_\sigma > 0$ ohms. Assume connected. Picture

$$\begin{array}{ccccc} \tilde{C}^0 & \xhookrightarrow{\ \delta\ } & C^1 & \twoheadrightarrow & H^1 \\ & & \times & & \\ \tilde{C}_0 & \overset{\partial}{\twoheadleftarrow} & C_1 & \hookleftarrow & H_1 \\ & & \downarrow & & \text{cycles} \\ & & \mathbb{R} & & \end{array}$$

duality pairing, diagonal type pairing
$$V_\sigma \cdot I_\rho = \begin{cases} 0 & \text{if } \sigma \neq \rho \\ V_\sigma I_\sigma & \text{if } \rho = \sigma \end{cases}$$

This pairing
$$V \cdot I = \sum_\sigma V_\sigma I_\sigma$$
is the power [redacted] dissipated by a configuration of voltage drops + currents.

At some point you must list the network circuit equations:

1) edge voltage drops given by a potential, a function on the nodes modulo constants.

This means restricting C^1 to $\delta \tilde{C}^0$

Is there some statement about 1-cycle) ? currents?

∂ take a f

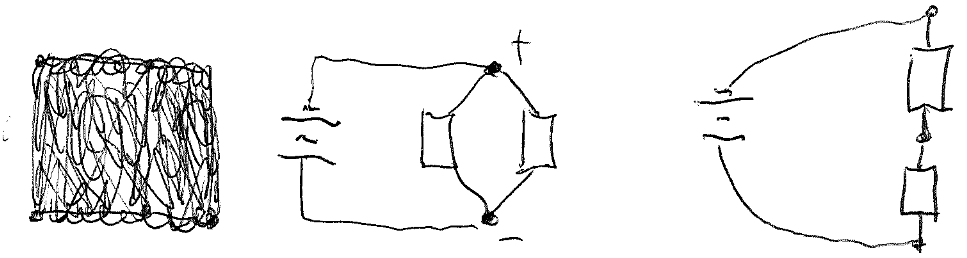
Try again to ~~understand~~ ~~circuit th~~ get a clear picture of the circuit equations.

Data. Γ graph connected, "metric" on edges. at least 2 nodes. Consider 1-port, specify two nodes. +, − nodes.

State of the network: for each σ V_σ, I_σ related by Ohm's Law.

$\{V_\sigma\}$ conservative − same as $\{V_x\}$ mod const.

$\{I_x\} = \partial\{I_\sigma\}$ supported at ± nodes.



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what's the difficulty? Start again: connected graph Γ ~~whose edges~~ a with "metric" given by pos. nos. $R_\sigma > 0$ for each edges. Given also a dist. pair of nodes + −, called external nodes for attaching a battery or current source. Then you (want to) be able to calculate the state of the network with the source attached. This state consists of V_σ, I_σ for each edge σ, better to say 1-cochain $\{V_\sigma\}$, 1-chain $\{I_\sigma\}$ satisfying Ohm: $V_\sigma = R_\sigma I_\sigma$, V is 1-coboundary, ∂I vanishes at each internal node.

Claim There's a vector space of such states of dim 1.

$$\mathbb{R}(\Gamma_0)/\mathbb{R} \rightarrow \mathbb{R}(\Gamma_1)$$

$$\mathbb{R}\Gamma_0 \leftarrow \mathbb{R}\Gamma_1$$

$$\begin{array}{ccccc} \mathbb{R} & \xleftarrow{([+]-[-])^t} & \tilde{C}^0 & \xrightarrow{\delta} & C^1 \\ & & \times & & \times \\ \mathbb{R} & \xrightarrow{[+]-[-]} & \tilde{C}_0 & \xleftarrow{\partial} & C_1 \\ & & \downarrow & & \downarrow \\ & & \mathbb{R} & & \mathbb{R} \end{array}$$

power pairing
$\langle V', I' \rangle = \sum V'_\sigma I'_\sigma$

---

this kinematics seems completely clear, nice interpretation via symplectic quotients. Next comes ~~dynamics~~ statics. Wait what is the conclusion? Answer might might be that

---

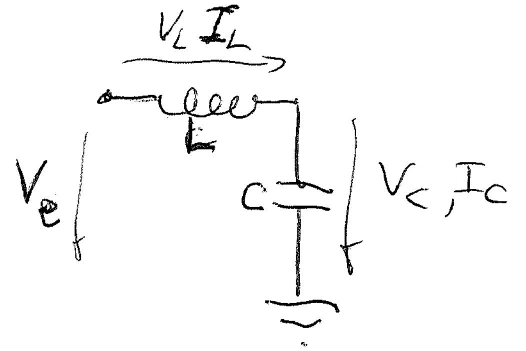
Geometric situation (kinematics)

$$\begin{array}{ccccc} \mathbb{R} & \overset{([+]-[-])^t}{\twoheadleftarrow} & \tilde{C}^0 & \hookrightarrow & C^1 \\ \\ \mathbb{R} & \xrightarrow{[+]-[-]} & \tilde{C}_0 & \twoheadleftarrow & C_1 \end{array}$$

symplectic ~~quotient~~ situation

puzzle about energy or power geometric pairing between voltage + current. How is this related to the energy going into a resistance.

Given an LC 1-port. Is there an associated partial unitary of rank 1? Reason: The LC 1-port is described by a rational function pos. res.



$$V_L = V_e - V_c$$

$$V_L = Ls\,I$$

$$I = Cs\,V_C$$

Q_s should be $\frac{1}{Ls} V_L^2 + Cs V_C^2$

$$\frac{1}{Ls}(V_e - V_C)^2 + Cs\,V_C^2 \qquad \text{stationing } V_c$$

$$\frac{1}{Ls}(V_e - V_C)(-1) + Cs\,V_C = 0$$

$$\frac{1}{Ls} V_e = \left(\frac{1}{Ls} + Cs\right) V_c$$

$$\boxed{V_C = V_e \frac{1}{1+LCs^2}}$$

$$V_L = V_e - V_c = V_e \left(1 - \frac{1}{1+LCs^2}\right)$$

$$\boxed{V_L = V_e \frac{LCs^2}{1+LCs^2}}$$

$$V_L = Ls\,I$$

$$I = V_e \frac{Cs}{1+LCs^2}$$

$$I = Cs\,V_C = V_e \frac{Cs}{1+LCs^2}$$

$$\frac{V_e}{I} = \frac{V_e}{V_e \frac{Cs}{1+LCs^2}} = \frac{1+LCs^2}{Cs} = Ls + \frac{1}{Cs}$$

~~[illegible]~~ Start with a connected R network with basepoint.

$$\tilde{C}^0/C^0_{int} \twoheadleftarrow \tilde{C}^0 \rightarrowtail C^1 \twoheadrightarrow H^1$$

you get a positive def q.f. on C^1 induces one on $\tilde{C}^0_{ext}$. Dual spaces

$$\tilde{C}^0_{ext} \twoheadleftarrow \tilde{C}^0 \xrightarrow{\delta} C^1$$

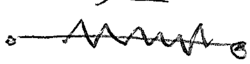
$$\tilde{C}_0^{ext} \rightarrowtail \tilde{C}_0 \xleftarrow{\partial} C_1$$

$$\tilde{C}^0 = C^0(X_\bullet; *)$$

You need to identify ~~[illegible]~~ induced ~~[illegible]~~ positive quad. forms with the ~~desired~~ solution of the circuit equations. The main part of the story is that ~~[illegible]~~ Voltage and Current spaces are naturally dual ~~[illegible]~~ via the power pairing (this is kinematics), and ~~then~~ you have the quadratic form induced by energy, also = the power/. ? NO somehow different - !

anyway what?

V, I



$V = IR$ $\qquad$ $\boxed{P = VI} = \frac{V^2}{R} = I^2 R$

Now

$$\tilde{C}^0 \rightarrowtail C^0 \xrightarrow{\delta} C^1$$
$$\downarrow R^{-1}$$
$$\tilde{C}_0 \twoheadleftarrow C_0 \xleftarrow{\partial} C_1$$

$I = R^{-1} V$

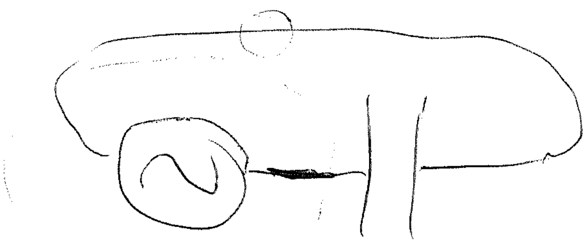
$$\overset{V}{\tilde{C}^0} \xrightarrow{\delta} C^1 \ni \delta V$$

$$\tilde{C}_0 \overset{\partial}{\twoheadleftarrow} C_1 \ni I = R^{-1}\delta V$$

Maxwell equations. $\vec{E}$ $\quad$ $\vec{B}$

$\nabla \cdot \vec{B} = 0$ $\qquad$ $\nabla \times \vec{E} + \partial_t \vec{B} = 0$

$\nabla \cdot \vec{E} = \rho$ $\qquad$ $\nabla \times B = J + \partial_t E$



Discuss what? Aim? Clean picture about LC ports. subquotient of polarized Euclidean space. You have to review this carefully in order to avoid a mistake. Also to [redacted] learn whether there are any complex hermitian aspects. You can look at a subquotient of a polarized complex Hilbert space. [redacted]

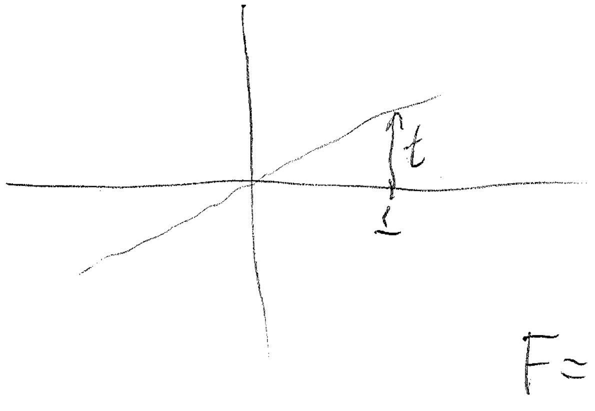
$$V \underset{j}{\overset{j^*}{\rightleftarrows}} \begin{pmatrix} H_+ \\ H_- \end{pmatrix} \qquad j = \begin{pmatrix} j_+ \\ j_- \end{pmatrix} \quad j^* = \begin{pmatrix} j_+^* & j_-^* \end{pmatrix}$$

j isom means $j^*j = j_+^*j_+ + j_-^*j_- = 1_V$

get proj. $jj^* = \begin{pmatrix} j_+ \\ j_- \end{pmatrix}\begin{pmatrix} j_+^* & j_-^* \end{pmatrix}$ on H with img jV

$$= \begin{pmatrix} j_+j_+^* & j_+j_-^* \\ j_-j_+^* & j_-j_-^* \end{pmatrix}$$

What [redacted] information is there? Just two involutions F, ε



$$1 + X = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}$$

$$g^{1/2} = \frac{1+X}{\sqrt{1-X^2}} = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix} \frac{1}{\sqrt{1+t^2}} \qquad \frac{1}{1+t^2}$$

$$F = g^{1/2}\varepsilon g^{-1/2} = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & +t \\ -t & 1 \end{pmatrix} a$$

$$= \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & +t \\ +t & -1 \end{pmatrix} \frac{1}{1+t^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{1+t^2}\begin{pmatrix} [redacted] 1-t^2 & 2t \\ 2t & 1-t^2 \end{pmatrix}$$

V (fin diml) subspace of $\begin{pmatrix} H_+ \\ H_- \end{pmatrix}$, $j = \begin{pmatrix} j_+ \\ j_- \end{pmatrix} : V \to \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$ the inclusion. What is the ~~geometry~~ structure lies behind this?

simply a Hilbert space rep (unitary) of the inf. dihedral group $\mathbb{Z}/2 * \mathbb{Z}/2$. i.e. two involutions F, ε on a Hilbert space

$$1_V = j^* j = (j_+^* \ j_-^*)\begin{pmatrix} j_+ \\ j_- \end{pmatrix} = j_+^* j_+ + j_-^* j_-$$

$$\text{projection on } jV = jj^* = \begin{pmatrix} j_+ \\ j_- \end{pmatrix}(j_+^* \ j_-^*) = \begin{pmatrix} j_+j_+^* & j_+j_-^* \\ j_-j_+^* & j_-j_-^* \end{pmatrix}$$

$$F = 2e - 1 = \begin{pmatrix} 2j_+j_+^* - 1 & 2j_+j_-^* \\ 2j_-j_+^* & 2j_-j_-^* - 1 \end{pmatrix}.$$

not useful.

Approach 1: Start with $H = \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$ and ε

Let F be another involution on H, ask what structure arises on H. Decomposition of H according to irreducible reps of the dihedral group.

$\frac{g + g^{-1}}{2}$ is in the central of the dihedral group.

$$\frac{F\varepsilon + \varepsilon F}{2} = \begin{pmatrix} j_+j_+^* - \frac{1}{2} & -j_+j_-^* \\ j_-j_+^* & -j_-j_-^* + \frac{1}{2} \end{pmatrix} + \begin{pmatrix} j_+j_+^* - \frac{1}{2} & j_+j_-^* \\ -j_-j_+^* & -j_-j_-^* + \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2j_+j_+^* - 1 & 0 \\ 0 & -2j_-j_-^* + 1 \end{pmatrix}$$

$g = F\varepsilon$ is unitary $\quad L_z = \{ \xi \mid g\xi = z\xi \}$

$$\Rightarrow g(\varepsilon\xi) = \varepsilon(g^{-1}\xi) = \varepsilon(z^{-1}\xi) = z^{-1}(\varepsilon\xi).$$

$\varepsilon : L_z \xrightarrow{\sim} L_{z^{-1}}$. Assume $H = L_z + L_{z^{-1}}$

$H = L_z + L_{z^{-1}}$ Let $\xi \in L_z$ then $\varepsilon\xi \in L_{z^{-1}}$

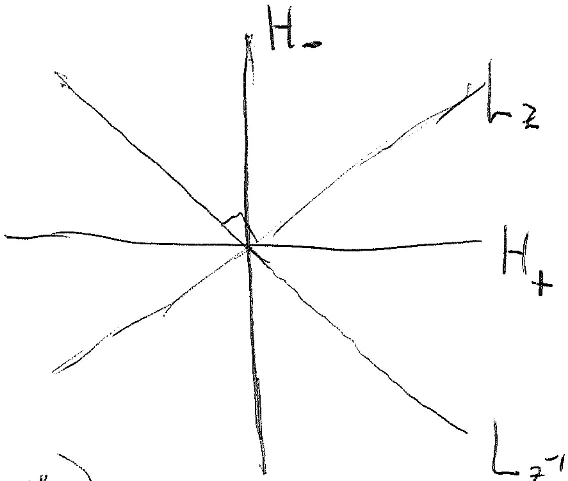
Suppose $H = \mathbb{C}\xi + \mathbb{C}\varepsilon\xi$ $\quad g\xi = z\xi$, $\quad g\varepsilon\xi = z^{-1}\varepsilon\xi$

$$(g + g^{-1})(\xi + \varepsilon\xi) = (z + z^{-1})\xi + (z^{-1} + z)\varepsilon\xi = (z + z^{-1})(\xi + \varepsilon\xi)$$

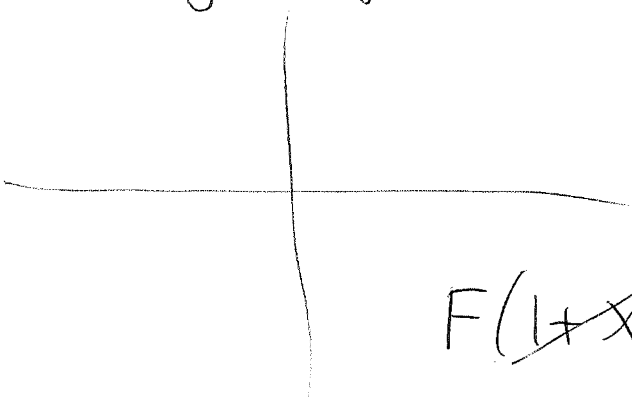
Assume $H = L_z + L_{z^{-1}}$ where $L_z = \{\xi \in H \mid g\xi = z\}$ same for g^{-1}, z^{-1}.

$\varepsilon : L_z \xrightarrow{\sim} L_{z^{-1}}$. Now $H = H_+ \oplus H_-$

So $\begin{pmatrix} L_z \\ L_{z^{-1}} \end{pmatrix} \simeq \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$



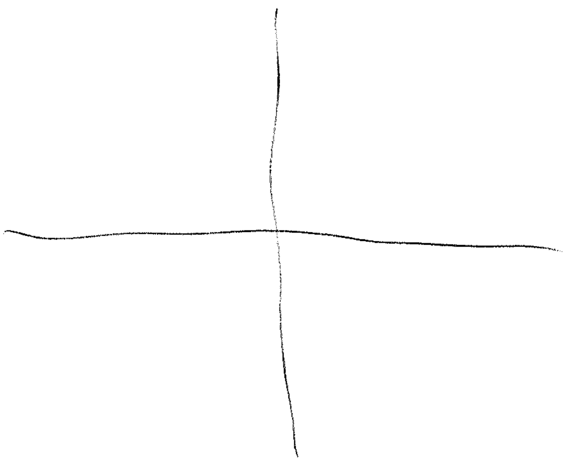
Better way to proceed is



$$F\begin{pmatrix} 1 & -T^* \\ T & 1 \end{pmatrix}$$

$$F(1+X) = (1+X)\varepsilon(1+X)^{-1} = \left(\frac{1+X}{1-X}\right)\varepsilon$$

(with $\frac{1+X}{1-X}$ circled and labelled g)



$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & -\bar{\omega} \\ \omega & 0 \end{pmatrix} \qquad X^2 = \begin{pmatrix} -|\omega|^2 & \\ & -|\omega|^2 \end{pmatrix}$$

$$1 + X = \begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix}$$

$$\frac{1+X}{\sqrt{1-X^2}} = \quad = \begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix} \frac{1}{\sqrt{1+|\omega|^2}}$$

$$\begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix}\begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix} = ($$

$$\begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix}\begin{pmatrix} 1 & -\bar{\omega} \\ \omega & 1 \end{pmatrix} = \begin{pmatrix} 1-|\omega|^2 & -2\bar{\omega} \\ 2\omega & 1-|\omega|^2 \end{pmatrix} \quad \frac{1}{1+|\omega|^2}$$

$$\frac{1+X}{1-X} = \begin{pmatrix} \frac{1-|\omega|^2}{1+|\omega|^2} & \frac{-2\bar{\omega}}{1+|\omega|^2} \\ \frac{2\omega}{1+|\omega|^2} & \frac{1-|\omega|^2}{1+|\omega|^2} \end{pmatrix}$$

It's sort of clear that the reality question occurs here with ω being real.

$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ on } \begin{pmatrix} H_+ \\ H_- \end{pmatrix} \qquad V \subset \begin{pmatrix} H_+ \\ H_- \end{pmatrix}$$

Given inv. F, ε on H let ξ_1 be an eigenvector for $F\varepsilon$ with eigenvalue z: $F\varepsilon\xi_1 = z\xi_1$, let $\xi_2 = \varepsilon\xi_1$, then

$F\varepsilon\xi_2 = F\varepsilon\varepsilon\xi_1 = F\xi_1$;

? $\quad g = F\varepsilon \qquad g\xi_1 = z\xi_1 \Longrightarrow$

$z(\varepsilon\xi_1) = \varepsilon(g\xi_1) = g^{-1}(\varepsilon\xi_1)$. Put $\varepsilon\xi_1 = \xi_2$

Put $W = \begin{pmatrix} \mathbb{C}\xi_1 \\ \mathbb{C}\xi_2 \end{pmatrix}$ $\quad \varepsilon = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad g = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{2} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} z & z \\ z^{-1} & -z^{-1} \end{pmatrix}\frac{1}{2} = \begin{pmatrix} \frac{z+z^{-1}}{2} & \frac{z-z^{-1}}{2} \\ \frac{z-z^{-1}}{2} & \frac{z+z^{-1}}{2} \end{pmatrix}$$

in new basis

$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad g = \begin{pmatrix} \cos & i\sin \\ i\sin & \cos \end{pmatrix} \qquad F = \begin{pmatrix} \cos & -i\sin \\ i\sin & -\cos \end{pmatrix}$$

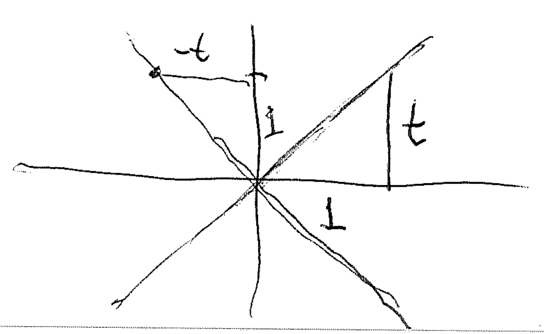
$$\varepsilon = \begin{pmatrix} 0 & \phi \\ \phi & 0 \end{pmatrix} \qquad g = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}$$

$$g\varepsilon = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & z \\ z^{-1} & 0 \end{pmatrix}$$

You learn what? You've started with an irred. repn of $(\mathbb{Z}/2)*(\mathbb{Z}/2)$, used the abelian normal subgroup gen. by $g = F\varepsilon$ to get the picture $\varepsilon = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad g = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}$ on $\mathbb{C}^2$

$$F = \begin{pmatrix} z & 0 \\ 0 & z^{-1} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & z \\ z^{-1} & 0 \end{pmatrix}$$

This is an irreducible repn over $\mathbb{C}$ $\quad z \neq \pm 1$.
but you want one over $\mathbb{R}$.



$$\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

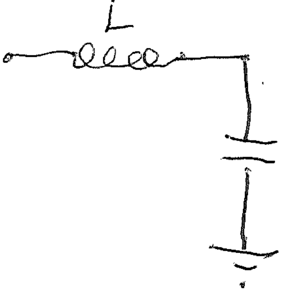
$$1 + X = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}$$

Γ connected LC network, with ground *.

$$\begin{array}{ccccc} \tilde{C}^0 & \xrightarrow{\delta} & C^1 & \longrightarrow & H^1 \\ & & \downarrow G_s & & \\ \tilde{C}_0 & \xleftarrow{\partial} & C_1 & \longleftarrow & H_1 \end{array}$$

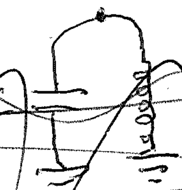
What is your aim? To link LC response to partial unitaries satisfying a reality condition. It seems you can do this for a 1-port using the characterization of response functions of LC 1-ports. Such a $Z(s)$ is a real rational function of s with simple poles on the imaginary axis having pos. residues

example



$$Z = Ls + \frac{1}{Cs}$$

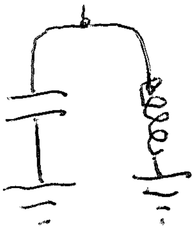
simple pole, pos. res at ∞, 0.



$$Z = \frac{1}{\frac{1}{Cs} + Ls} = \frac{L^{-1}s}{s^2 + \frac{1}{LC}}$$

$$\omega^2 = \frac{1}{LC} \qquad \frac{1}{s - i\omega} + \frac{1}{s + i\omega} = \frac{2s}{s^2 + \omega^2}$$

$$\frac{V_C}{I_C} = \frac{1}{Cs}$$

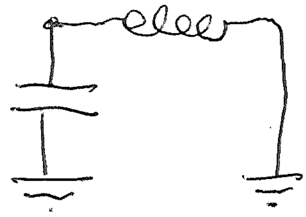


$$V = V_C = V_L \qquad \frac{V_L}{I_L} = Ls$$

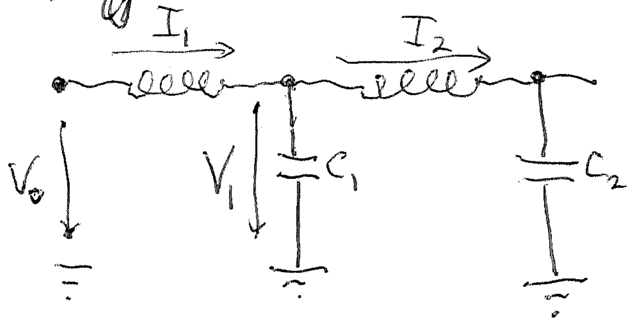
$$I = I_C + I_L$$

$$\frac{V}{I_C + I_L} = \frac{V}{CsV + \frac{V}{Ls}} = \frac{1}{Cs + \frac{1}{Ls}} = \frac{Ls}{LCs^2 + 1}$$

$$= \frac{C^{-1}s}{s^2 + \frac{1}{LC}} = C^{-1}\frac{s}{s^2 + \omega^2}$$



discrete transmission line



$$V_0 - V_1 = L_1 s I_1$$

$$I_1 - I_2 = C_1 s V_1$$

$$V_1 - V_2 = L_2 s I_2$$

$$V_0/I_1 = L_1 s + V_1/I_1$$

$$= L_1 s + \frac{1}{C_1 s +}$$

$$\frac{V_0}{I_1} = L_1 s + \frac{V_1}{I_1}$$

$$\frac{I_1}{V_1} = C_1 s + \frac{I_2}{V_1}$$

$$\frac{V_1}{I_2} = L_2 s + \frac{V_2}{I_2}$$

continuous

$$V_x - V_{x+dx} = \lambda_x dx\, s I_x$$

$$I_x - I_{x+dx} = \gamma_x dx\, s V_x$$

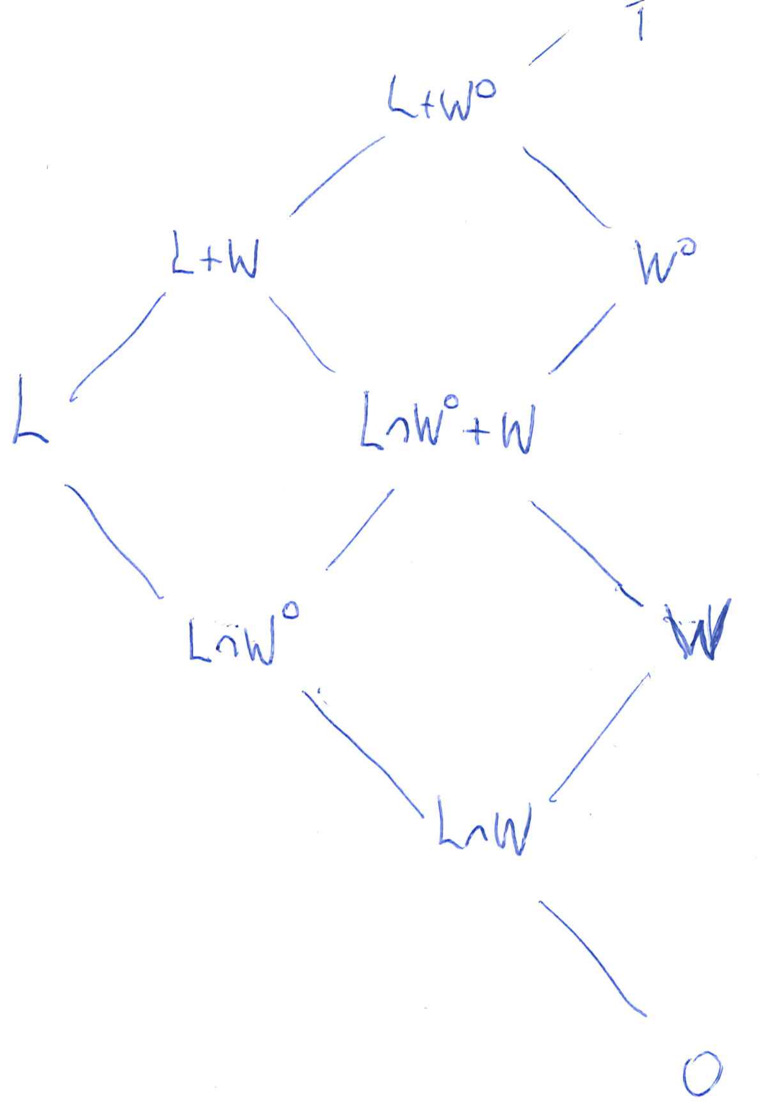
$$\partial_x V + s\lambda(x) I = 0$$

$$\partial_x I + s\gamma(x) V = 0$$

need to adjust x so that signal speed $= 1$.

$$\begin{pmatrix} V_0 \\ I_1 \end{pmatrix} = \begin{pmatrix} L_1 s & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ V_1 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ V_1 \end{pmatrix} = \begin{pmatrix} C_1 s & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$



$$(L \cap W^\circ + W)^\circ = (L+W)\cap W^\circ = (L\cap W^\circ) + W.$$

Is it true that $\dfrac{L\cap W^\circ + W}{W}$ is Lagrangian in W°/W

$$\| $$

~~$(L+W)\cap W^\circ$~~ $L\cap W^\circ / L\cap W$

---

count dims.

Suppose we have $T \supset W^\circ \supset W \supset 0$. ~~and we choose a fixed~~ Then we seem to have a retraction of Lagrangian ~~of~~ subspaces of T onto the subspace of L between W° and W i.e. $W^\circ \supset L \supset W$, namely

$L \longmapsto L\cap W^\circ + W = (L+W)\cap W^\circ$. This is a retraction of the symplectic Grass of T onto the symplectic Grass of W°/W. Fix L_0 $W \subset L_0 \subset W^\circ$

tangent space inclusion $\quad \mathrm{Hom}(L_0, T/L_0) \longleftarrow \mathrm{Hom}(L_0/W, W^\circ/L_0)$

there

tangent space to $SG(T)$ $\quad Hom(L_0, T/L_0)$ $\leftarrow$ dim $\frac{n(n+1)}{2}$

$\updownarrow$

tangent space to $SG(W^0/W)$ $\quad Hom(L_0/W, W^0/L_0)$

You seem to have a retraction. ~~Certainly you have a~~

You have obvious $SG(W^0/W) \hookrightarrow SG(T)$,

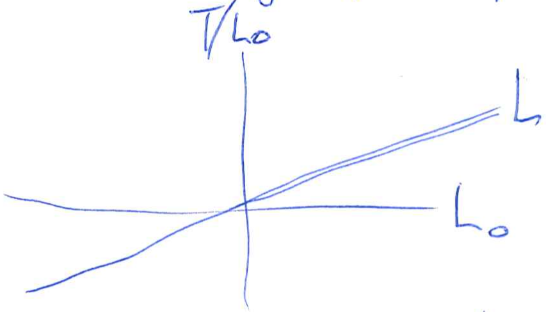
what is the ~~new m~~ retraction going the other way

$L_0 \longrightarrow T/L_0$ ~~View this as a~~

Suppose chosen a Lagrang. comp. to L_0.

$L \cap W^0_A \qquad W^0_A/L_0$

$$\begin{array}{ccc} & & W^0_A/L_0 \\ & & \downarrow \\ L & \longrightarrow & T/L_0 \\ \downarrow & & \searrow \\ L_0 & & T/W_0 \end{array}$$



$L \cap W^0$
$+$
W

Choose splittings $\quad T \supset W^0 \supset L_0 \supset W \supset 0$

So we will assume $\quad T = L_0 \oplus L_0^*$

where $\quad L_0 = V \oplus V'$

Look at

$$\begin{array}{ccc} W & & \\ V & \overset{\iota}{\hookrightarrow} & L_0 \\ & & \oplus \\ V^* & \overset{\iota}{\twoheadleftarrow} & L_0^* \end{array} \qquad V^0$$

First thing you need to do is to check that the retraction takes Γ_A to $\Gamma_{\iota^t A \iota}$

$W = \begin{pmatrix} V \\ 0 \end{pmatrix} \qquad W^0 = \begin{pmatrix} L_0 \\ V^0 \end{pmatrix} \qquad \Gamma_A = \left\{ \begin{pmatrix} x \\ Ax \end{pmatrix} \middle| x \in L_0 \right\}$.

$\Gamma_A \cap W^0 = \left\{ \begin{pmatrix} x \\ Ax \end{pmatrix} \middle| Ax \in V^0 \right\}$

$$T = \begin{matrix} L \\ \oplus \\ L^* \end{matrix} \qquad W = \begin{matrix} V \\ \oplus \\ 0 \end{matrix} \qquad W^\circ = \begin{matrix} L \\ \oplus \\ V^\circ \end{matrix}$$

$\Gamma = \Gamma_A = \binom{1}{A} L \subset T$. Now want

$\Gamma' = (\Gamma + W) \cap W^\circ = (\Gamma \cap W^\circ) + W$. This Γ' contains W and is contained in W°. $W \subset \Gamma' \subset W^\circ$. Assume $\Gamma' = \Gamma_B$) does this make sens? $W^\circ/W = \begin{matrix} L/V \\ \oplus \\ V^\circ \end{matrix}$

It would seem that we take $A : L \to L^*$ then cut it down to $\underline{A^{-1}(V^\circ)}$ and extend. You have a quadratic form on L ?

Curious. You have $L = L_0$ $\quad T = \begin{matrix} L \\ \oplus \\ L^* \end{matrix}$

$$W = \begin{matrix} V \\ \oplus \\ 0 \end{matrix} \qquad W^\circ = \begin{matrix} L \\ \oplus \\ V^\circ \end{matrix} \qquad W^\circ/W = \begin{matrix} L/V \\ \oplus \\ V^\circ = (L/V)^* \end{matrix}$$

Now take $\Gamma_A = \left\{ \binom{x}{Ax} \middle| x \in L \right\}$. Look at $\Gamma_A \cap W^\circ + W$

$$\Gamma_A \cap W^\circ = \left\{ \binom{x}{Ax} \middle| \; x \in A^{-1}V^\circ \text{ i.e.} \right\} = \left\{ \binom{A^{-1}y}{y} \middle| \; y \in V^\circ \right.$$

the conjecture is that this $(\Gamma_A \cap W^\circ + W)/W$ is the graph of the induced quadratic form on L/V.

What next?? Back to Lagrangian subbundles

You have $T = V \oplus V'$ symplectic.

Electrical ~~situation~~ have $0 \to \mathcal{L} \to \mathcal{O} \otimes T \to \check{\mathcal{L}} \to 0$

what to hope for? ~~Electrical~~ An LC network gives: $T = H^0(\mathcal{O}(1)) \otimes V$ where V has a given nondeg quad. form which is combined with the canon. skew form on $H^0(\mathcal{O}(1))$ to get a symp form.

So what are you doing? ~~You have a coherent sheaf.~~
You ~~[illegible]~~ form over $SG(T)$ the fibre bundle of flags $\ell \subset L$. ~~[illegible]~~. Symp flag variety has dim. $3+1=4$. ~~[illegible]~~

Basic idea is that the map $L \mapsto L \cap W^\circ + W$ is rational, e.g. regular ~~outs~~ where $L \cap W = 0$. YES.
closed subset of L $\ni W \subset L \subset W^\circ$ is of dim 1.
So you have ~~a Lagrangian~~ the variety $SG(T)$ of dim 3 and the variety $SG(W^\circ/W)$ of dim 1, and a rational map $SG(T) \longrightarrow SG(W^\circ/W)$.

$\dim SG(T) = n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$

$$\overset{2n}{T} \supset \overset{2n-r}{W^\circ} \supset \overset{r}{W} \supset 0$$

What are you trying to do?

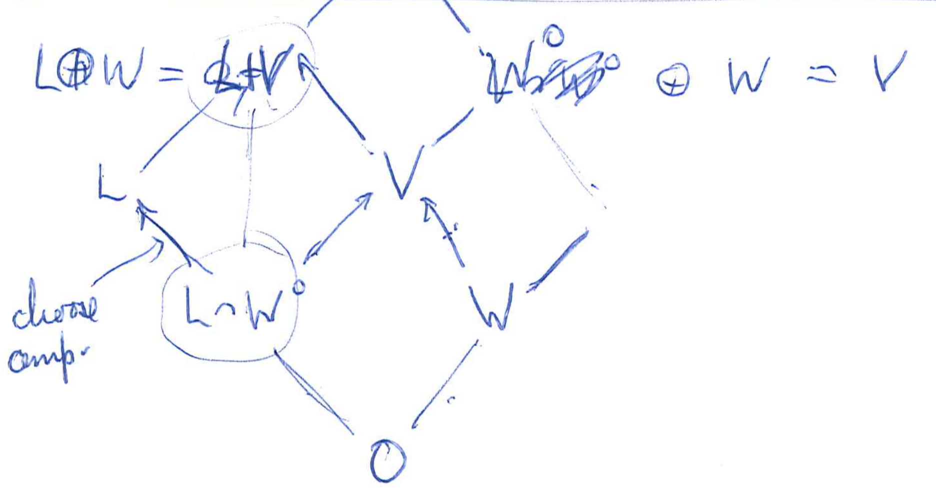
~~Consider~~ Consider $\{L \in SG(T) \mid L \cap W = 0\} = U$

For $L \in U$ $\quad L + W^\circ = T$ so $L \cap W^\circ = Z$ has dim $n-r$

$$\tilde{T} = \left\{ (\overset{n}{L}, Z, \overset{n}{V}) \;\middle|\; \begin{array}{ll} L, V \in SG(T), & V \supset W \\ Z \subset L \cap V & \dim Z = n-r \end{array} \right\} \quad \text{OKAY}$$

$$\dim \tilde{T} = \underset{V}{\frac{(n-r)(n-r+1)}{2}} + \underset{Z}{(n-r)r} + \frac{r(r+1)}{2}$$

$$= \frac{(n-r)^2 + 2(n-r)r + r^2}{2} + \frac{n-r+r}{2} = \frac{n^2+n}{2}$$



You should be dealing with a real form of $P(\mathbb{C}^2)$ with $SL(2,\mathbb{C})$ symmetry. Thus you could look at $P(\mathbb{R}^2)$ with $SL(2,\mathbb{R})$ action

You remember something about spinors. Yes

$$\mathcal{O}(-1)^{\otimes 2} = \mathcal{O}(-2) \simeq \Omega^1.$$

$(cz+d)a\delta z - (az+b)c\delta z$

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad g^*(f\delta z) = f\left(\frac{az+b}{cz+d}\right)\delta\left(\frac{az+b}{cz+d}\right) = \frac{ad-bc}{(cz+d)^2}\delta z$$

Consider a rational section of $\mathcal{O}(-1)$: $f(z)\begin{pmatrix} z \\ 1 \end{pmatrix}$. Then

$$g^* f\begin{pmatrix} z \\ 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} f\left(\frac{az+b}{cz+d}\right)\begin{pmatrix} \frac{az+b}{cz+d} \\ 1 \end{pmatrix} \leftarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} z \\ 1 \end{pmatrix}\frac{1}{cz+d}$$

$$= \frac{1}{cz+d} f\left(\frac{az+b}{cz+d}\right)\begin{pmatrix} z \\ 1 \end{pmatrix}$$

$$0 \longrightarrow \mathcal{O}(-1) \longrightarrow \mathcal{O}\otimes\mathbb{C}^2 \longrightarrow \mathcal{O}(1) \longrightarrow 0$$

Canon. isom. $\mathcal{O}(-1)\otimes_{\mathcal{O}}\mathcal{O}(1) = \mathcal{O}\otimes\Lambda^2\mathbb{C}^2$

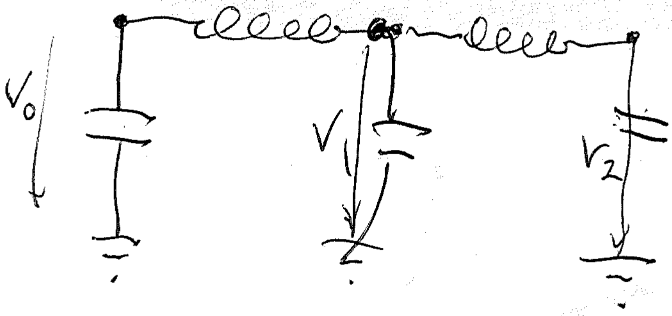
tang bundle $\hookrightarrow \Theta = \mathrm{Hom}_{\mathcal{O}}(\mathcal{O}(-1), \mathcal{O}(1)) = \mathcal{O}(-2)$

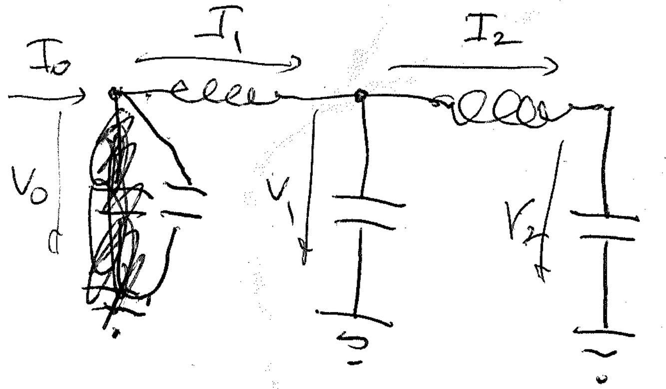
Idea from the ledger.

Consider $\mathbb{C}^2$ with action of $SU(1,1)$.

In other words you treat $\mathbb{C}^2$ as a Krein space

Now consider





$$I_0 - I_1 = C_0 s V_0$$

$$V_0 - V_1 = L_1 s I_1 \qquad I_1 - I_2 = C_1 s V_1$$

$$V_1 - V_2 = L_2 s I_2 \qquad I_2 - I_3 = C_2 s V_2$$

$$\frac{V_0}{I_0} = \frac{V_0}{I_1 + C_0 s V_0} = \frac{1}{C_0 s + \frac{I_1}{V_1 + L_1 s I_1}}$$

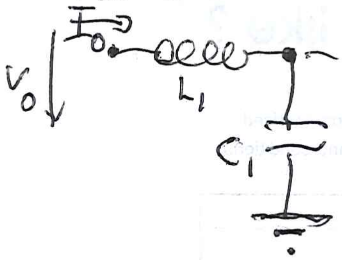
$$= \frac{1}{C_0 s + \frac{1}{L_1 s + \frac{V_1}{I_1}}} \qquad \text{etc.}$$

Converse - suppose $f(s)$ rational real assume maps RHP into itself.

What you want to do is to link the scattering picture to LC circuits somehow. There should be something you could do. Take an LC 1-port. This should have an obvious type of response – real rational function of s, purely imaginary poles, and there should be an equivalent LC ladder obtained from the partial fraction expansion. It might be better to use a string with finite point masses.

$$Z_0 = L_1 s + \cfrac{1}{C_1 s + \cfrac{1}{L_2 s + \cfrac{1}{\;}}}$$

Review



$$Z_0 = L_1 s + \cfrac{1}{C_1 s + \cfrac{1}{Z_1}}$$

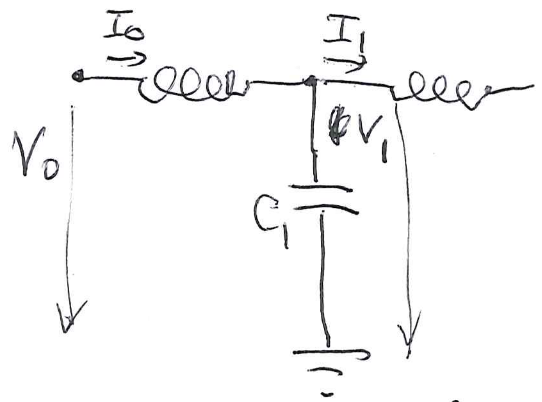
Idea is that you get a real rational fn $Z_0(s)$ such that $Re(s) > 0 \Rightarrow Re(Z_0(s)) > 0$.

This should restrict the partial fraction expansion to terms $\frac{1}{s^2+\omega^2}$

$$V_0 = L_1 s I_{L_1} + V_1$$

$$V_1 = \frac{1}{C_1 s} I_{C_1}$$

$$I_0 = I_{C_1} + I_{L_2}$$



$V_0 = L_1 s I_0 = L_1 s (C_1 s V_1 +$

You want to start at the scattering end, but maybe this is too hard.

Maybe begin with an LC circuit. This is a complicated system of constant coeffs. DE's. Solutions form a module over the time transl. gp $\mathbb{R}$.

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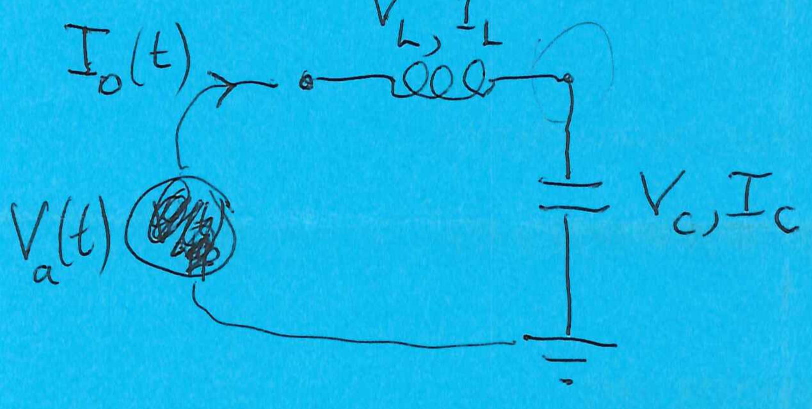
functions of $s \in i\mathbb{R}$. some ring of

You would like to understand the variational approach. IDEA can you impose constraints via ~~statistical mechanics~~. better: Lagrange multipliers



$$V_L = L\partial_t I_L$$

$$I_c = C\partial_t V_c$$

constraints $I_0 = I_L = I_c$

$$V_a = V_L + V_c$$

<u>Problem:</u> How does this involve a variational principle?

$$H = \mathbb{C}' \oplus \mathbb{C}$$

$\Downarrow$ $\quad V_L + V_c \swarrow$ $\quad \uparrow$

$H/ \qquad \mathbb{C} \qquad \left\{ \begin{pmatrix} I_L \\ I_c \end{pmatrix} \middle| I_L = I_c \right\}$

$\{V_{in}\} \qquad \{I_{out}\}$

$$V_L I_L + V_c I_c = \underbrace{(V_L + V_c)}_{V_{in}} I_{out}$$

today: Legendre transform, Lagrange multipliers .

~~[illegible]~~

begin with variables x, L x ind

Introduce ξ new ind L dep.

+ Put $H = x\xi - L$. At the moment H depends on x, ξ. Keep ξ fixed look for a stationary point for H as a fn of x. $0 = \partial_x H = \xi - \partial_x L$

In good cases [illegible] $\xi = \partial_x L$, ~~can solved~~

eg. $\partial_x \xi = \partial_x^2 L \neq 0$, then can use implicit function thm. to view ξ as

---

x, L

partial unitary $X \overset{a}{\underset{b}{\rightrightarrows}} Y$ $\quad a^*a = b^*b = 1_X$

$$Y = aX \oplus V_+ = V_- \oplus bX \qquad V_+ = \mathrm{Ker}\, a^* \quad V_- = \mathrm{Ker}\, b^*$$

Then you extend to a Hilbert space + unitary of by treating V_- as incoming and V_+ as outgoing.

$$\oplus\, u^{-2}V_- \oplus u^{-1}V_- \oplus X \oplus V_+$$

$$V_- \oplus uX \oplus uV_+ \oplus u^2V_+ \oplus \cdots$$

$$H^2_-(S^1, V_-) \oplus X \oplus H^2(S^1, V_+)$$

Maybe you need to explore Pick functions: analytic functions from D to UHP

Derive Poisson kernel solving Dirichlet problem for D

$u(z)$ holom. for $|z| < 1+\varepsilon$. Look

$$u(re^{i\theta}) = \sum_{n\geq 0} a_n r^n e^{in\theta} \quad \Big| \quad a_n = \int e^{-in\theta} u(re^{i\theta}) \frac{d\theta}{2\pi}$$

$u(z)$ holom. in D

$$u(z) = \sum a_n z^n \qquad a_n = [redacted] \oint z^{-n} u(z) \frac{dz}{2\pi i z}$$

$$u(re^{i\theta}) = \sum_{n\geq 0} a_n r^n e^{in\theta} \qquad a_n = \int_0^{2\pi} e^{-in\theta} u(re$$

leave r out of it. The important thing is to express $u(z)$ in term of the Imag part of u on the boundary.

So what do you aim for? First of all some sort of sheaf on $\mathbb{P}^1$

$$0 \longrightarrow \mathcal{O}(-1)\otimes X \longrightarrow \mathcal{O}\otimes Y \longrightarrow E \longrightarrow 0$$

<u>But</u> this doesn't seem to involve $Y \oplus Y$

Go back to (abstract) LC circuit

$$\begin{array}{ccccc}
\tilde{C}^0_{ext} & \twoheadleftarrow & \tilde{C}^0 & \hookrightarrow & C^1 \\
\oplus & & \oplus & & \oplus \\
\tilde{C}_{0,ext} & \rightarrowtail & \tilde{C}_0 & \twoheadleftarrow & C_1
\end{array}$$

$$\begin{array}{ccccc}
V_2/V_1 & \twoheadleftarrow & V_2 & \rightarrowtail & D \\
\oplus & & \oplus & & \oplus \\
V_1^*/V_2^* & \rightarrowtail & V_2^* & \twoheadleftarrow & D^* \\
\cup & & \| & & \\
V_1^0/V_2^0 & & D^*/V_2^0 & &
\end{array}$$

HERE KINEMATICS

NEED DYNAMICS which seems to amount to a Lagrangian subspace L_s of $\begin{smallmatrix} D \\ \oplus \\ D^* \end{smallmatrix}$ depending on a frequency parameter s. So what happens is that you project L_s to a Lagrangian subspace in the symplectic quotient.

$$W = \begin{smallmatrix} V_1 \\ \oplus \\ V_2^0 \end{smallmatrix} \qquad W^\perp = \begin{smallmatrix} V_2 \\ \oplus \\ V_1^0 \end{smallmatrix} \qquad W^\perp/W = \begin{smallmatrix} V_2/V_1 \\ \oplus \\ V_1^0/V_2^0 \end{smallmatrix}$$

Repeat $T^4 \supset \underset{\dim 3}{W^\perp}$ [illegible] $\supset \underset{\dim 1}{W} \supset 0$

$L = L^\perp$ Lagrangian and sat $L \cap W = 0$

$$\begin{array}{ccc} & T & \\ L+W & & W^\perp \\ L & (L \cap W^\perp)+W & \\ L\cap W^\perp & & W \\ & 0 & \end{array}$$

The real question is how to describe the possible L's. The line $L \cap W^\perp$ is equivalent to the Lagrangian subspace $L\cap W^\perp + W/W = (L+W)\cap W^\perp/W$ in $SG(W^\perp/W)$

$T = \begin{pmatrix} D \\ D^* \end{pmatrix}$ $\quad V_1 \subset V_2 \subset D \quad$ $W = \begin{pmatrix} V_1 \\ V_2^\circ \end{pmatrix}, W^\perp = \begin{pmatrix} V_2 \\ V_1^\circ \end{pmatrix}$

$L = \begin{pmatrix} 1 \\ Q \end{pmatrix} D \subset \begin{pmatrix} D \\ D^* \end{pmatrix}$

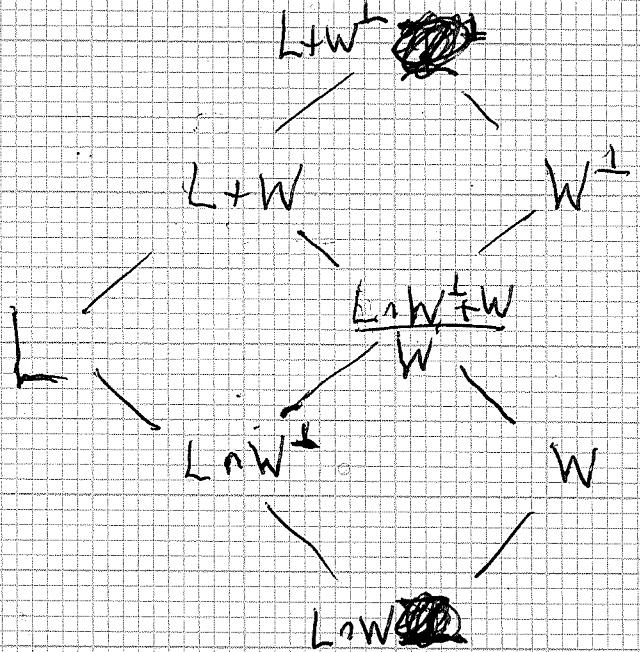
$$L \cap W^\perp = \left\{ \begin{pmatrix} \xi \\ Q\xi \end{pmatrix} \;\middle|\; \xi \in V_2,\ Q\xi \in V_1^\circ \right\}$$

$$\begin{array}{ccccc} V_1 & \rightarrowtail & V_2 & \rightarrowtail & D \\ & & \downarrow & & \downarrow Q \\ D^*/V_1^\circ & \twoheadleftarrow & D^*/V_2^\circ & \twoheadleftarrow & D^* \end{array}$$

interested in $\xi \in V_2$ such that $Q\xi \in V_1^\circ$ probably means that ξ is stationary wrt variations $\xi + \delta v_1$

making $L \longmapsto (L+W)\cap W^\perp/W = L\cap W^\perp + W/W$
into some sort of correspondence

Take T^4, W line. Generic situation is when $L\cap W = 0$ equiv. $L + W^\perp = 0$. Singular case is when $L\cap W \neq 0$ i.e. $W \subset L \subset W^\perp$



gen. case $L\cap W = 0$

sing case $L\cap W \neq 0$.

You want to introduce the variety of $\ell \subset L$. This is a $\mathbb{P}^1$ bundle over $SG(T)$

$W^\perp$

$T^4 \supset W^\perp \supset W \supset 0$ $\quad L \subset T$

generic case $L\cap W = 0$ equiv $L + W^\perp = T$. in this case $L\cap W^\perp$ is a line in L and one has a point $L\cap W^\perp \subset L$ in the flag bundle over $\underbrace{SG(T)}_{\text{dim } 3}$, call that $\underbrace{SG'(T)}_{\text{dim } 4}$. Do you have a map $SG'(T) \longrightarrow SG(W^\perp/W)$

T symplectic, W isotropic, $W^\perp/W$ the symplectic quotient of T. Example

$T = \begin{pmatrix} D \\ D^* \end{pmatrix}$ with $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $W = \begin{pmatrix} V_1 \\ V_2^0 \end{pmatrix}$ where $V_1 \subset V_2 \subset D$

Then $W^\perp = \begin{pmatrix} V_1 \\ 0 \end{pmatrix}^\perp \cap \begin{pmatrix} 0 \\ V_2^0 \end{pmatrix}^\perp = \begin{pmatrix} D \\ V_1^0 \end{pmatrix} \cap \begin{pmatrix} V_2 \\ D^* \end{pmatrix} = \begin{pmatrix} V_2 \\ V_1^0 \end{pmatrix}$

Here $V_i^0 = \{\lambda \in V_i^* \mid \lambda V_i = 0\}$, and $W^\perp$ refers to annihilator for the symplectic form. Note

$$W^\perp / W_0 = \begin{pmatrix} V_2/V_1 \\ \\ V_1^0/V_2^0 \end{pmatrix} = \begin{pmatrix} V_2/V_1 \\ \\ (V_2/V_1) \end{pmatrix}$$

Note that the W's of this form are the isotropic subspaces preserved by $\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Now add to T, W as above a Lagrangian subspace $L \subset T$. In above, Example: $L = \Gamma_Q = \begin{pmatrix} 1 \\ Q \end{pmatrix} D$ where $Q : D \to D^*$ is symmetric. Such "quadratic forms" Q yield all L projecting nonsingularly on D. A generic Q should be nondegenerate when restricted to D, V_1, V_2 and therefore to yield a splitting of the filtration $0 \subset V_1 \subset V_2 \subset D$ and also nondegenerate forms on the layers.

$$\begin{array}{ccccccc} 0 \subset & V_1 & \subset & V_2 & \subset & D \\ & \downarrow \cong & & \downarrow \cong & & \downarrow \cong \\ 0 \twoheadleftarrow & V_1^* & \twoheadleftarrow & V_2^* & \twoheadleftarrow & D^* \end{array}$$

Thus quadratic form induced by Q on V_2/V_1 yields a Lagrangian subspace of $W^\perp/W$.

Go over unit circle stuff where you use hermitian forms.

$\mathbb{P}_1\mathbb{C}$ $\quad SU(1,1)$ symmetries of $\mathbb{C}^2$ equipped with the hermitian form

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}^* \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \bar{z}_1 & \bar{z}_2 \end{pmatrix} \begin{pmatrix} z_1 \\ -z_2 \end{pmatrix} = |z_1|^2 - |z_2|^2$$

$g \in SU(1,1)$ means $g^* \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$g^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

assume $\det(g) = 1$. $\quad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad g^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$g^* = \begin{pmatrix} \bar{a} & \bar{c} \\ \bar{b} & \bar{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} d & b \\ c & a \end{pmatrix}$$

$\therefore \quad \bar{a} = d, \ \bar{b} = c$

$$\boxed{g = \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix} \qquad |a|^2 - |b|^2 = 1}$$

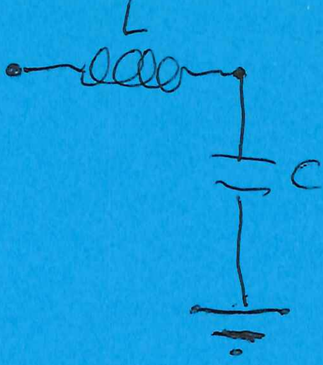
hermitian form above applied to $\mathbb{P}(\mathbb{C}^2)$ vanishes on $|z| = 1$. $\quad > 0$ on $|z| < 1$

< 0 — $|z| > 1$. $\qquad z = \frac{z_1}{z_2}$

Y complex v.s. with hermitian form.

The idea maybe is that you go between $\mathbb{P}_1(\mathbb{R}) \subset \mathbb{P}_1\mathbb{C}$ and $\{|z| = 1\}$. Over $\mathbb{P}_1\mathbb{C}$ you have a canonical $\mathcal{O}(-1) \longrightarrow \mathcal{O} \otimes \mathbb{C}^2 \longrightarrow \mathcal{O}(1)$. If we restrict to the reals then ? ?

Example.



$$\begin{pmatrix} C^1_\phi \\ \\ C_1 \end{pmatrix} = T^4$$

no loops.

Constraint $I_L = I_C$

You have variables V_L V_C I_L I_C making a symplectic space T of dim 4 where V_L, I_L are "dual variables"

V_C, I_C ———————— .

Constraint $I_L = I_C$ is a linear fnl. on T represented by $V_L + V_C$ (up to signs)

[redacted]

Next have Lagrangian subspace L_5.

$$T = \begin{pmatrix} D \\ D^* \end{pmatrix}, \quad L = \begin{pmatrix} 1 \\ Q \end{pmatrix} D, \quad W = \begin{pmatrix} V_1 \\ V_2^0 \end{pmatrix} \subset W^\perp = \begin{pmatrix} V_2 \\ V_1^0 \end{pmatrix}$$

$$L \cap W = \left\{ \begin{pmatrix} \xi \\ Q\xi \end{pmatrix} \;\middle|\; \begin{matrix} \xi \in V_1 \\ Q\xi \in V_2^0 \end{matrix} \right\}$$

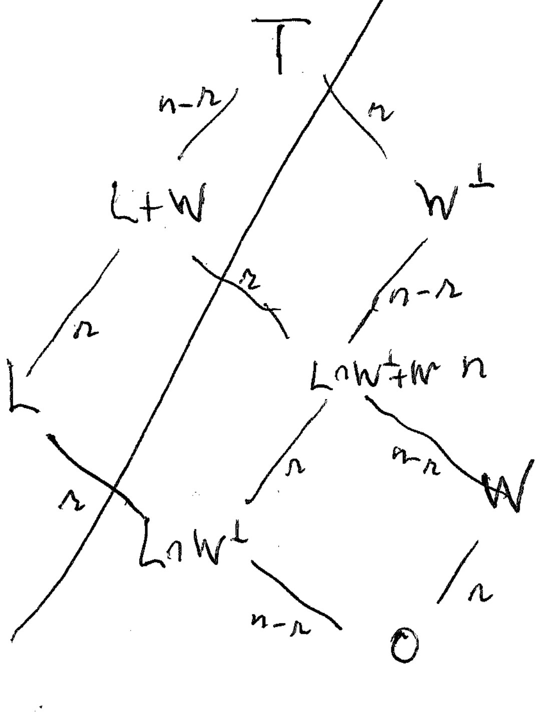
$\therefore$ $L \cap W$ is $\{\xi \in V_1 \mid V_2 Q \xi = 0\}$

$$\begin{array}{ccc} V_1 & \hookrightarrow & D \\ & & \downarrow Q \\ & & D^* \longrightarrow D^*/V_2^0 \\ & & \qquad\quad \shortparallel \\ & & \qquad\quad V_2^* \end{array}$$

$$\begin{array}{ccc} & V_1 \hookrightarrow & D \\ & & \downarrow Q \\ V_2^* = D^*/V_2^0 & \twoheadleftarrow & D^* \end{array}$$

T^{2n}, L^n, $W \subset W^\perp$ | $L \cap W^\perp + W = (L+W) \cap W^\perp$

$$L \cap W = 0 \Leftrightarrow L + W^\perp = T$$



Question: Can you fit the transmission line into your subquotient of a polarized Hilb space picture?

transmission line = simplest Dirac eqn., 2 diml space-time left + right movers, harmonic oscillator appears upon bosonization

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Idea (from '98) The response function arising from an LC circuit with external nodes should be a kind of Gelfand R.. quasi-determinant.

Q: Is there some link between response functions (impedance function $Z(s)$) for LC circuits (more generally subquotients of a polarized Hilb spaces) and response functions (scattering operators $S(z)$) for partial unitary operators?

$|h_n| < 1 \qquad k_n = \sqrt{1-|h_n|^2}$

$$\begin{pmatrix} p_n \\ q_n \end{pmatrix} = \frac{1}{k_n} \begin{pmatrix} 1 & h_n \\ \bar{h}_n & 1 \end{pmatrix} \begin{pmatrix} z & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_{n-1} \\ q_{n-1} \end{pmatrix}$$

form you want to take

$$\begin{pmatrix} p_x \\ q_x \end{pmatrix} = \begin{pmatrix} 1 & h_x dx \\ \bar{h}_x dx & 1 \end{pmatrix} \begin{pmatrix} 1+ikdx & 0 \\ 0 & 1-ik^{dx} \end{pmatrix} \begin{pmatrix} p_{x-dx} \\ q_{x-dx} \end{pmatrix}$$

$$\begin{pmatrix} 1+ikdx & h_x dx \\ \bar{h}_x dx & 1-ikdx \end{pmatrix}$$

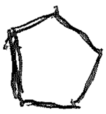
$$\partial_x \begin{pmatrix} p_x \\ q_x \end{pmatrix} = \begin{pmatrix} i\xi & h_x \\ \bar{h}_x & -i\xi \end{pmatrix} \begin{pmatrix} p_x \\ q_x \end{pmatrix}$$

There is something one can do for h_x real.

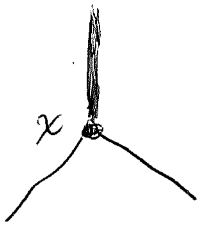
It seems ~~that~~ it might be possible to [illegible] link partial unitaries

Power? $P = VI$ in a direct current R circuit so you should proceed following Bott-Weyl.

graph Γ with resistance R_σ assigned to each edge σ. Orient edges. ~~[illegible]~~s variables V_σ, I_σ for each edge, subject to the relations

$\{V_\sigma\}$ conservative equiv $\sum V_\sigma$ along any cycle is zero. 

$\forall$ internal node



$$\sum_{x \to \sigma} I_\sigma = 0 \qquad x \text{ internal}$$

Ohm's Law $\qquad V_\sigma = R_\sigma I_\sigma \qquad \forall$ edge

$$\begin{array}{ccccc} \tilde{C}^0_{ext} & \longleftarrow & \tilde{C}^0 & \xrightarrow{\delta} & C^1 \\ & & & & \downarrow \tfrac{1}{R} \\ \tilde{C}_{ext,0} & \rightarrowtail & \tilde{C}_0 & \xleftarrow{\partial} & C_1 \quad Z_1 \end{array}$$

$\tilde{C}_0 = \mathbb{C}[\text{all nodes}]/\mathbb{C}[*]$

$\tilde{C}^0 =$ all functions on the nodes $= 0$ on $*$.

~~free~~ Let the internal voltage nodes be free.

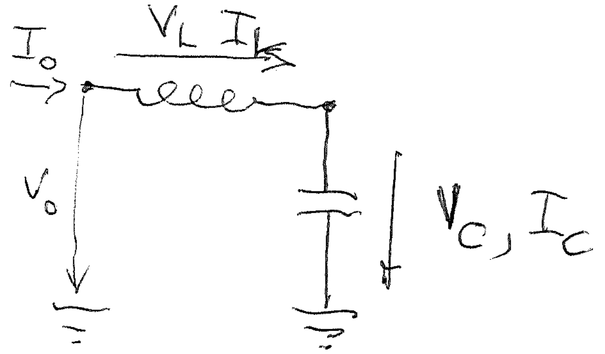
By positivity of the quadratic form on C^1 you get an induced positive form on the ~~external~~ ~~[illegible]~~ applied voltage. ~~[illegible]~~

Why does this imply that there's a unique solution of the equations:

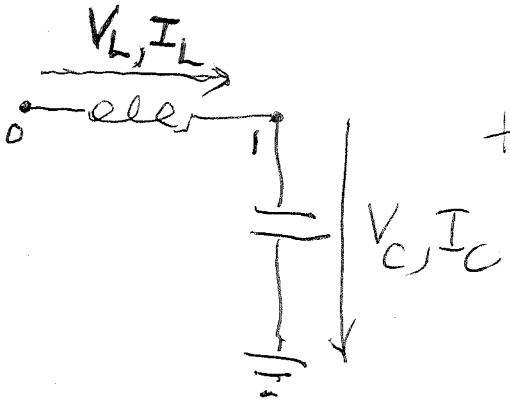
$(\delta V)_\sigma = R_\sigma I_\sigma \qquad V_\sigma$

$\partial I_1 = 0 \quad$ on internal nodes.

Start where?



$$V_L = L\,\partial_t I_L$$
$$I_C = C\,\partial_t V_C$$
$$I_0 = I_L = I_C$$
$$V_0 = V_L + V_C$$



three vertices, two edges.

$$V_L = L\,\partial_t I_L$$
$$I_C = C\,\partial_t V_C$$

assoc. to node 0 are V_0, I_0

—— node 1 — V_1, I_1

δ, ∂ relations

$$V_L = V_0 - V_1 \qquad I_0 = I_L$$
$$V_C = V_1 \qquad I_L = I_C$$

Node 1 internal

$$\tilde{C}^0_{ext} \longleftarrow \tilde{C}^0 \overset{\sim}{\longleftrightarrow} C^1$$

$$(V_0) \qquad (V_0, V_1) \qquad (V_L, V_C)$$

$$\left(\frac{1}{Ls}(V_0 - V_1)^2 + Cs V_1^2 , \quad \frac{1}{Ls} V_L^2 + Cs V_C^2 \right)$$

$$0 = \frac{1}{Ls}(V_0 - V_1)(-1) + Cs V_1 \Rightarrow \frac{1}{Ls} V_0 = \left(\frac{1}{Ls} + Cs\right) V_1$$

$$V_1 = \frac{V_0}{1 + LCs^2} \qquad V_0 - V_1 = V_0\left(1 - \frac{1}{1+LCs^2}\right) = V_0 \frac{LCs^2}{1+LCs^2}$$

from an LC 1-port you get an impedance $Z(s)$ of a specific form, e.g.

$a_\omega \frac{s}{s^2+\omega^2}$ $\qquad a_\omega > 0$.

Z maps RHP to itself, $i\mathbb{R}$ to $i\mathbb{R}$

$V \hookrightarrow H_+ \oplus H_-$ via $\begin{pmatrix} j_+ \\ j_- \end{pmatrix}$

here have $s\|\xi_+\|^2 + s^{-1}\|\xi_-\|^2$

on V you get sj

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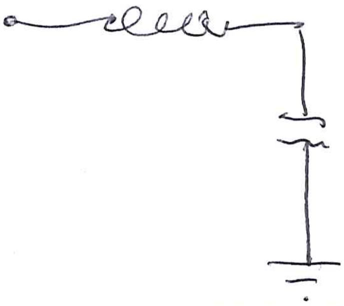
need to work on an example



IDEA. Hyman Bass defines a ζ function of a graph.
But a graph is a correspondence of some sort,
Is there a ζ function for a correspondence?
Iterate the correspondence n-fold intersect with Δ?

explain what you mean by variational principle

how to proceed

You have [redacted]

a symplectic quotient of

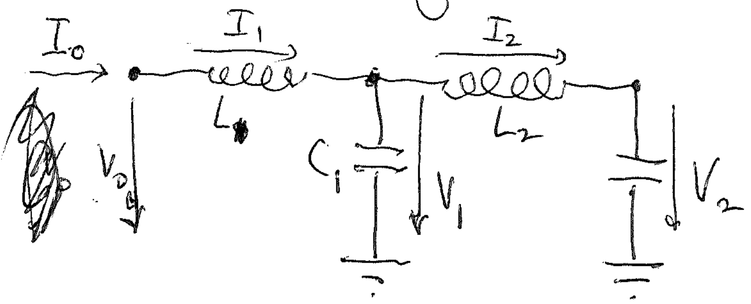
$C' \oplus C_1$ which arises

from a subquotient of C' — this is kinematics. There is quadratic form Q_s on C_1 — this is dynamics. the response fn is the induced quad form on the subquotient (ext nodes)

Confusing part. Why symplectic reduction of a Lagrangian subspace is related to a variational principle.

List problems as of July 15, 02

continuous version of LC circuit.



$$I_0 = I_1$$

$$V_0 - V_1 = L_1 s I_1$$
$$I_1 - I_2 = C_1 s V_1$$
$$V_1 - V_2 = L_2 s I_2$$

$$\frac{V_0}{I_1} = \frac{L_1 s I_1 + V_1}{I_1} = L_1 s + \frac{V_1}{I_1}$$

$$\frac{I_1}{V_1} = C_1 s + \frac{1}{L_2 s + V_2/I_2}$$

Here the variables you consider are $V_0, I_1, V_1, I_2, V_2, \ldots$
What is your aim? For an LC 1-port you have a symplectic quotient ~~reduction~~ picture

$$\{V_e\} \twoheadleftarrow \tilde{C}^0 \xrightarrow{\ \delta\ } C^1 \qquad \tilde{C}^0 = \{V_x\}$$

DR diffbl

$$\{I_e\} \rightarrowtail \tilde{C}_0 \xleftarrow{\ \partial\ } C_1$$

$$-\partial_x V = \lambda s I$$
$$-\partial_x I = \gamma s V$$

You ought to clarify the power/energy situation

56×38 cm

Sharp R-963
37×55,5×54
John Lewis

Whirlpool MY 277
32×53×48
Dixons

R953 ?

$$\frac{1}{k}\begin{pmatrix}1 & h\\ h & 1\end{pmatrix}\begin{pmatrix}z^{1/2} & 0\\ 0 & z^{-1/2}\end{pmatrix} \qquad \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}^{-1} = \frac{1}{+2}\begin{pmatrix}+1 & +1\\ +1 & -1\end{pmatrix}$$

$$\begin{pmatrix}1 & h\\ h & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1+h & 0\\ 0 & 1-h\end{pmatrix}$$

~~$\begin{pmatrix}z^{1/2} & 0\\ 0 & z^{-1/2}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} =$ [illegible] $= \begin{pmatrix}z^{1/2} & z^{1/2}\\ z^{-1/2} & -z^{-1/2}\end{pmatrix}$~~

$$\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}z^{1/2} & 0\\ 0 & z^{-1/2}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}z^{1/2} & z^{-1/2}\\ z^{1/2} & -z^{-1/2}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$= \begin{pmatrix}\frac{z^{1/2}+z^{-1/2}}{2} & \frac{z^{1/2}-z^{-1/2}}{2}\\ \frac{z^{1/2}-z^{-1/2}}{2} & \frac{z^{1/2}+z^{-1/2}}{2}\end{pmatrix} = \begin{pmatrix}\cos\frac{\theta}{2} & i\sin\frac{\theta}{2}\\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}$$

$$\sim \begin{pmatrix}1 & i\tan\frac{\theta}{2}\\ i\tan\frac{\theta}{2} & 1\end{pmatrix} = \begin{pmatrix}1 & s\\ s & 1\end{pmatrix}$$

~~[illegible]~~ Hope that this transformation provides the link between partial unitaries and LC circuits. The first part of the problem is the frequency parameter z on the unit circle versus s on $i\mathbb{R}$.

$$s = \frac{z^{1/2}-z^{-1/2}}{z^{1/2}+z^{-1/2}} = \frac{z-1}{z+1} \qquad z = \frac{1+s}{1-s}$$

$$s = \frac{z+1-2}{z+1} = 1 - \frac{2}{z+1}$$

$$\frac{-s+1}{2} = +\frac{1}{z+1} \qquad z+1 = \frac{2}{1-s} \qquad z = \frac{2}{1-s} - 1 = \frac{2-1+s}{1-s} = \frac{1+s}{1-s}$$

$$\tilde{C}^0_{ext} \twoheadleftarrow \tilde{C}^0 \xhookrightarrow{\ \delta\ } C^1$$

On $\tilde{C}^0$ one has the quad form $\delta V^t R^{-1} \delta V$, the restriction of $(V^1)^t R^{-1} V^1$ on C^1. Next you descend $\delta V^t R^{-1} \delta V$ from $\tilde{C}^0$ to the quotient $\tilde{C}^0_{ext} = \tilde{C}^0 / C^0_{int}$

$$\begin{array}{ccccc} C^0_{int} & \hookrightarrow & \tilde{C}^0 & \twoheadrightarrow & \tilde{C}^0_{ext} \\ & & \downarrow & & \\ C_{int,0} & \twoheadleftarrow & \check{C}_0 & \hookleftarrow & \tilde{C}_{ext,0} \end{array}$$

Given $V^0 \in \tilde{C}^0$ you project $V^0 \perp$ to C^0_{int}

do perturbation: Given $X \underset{b}{\overset{a}{\rightrightarrows}} Y$ $\quad a^*a = 1 = b^*b$

$\xi_\pm$ unit vector basis for $V_\pm$. $\quad \underbrace{ba^*}_{c_0} + \underbrace{\xi_- h \xi_+^*}_{\delta} = c_h$

$$(\lambda - c_h)^{-1} = (\lambda - c_0 - \delta)^{-1} = (\lambda - c_0)^{-1} + (\lambda - c_0)^{-1}\delta(\lambda - c_0)^{-1} + \cdots$$

$\delta = \xi_- h \xi_+^*$

$(\lambda - ba^*)^{-1} + (\lambda - ba^*)^{-1}\xi_- h \xi_+^*(\lambda - ba^*)^{-1} + \cdots$ You are missing something

$$(\lambda a - b)x = -v_+ + v_- \qquad x = a^*(\lambda - ba^*)^{-1}v_-$$

$$(\lambda - a^*b)x = a^* v_-$$

$$-(\lambda a - b)a^*(\lambda - ba^*)^{-1}v_-$$

$$G_h = G_0 + G_0 \delta G_0 + \cdots \qquad v_+ = \lambda(1 - aa^*)(\lambda - ba^*)^{-1}v_-$$

$$\xi_+^* G_h \xi_- = \xi_+^* G_0 \xi_- + \xi_+^* G_0 \xi_- h \xi_+^* G_0 \xi_- + \xi_+^* G_0 \xi_- h \xi_+^* G_0 \xi_- h \xi_+^* G_0 \xi_-$$

$$= (\xi_+^* G_0 \xi_-)\frac{1}{1 - h\,\xi_+^* G_0 \xi_-}$$

$$\xi_+^* G_0 \xi_- = \xi_+^*(\lambda - ba^*)^{-1}\xi_- = \xi_+^*(1 - \lambda^{-1}ba^*)^{-1}\xi_- = S_0(\lambda^{-1})$$

$$\xi_+^* G_h \xi_- = S_0(\lambda^{-1})\frac{1}{1 - hS_0(\lambda^{-1})} \qquad \frac{S_0}{1 - hS_0} = S_h$$

$$\| \quad S_h(\lambda^{-1})$$

$$\xi_+^*(\lambda - c_h)^{-1}\xi_-$$

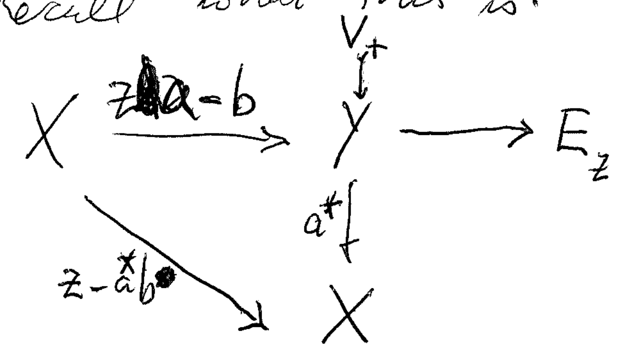
$$1 + \frac{hS_0}{1 - hS_0} = 1 + hS_h$$

$$\frac{1}{1 - hS_0} = 1 + hS_h$$

You're missing the meaning of $\xi_+^* G_h \xi_-$

c_h is a perturbation of ba^*

$S(z)$ recall what this is.



$$y - \overbrace{y(z)}^{V_+} = (za-b)x$$

$$a^*y = (z-a^*b)x$$

$$x = (z-a^*b)^{-1}a^*y = a^*(z-ba^*)^{-1}y$$

$$y(z) = y - (za-b)a^*(z-ba^*)^{-1}y$$

$$= ((z-ba^*) - (za-b)a^*)(z-ba^*)^{-1}y = (1-aa^*)\frac{z}{z-ba^*}y$$

Maybe there's a simpler version with

$$W = \begin{pmatrix} a \\ b \end{pmatrix} X \subset \begin{pmatrix} Y \\ Y \end{pmatrix} \qquad W^\perp = W \oplus \begin{pmatrix} V_+ \\ V_- \end{pmatrix}$$

You want $\begin{pmatrix} 1 \\ z \end{pmatrix} Y \cap W^\perp$

[illegible]

$$\begin{pmatrix} y \\ zy \end{pmatrix} = \begin{pmatrix} ax \\ bx \end{pmatrix} + \begin{pmatrix} v_+ \\ v_- \end{pmatrix}$$

$$z(ax + v_+) = bx + v_-$$

$$\boxed{(za-b)x = -zv_+ + v_-}$$

$$(z-a^*b)x = a^*v_-$$

$$x = a^*(z-ba^*)^{-1}v_-$$

$$zv_+ = v_- - (za-b)a^*(z-ba^*)^{-1}v_- = (z - ba^* - (zaa^* - ba^*)) \times (z-ba^*)^{-1}v_-$$

(inserted: $(z-ba^*)(z-ba^*)^{-1}$)

$$zv_+ = z(1-aa^*)(z-ba^*)^{-1}v_-$$

[illegible]