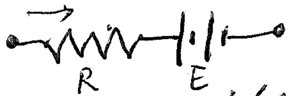
α1 Thevenin theory for R networks: The basic idea is to introduce on each edge an ideal e.m.f. in series with resistance for the edge. (ideal e.m.f. = battery with 0 internal resistance.) These e.m.f.'s yield inhomogeneous forcing terms for the circuit equations. Similar to the initial values of the "dominant" variables V_C, I_L for an LC network.

For each edge you have , so that Ohm's law says that the voltage drop V for the edge is

C^1
$\uparrow R$
C_1

$$\boxed{V = -RI + E.}$$

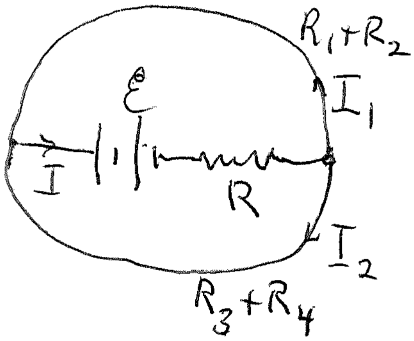
(Maybe V_{ap} instead of ϵ?)

Thus you have $2e$ variables subject to e Ohm constraints and $v-1+l = e$ Kirchhoff constraints.

The point of the Thevenin theory ~~should~~ might be that the graph does not change when external sources are applied.

Start again with a connected network such that each edge consists of an ideal e.m.f. in series with a resistance. View these edge emf's as inhomogeneous terms added to the Ohm's Law equations for the edges: $V = -RI + E$. Combining these e equations with the $e = v-1+l$ Kirchhoff constraints should yield (Weyl's positivity argument) a unique solution $(V, I) \in C^1 \oplus C_1$ for any E.

p1

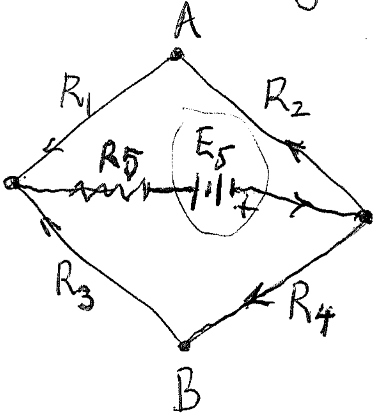


~~RI_1+RI_2~~

$RI + (R_1+R_2)I_1$?

$$\mathcal{E} = R + \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

You need to understand Thevenin theory. It ~~should~~ might involve superposition.

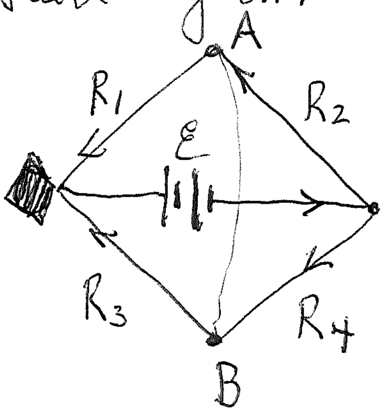


$4-1+2 = 5$ five linear eqns.

$V_j = R_j I_j \qquad j=1,2,3,4$

$V_5 = R_5 I_5 + E_5$

Start again. Thevenin theory says that the port A,B is equivalent to a pure emf



$$\mathcal{E}_0 = \left(\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right)\mathcal{E}$$

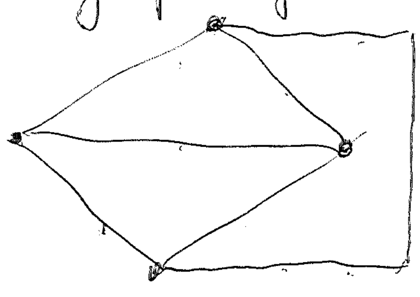
in series with a resistance

$$R_0 = \frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4}$$

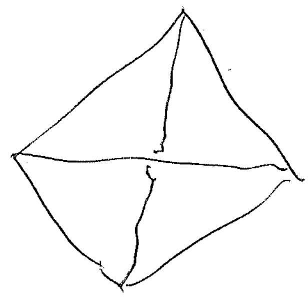
You have to make clear what this means.

$V_j = R_j I_j \qquad j=1,2,3,4 \qquad V_5 = \mathcal{E}_0$

It seems that you have to add an edge to the graph joining A to B.



$4-1+3 = 6$



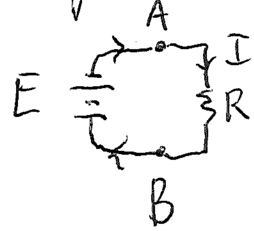
It's possible that a Wheatstone bridge is too hard to begin with. Look for something simpler, namely where one of the external nodes is the ground.

---

idea. Begin with a closed connected R network having no internal emf's. Attach a battery, an external emf, from node A to node B. What happens? More precisely, what linear equations describe, calculate, determine the state of the network, i.e. $(V, I) \in C^1 \oplus C_1$.

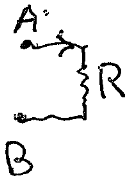
There are two pictures depending on whether you use the original graph or the augmented graph including the external emf. (1) In the former you having to alter the Kirchhoff current condition to allow node currents going into A and out of B. Also you must restrict the potential drop from A to B to be the voltage of the battery. (2) The latter picture consists of the augmented graph ~~with the new~~ including the battery, which will be treated as a forcing term.

You should check these two pictures are equivalent simple examples. A State is (V_R, I_R). The equations are



$V_R = E \qquad V_R = RI_R$.

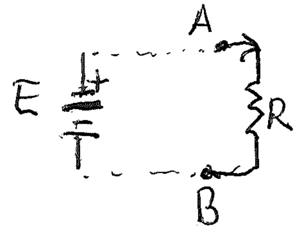
This picture is misleading. The graph should be



A state should consist of V_R, I_R and $\overset{I_A}{V_A}\ \overset{I_B}{V_B}$. Equations are

$$V_A - V_B = V_R \qquad V_R = RI_R \qquad I_A = I_R = -I_B$$

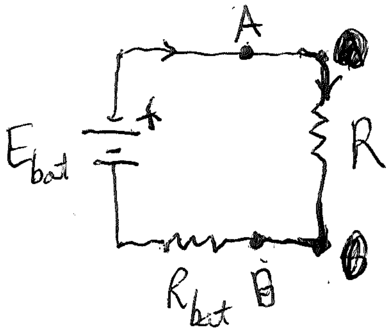
$$V_A - V_B = E \qquad ?$$

81  have^6 variables V_R, I_R $\quad V_A, I_A$ $\quad V_B, I_B$

have eqns $V_R = RI_R$, $V_A - V_B = V_R$

$I_A = I_R = -I_B$

You probably do not want V_A, V_B separately, rather it seems that there are 5 variables V_R, $V_A - V_B$, I_R, I_A, I_B subject to the ^{5} relations $V_R = V_A - V_B$, $I_R = I_A = -I_B$, $V_R = RI_R$, $V_R = E_{bat}$. Next try connecting a battery having internal resistance R_{bat}. Consider the variables



V_R, $V_A - V_B$, I_A, I_R, I_B, I_{bat}

$V_A - V_B = V_R = RI_R$

$I_{bat} = I_A = I_R = -I_B$

$E_{bat} = V_R + R_{bat} I_{bat}$

$E_{bat} = RI_R + R_{bat} I_R = (R + R_{bat}) I_{bat}$

response $I_{bat} = I_A = I_R = -I_B$ is $\dfrac{E_{bat}}{R + R_{bat}}$

The case above, where you look first at a network (only R's) with external nodes A, B - is probably harder than changing the graph by attaching a battery edge (consisting of pure emf + R_{bat}).

IDEA You haven't considered the old idea that the pair A, B determines a subquotient $C \twoheadleftarrow \overline{C}^0 \to C'$, such that the quadratic form $R^{-1}: C^1 \to C_1$ induces a quadratic form on $\mathbb{C} = \{\varphi(A)-\varphi(B) \mid \varphi \in \bar{C}^0\}$, which should [redacted] yield the current response to the applied voltage [redacted] from A to B.

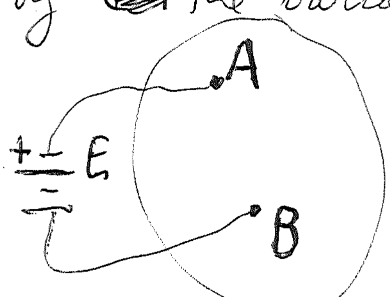
At this point you should have enough ideas to decypher, decipher the Thevenin theory for R-networks. Besides the ~~equivalences~~ Thevenin equivalent edge for a circuit with two external nodes, [redacted] which involves induced quadratic forms on a subquotient, you also have the idea of augmenting the graph by an edge with pure emf.

Start again with a connected R-network [redacted] equipped with 2 nodes A, B ("external)

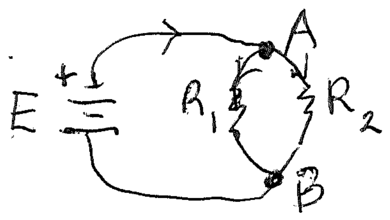
$$\begin{array}{ccccc} \varphi(A)-\varphi(B) & \longleftarrow & \varphi & & \\ \mathbb{C} & \twoheadleftarrow & \bar{C}^0 & \subset & C^1 \\ & & & & {\scriptstyle S}\downarrow{\scriptstyle R^{-1}} \\ \mathbb{C} & \longrightarrow & \bar{C}_0 & \twoheadleftarrow & C_1 \end{array}$$

Instead let's look at attaching a new edge from A to B containing an emf. You [redacted] now have a new graph, hence another V, I to be added to $C^1 \oplus C_1$. ~~So~~

additively our state space $C^1 \oplus C_1$ has increased by [redacted] the variables V_E, I_E. You probably want to treat the [redacted] emf E as a forcing term; an inhomogeneous term added [redacted] to the Ohm condition for the edge. [redacted] Thevenin theory should [redacted] give a resistance for what a sensitive ohmmeter would measure between A and B.



Example.



$2-1+2 = 3$ edges) variables

$V_{R_1}, V_{R_2}, I_{R_1}, I_{R_2}, V_{bat}, I_{bat}$

§1

$$V_1 \overset{①}{=} R_1 I_1\,,\quad V_2 \overset{②}{=} R_2 I_2\,,\quad \underset{\overset{\|}{E}}{V_{bat}} \overset{③}{=} V_1 \overset{④}{=} V_2$$

$$I_{bat} \overset{⑤}{=} I_1 + I_2$$

$$V_{bat} = R_1 I_1 = R_2 I_2\,,\qquad I_{bat} = I_1 + I_2 = V_{bat}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V_{bat} = I_{bat}\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right) = I_{bat}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Aim for a better understanding. Take a connected R-network with 2 "external" nodes A, B, then attach a pure emf between these nodes.

$$\bar{C}^0 \hookrightarrow C^1$$

Puzzle How is

$$\begin{tikzcd}
\mathbb{R} & \bar{C}^0 \arrow[l, two heads] \arrow[r, hook] & C^1 \\
\mathbb{R} \arrow[r, hook] & \bar{C}_0 & C_1 \arrow[l] \arrow[u, "R"]
\end{tikzcd}$$

$\varphi(A)-\varphi(B) \leftarrow\!\shortmid\ \varphi$

$1 \mapsto [A]-[B]$

linked to attaching a battery between A, B

Pushout?

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] \arrow[d] & C^1 \arrow[r] \arrow[d] & H^1 \arrow[d, equal] \\
\mathbb{R} \arrow[r, hook] & \hat{C}^1 \arrow[r] & H^1
\end{tikzcd}$$

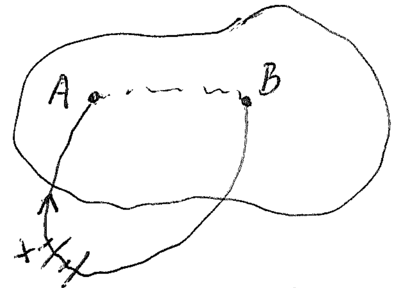
Shouldn't there be a new loop? Yes, so pushout seems wrong.

Let's try to solve the puzzle by working from the augmented graph. You start with the symplectic phase space $C^1 \oplus C_1$ for the initial graph and add a symplectic 2 plane i.e. the phase space of the attached edge.

You begin with a connected R-network. ~~[illegible]~~ Let $A \neq B$ be two nodes; attach an oriented edge joining A to B. Let C be the complex of chains for the original ~~network~~ graph X, ~~let~~ $\hat{C}_\circ$ be the complex of chains ~~for~~ the augmented ~~graph~~ Y. So it's clear that one has an exact seq. of complexes

$$0 \longrightarrow C_\circ \longrightarrow \hat{C}_\circ \longrightarrow \mathbb{R}[1] \longrightarrow 0.$$

Because X [redacted] is connected [redacted] there is a path in ~~X~~ joining A to B which can be combined with the new edge to get a 1-cycle:

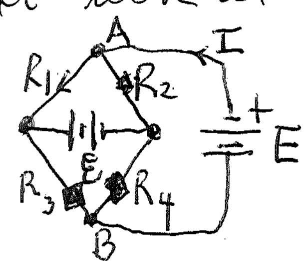


$$0 \to H_1(X) \longrightarrow H_1(Y) \longrightarrow \mathbb{R} \longrightarrow 0$$

The number l of ind loops increases by 1; the number of nodes stays the same.

Now [redacted] how do we find the current flow induced by the attached emf? The ~~applied~~ emf E is a forcing term, an inhomogeneous term added to the [redacted] homogeneous linear equations of the network. [redacted] It's clear that the [redacted] edge currents depend linearly on the applied emf E, ~~[illegible]~~ in particular the current through the attached edge is proportional to E, whence you have a resistance governing the current response for the "port" A, B.

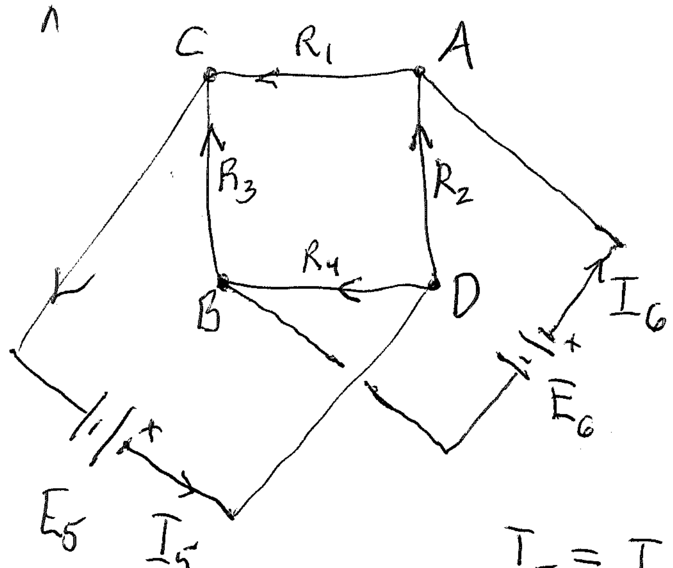
Next look at a network with internal emfs.



$4-1+3=6$ [redacted]

There should be 12 equations corresp. to the 12 obls. $V_1, V_2, V_3, V_4, \mathcal{E}, E$ and the assoc. current

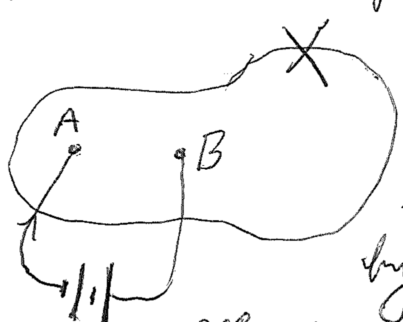
01 You are looking at the general Wheatstone bridge | without internal resistances in the two batteries | with ideal batteries.



$$I_5 = I_2 + I_4$$
$$I_5 = I_1 + I_3$$
$$I_6 = I_1 - I_2$$
$$I_6 = -I_3 + I_4$$

---

back to R-networks, try not to get stuck with complicated networks, instead you want to understand the general theory. The importance case seems to be augmenting a graph by an edge in order to handle an applied emf between two nodes.



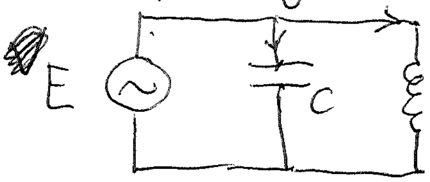
Because the original network X is connected the new graph has an extra loop, which is given by the new edge together with, followed by a path in X from A to B. It seems that ultimately you need the full state space consisting of V, I for all edges. Why??

Maybe not. You would prefer to work with the space C^1 of 1-cochains equipped with the symmetric quadratic form given by the energy of a configuration of edge voltages.

u1 [redacted] Let's try to get a better understanding of the Thevenin equivalent [redacted] circuit.

Maybe you want to look first at the case of a pure resistance network with attached pure emf between A, B. You really need to [redacted] reconcile the augmented circuit equations with the idea that you are changing the Kirchhoff constraints by allowing current to flow into the node A and out of the node B, and also restricting the voltage difference $\varphi(A)-\varphi(B)$ to be the value of the emf. There's a variational situation (Lagrange multipliers?) to be understood in which the node current being 0 is linked to the corresponding voltage being a free variable when minimizing the energy.

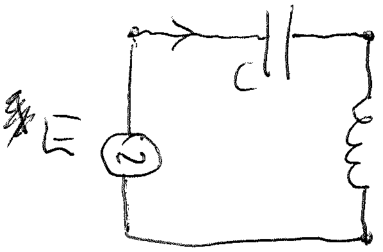
Examples of variational method using only voltage variables



$$\text{Energy} = \frac{1}{2}CsV_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2 = \frac{1}{2}\left(Cs+\frac{1}{Ls}\right)V_E^2$$

The current should be $\frac{\partial}{\partial E}(\text{Energy})$

so $I = \left(Cs+\frac{1}{Ls}\right)V_E$, $\therefore \frac{V_E}{I} = \frac{Ls}{LCs^2+1}$



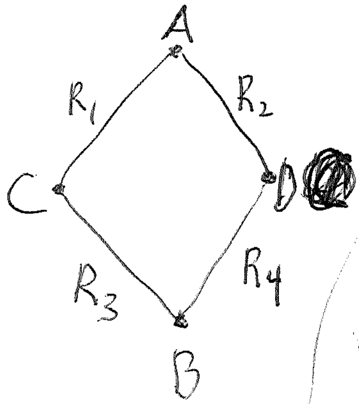
$\text{Energy} = \frac{1}{2}CsV_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2$ again, but now you want the critical point subject to $V_E = V_C + V_L$. Use Lagrange multiplier: $F = \frac{1}{2}CsV_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2 + \lambda(V_E - V_C - V_L)$. Then

$$\frac{\partial F}{\partial V_C} = CsV_C - \lambda = 0, \quad \frac{\partial F}{\partial V_L} = \frac{1}{Ls}V_L - \lambda = 0, \quad V_E = V_C + V_L$$

$\therefore V_C = \frac{\lambda}{Cs}$, $V_L = Ls\lambda$, $V_E = \left(\frac{1}{Cs}+Ls\right)\lambda$. Now eliminate λ: $\lambda = CsV_C = \frac{1}{Ls}V_L = \frac{V_E\,Cs}{LCs^2+1}$ $\therefore$

$$V_C = \frac{V_E}{1+LCs^2}, \quad V_L = \frac{V_E\,LCs^2}{1+LCs^2}$$

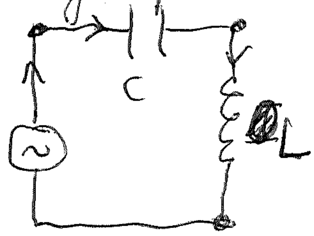
K-1 Wheatstone bridges. What is the situation?



4 edges, 4 nodes, 1 loop

~~can you improve glitch~~ ~~treat~~

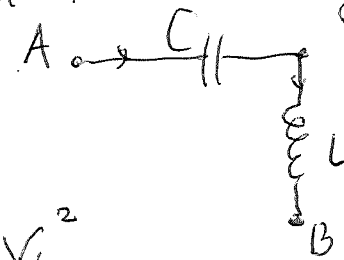
You feel that the Lagrange multiplier calculation should be related to augmenting the graph by the pure emf edge. Consider again



3 nodes
1 loop
3 edges.

You have 3 voltage variables V_E, V_C, V_L 1 Kirchhoff constraint $V_E = V_C + V_L$ and energy $\frac{1}{2} Cs V_C^2 + \frac{1}{2}\frac{1}{Ls} V_L^2$

(Review what you already did in the voltage picture: you have the circuit

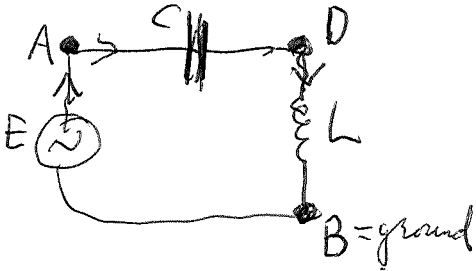


and you apply V_E from A to B.

Then you have Energy $= \frac{1}{2} Cs V_C^2 + \frac{1}{2}\frac{1}{Ls} V_L^2$

and Kirchhoff: $V_E = V_C + V_L$ and you use the Lagrange multiplier method in which λ turns out to be [redacted] $\lambda = I_E = I_C = I_L$.)

Now consider the augmented graph:

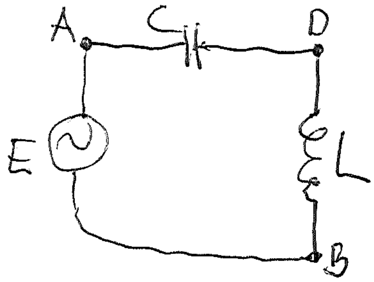


You want to [redacted] set up ~~things~~ using only voltage variables and energy quadratic form. (V_E, V_C, V_L). (Note that the orientation of the edges does not seem to matter ??)

$$\underbrace{C^0(Y)}_{\text{dim } 2} \xrightarrow{\delta} \underbrace{C^1(Y)}_{\text{dim } 3}$$

[redacted] ?

λ1



$$\tilde{C}^0(Y) \xrightarrow{\delta} C^1(Y) \longrightarrow H^1(Y)$$

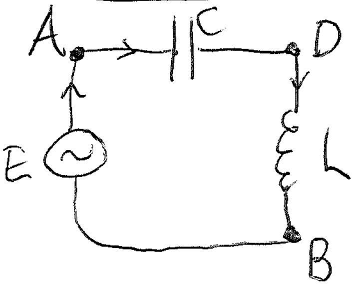
$$C^1(Y) = \{(V_E, V_C, V_L)\} \mapsto V_E - V_C - V_L = 0$$

$\tilde{C}^0(Y)$ has basis $\varphi(A)-\varphi(B),\ \varphi(D)-\varphi(B)$

$$\underbrace{\varphi(A)-\varphi(D)}_{V_C},\ \underbrace{\varphi(D)-\varphi(B)}_{V_L}$$

Energy $(V_C, V_L) \longmapsto \frac{1}{2}CsV_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2$, this is a quadratic form on $\tilde{C}^0(Y)$. Something is wrong.

Start again. 3 nodes, 3 edges, 1 loop.



~~Start with~~ a configuration space $C^1(Y)$ of dimension 3 described by the 'independent' vbls V_E, V_C, V_L. On this space $C^1(Y)$ you have the power form $\frac{1}{2}CsV_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2$, and you have the Kirchhoff constraint $V_E = V_C + V_L$ $\big(V_E = \varphi(A)-\varphi(B),\ V_C = \varphi(A)-\varphi(D),\ V_L = \varphi(D)-\varphi(B)\big)$.

What does it mean to solve the circuit equations? So far you only have the voltage variables, which you think should be enough. ~~What's going~~

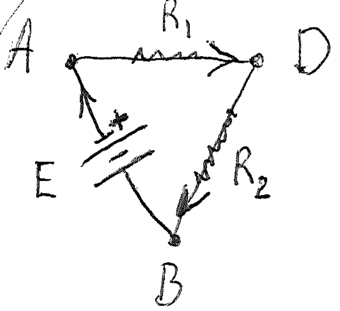
Let's find the currents. $I_C = \partial_{V_C}(\text{power}) \overset{1}{=} CsV_C$

$I_L = \partial_{V_L}(\text{power}) \overset{2}{=} \frac{1}{Ls}V_L$. Kirchhoff current law:

$I_E \overset{3}{=} I_C \overset{4}{=} I_L$. Kirchhoff voltage law: $V_E \overset{5}{=} V_C + V_L$

Now have 5 variables (because V_E is fixed) subject to 5 equations.

μ1



have oriented the edges

$$V_1 = \varphi(A) - \varphi(D)$$
$$V_2 = \varphi(D) - \varphi(B)$$
$$V_1 + V_2 = \varphi(A) - \varphi(B)$$

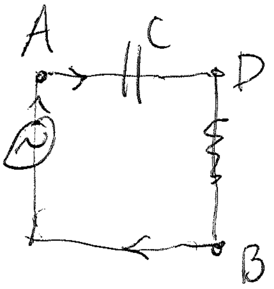
Kirchhoff ~~current~~ law says voltage

$$I_E = I_1 = I_2$$
$$V_E = V_1 + V_2$$

$$V_E = \varphi(B) - \varphi(A)$$

The point is that if $\varphi(A) > \varphi(D) > \varphi(B)$ then V_E in the orientation clockwise must be < 0. in fact $= \varphi(B) - \varphi(A)$. Thus the Kirchhoff constraints are

$$I_E = I_1 = I_2 \quad \text{and} \quad V_E + V_1 + V_2 = 0.$$



Energy is $\frac{1}{2} Cs V_C^2 + \frac{1}{2} \frac{1}{Ls} V_L^2$

$$F = \frac{1}{2} Cs V_C^2 + \frac{1}{2} \frac{1}{Ls} V_L^2 - \lambda(V_E + V_C + V_L)$$

$$0 = \frac{\partial F}{\partial V_C} = Cs V_C - \lambda \qquad V_C = \frac{\lambda}{Cs}$$

$$0 = \frac{\partial F}{\partial V_L} = \frac{1}{Ls} V_L - \lambda \qquad V_L = Ls \lambda$$

$$0 = \frac{\partial F}{\partial \lambda} = V_E + V_C + V_L \qquad -V_E = \lambda\left(\frac{1}{Cs} + Ls\right)$$

$$\lambda = \frac{-V_E}{\frac{1}{Cs} + Ls} \qquad \lambda = Cs V_C$$

$$\lambda = Cs V_C = \frac{1}{Ls} V_L = \frac{-V_E}{\frac{1}{Cs} + Ls} \qquad V_C = \frac{-V_E}{CLs^2 + 1}$$

1

$\lambda = \frac{(-V_E)}{\frac{1}{Cs}+Ls}$, $V_C = \frac{(-V_E)}{LCs^2+1}$, $V_L = \frac{(-V_E)LCs^2}{LCs^2+1}$

$$\cancel{E = \frac{1}{2} Cs \frac{(-V_E)^2}{(LCs^2+1)^2} + \frac{1}{2}\frac{1}{Ls}\frac{(-V_E)^2(LCs^2)^2}{(LCs^2+1)^2}}$$

~~Start again with~~ $\cancel{I_E = I_1 = I_2}$, $\cancel{V_E + V_1 + V_2 = 0}$

$$\cancel{\frac{V_E}{I_E} + Cs + \frac{1}{Ls} = 0} \qquad \cancel{\frac{-V_E}{Cs + \frac{1}{Ls}}}$$

$$\cancel{\frac{-V_E}{I_E} = Cs + \frac{1}{Ls}} \qquad \cancel{V_c}$$

Start again with $V_E + V_C + V_L = 0$ and $I_E = I_C = I_L$. Then $\frac{V_E}{I_E} + \frac{V_C}{I_C} + \frac{V_L}{I_L} = 0$

so $\boxed{\frac{-V_E}{I_E} = \frac{1}{Cs} + Ls}$

The power through the E-edge seems to be

$$(-V_E) = \left(\frac{1}{Cs} + Ls\right) I_E$$

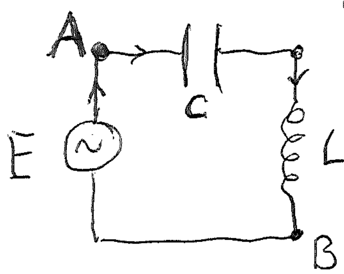
$$-V_E I_E = \left(\frac{1}{Cs} + Ls\right) I_E^2$$

$$I_E = \frac{(-V_E)}{\frac{1}{Cs}+Ls} \Longrightarrow \boxed{-V_E I_E = \frac{V_E^2}{\frac{1}{Cs}+Ls}}$$

It seems that the power through a pure emf is <0 since the resistor is >0. ?

Note: Although we use C, L components, putting $s=1$ makes them behave like resistors with DC voltage and current.

31/ Review what you learned about



In the first picture you have 3 nodes, 2 edges, 0 loops. You don't count the edge containing E. Variables are V_C, V_L subject to the constraint $V_E + V_C + V_L = 0$ with the power $= \frac{1}{2}Cs V_C^2 + \frac{1}{2}\frac{1}{Ls}V_L^2$. V_E is fixed.

In the second picture in which you have added the edge containing E you have 3 nodes 1 loop and $(3-1)+1 = 3$ edges. The variables describing a configuration are V_E, V_C, V_L with the same power. There is 1 Kirchhoff constraint $V_E + V_C + V_L = 0$. So you get the same variational problem.

Next the phase space picture, where in addition to the variables V_C, V_L you have the currents $I_C = \frac{\partial}{\partial V_C}(\text{Power}) = CsV_C$, $I_L = \frac{1}{Ls}V_L$, and also I_E subject to the Kirchhoff current condition $I_E = I_C = I_L$. Then (dividing by this current)

$$\boxed{\frac{V_E}{I_E} + \frac{1}{Cs} + Ls = 0}$$

or

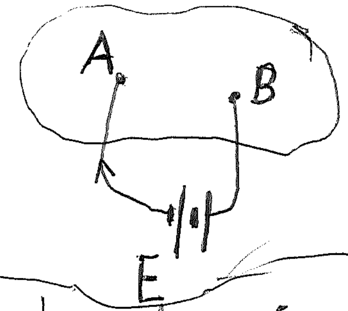
$$I_E = -\frac{V_E}{\frac{1}{Cs}+Ls}$$

so the power going into the E edge is

$$V_E I_E = -\frac{V_E^2}{\frac{1}{Cs}+Ls}$$

where the − sign means the power is actually going out of the "battery".

01 [redacted] Return to the voltage picture of a R-network (connected) with a single pair of external nodes A, B.



$$\begin{array}{ccc} \varphi \in \bar{C}^0(X) & \xrightarrow{\delta} & C^1(X) \\ \downarrow & & \downarrow \\ E = \varphi(A)-\varphi(B) & & R \end{array}$$

You ultimately want a quadratic function of E.

I think what you want to do is to handle the constraint $E - \varphi(A) + \varphi(B)$ via a Lagrange multiplier combined with the power quadratic form on $\bar{C}^0(X)$. The Lagrange multiplier λ will probably turn out to be the current going into A and out of B. [redacted] Lagrange functional on $\varphi \in \bar{C}^0(X)$ is $F = \frac{1}{2}\varphi^t \delta^t R^{-1} \delta\varphi + \lambda(E - \varphi(A) + \varphi(B))$

$$\frac{\partial F}{\partial \varphi} = (\delta^t R^{-1}\delta)\varphi - \lambda[A] + \lambda[B] = 0.$$

$$\frac{\partial F}{\partial \lambda} = E - \varphi(A) + \varphi(B) = 0.$$

says that the current response to the node potential φ is $\lambda([A]-[B])$, i.e. [redacted] λ in at A and λ out at B. [redacted] The power of the critical voltage configuration is

$$\varphi^t(\delta^t R^{-1}\delta)\varphi = \lambda(\varphi(A) - \varphi(B)) = \lambda E$$

Puzzle about the $\frac{1}{2}$. NO The critical point (φ, λ) should be determined by $(\delta^t R^{-1}\delta)\varphi = \lambda([A]-[B])$, which determines φ as a linear homogeneous function of λ, together with the other condition $E = \lambda(\varphi(A) - \varphi(B))$

π1 Let's try again with Lagrange multiplier

$$F = \frac{1}{2}\varphi^t(\delta^t R^{-1}\delta)\varphi + \lambda\left(E - \varphi(A) + \varphi(B)\right)$$

$$\frac{\partial F}{\partial \varphi} = (\delta^t R^{-1}\delta)\varphi - \cancel{\lambda\varphi(A)}\ \lambda[A] + \lambda[B] = 0$$

to solve (understand) $\quad (\delta^t R^{-1}\delta)\varphi = \lambda([A]-[B])$

$$E = \varphi(A) - \varphi(B)$$

Somehow you want to eliminate λ so that φ, the critical node potential, becomes a function of E. The first equation is equiv. to

$$\psi^t(\delta^t R^{-1}\delta)\varphi = \lambda(\psi(A) - \psi(B)) \quad \forall \psi \in C^0(X)$$

$\delta^t R^{-1}\delta$ is a symmetric bilinear form, nondegenerate invertible

$$\varphi = \lambda\ (\delta^t R^{-1}\delta)^{-1}([A]-[B])$$

$$E = \varphi(A) - \varphi(B) = \lambda\underbrace{([A]-[B])^t(\delta^t R^{-1}\delta)^{-1}([A]-[B])}_{Q}$$

$\lambda = \dfrac{E}{Q}$. What is the critical value of F?

$$\frac{1}{2}\varphi^t(\delta^t R^{-1}\delta)\varphi = \frac{1}{2}\lambda([A]-[B])^t(\delta^t R^{-1}\delta)^{-1} \times$$

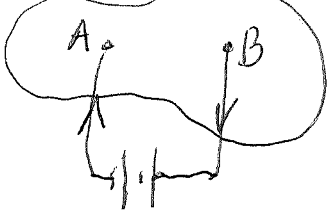
$$(\delta^t R^{-1}\delta)$$ [redacted] $$\lambda(\delta^t R^{-1}\delta)^{-1}([A]-[B])$$

$$= \frac{1}{2}\lambda^2 Q = \frac{1}{2}\frac{E^2}{Q}$$

The corresponding current is $\dfrac{\partial}{\partial E}\left(\dfrac{1}{2}\dfrac{E^2}{Q}\right) = \dfrac{E}{Q}$

$\boxed{E \text{ should be } V_E}$

ρ1 Review the case where the graph is augmented by the E-edge ~~going~~ from A to B. Here



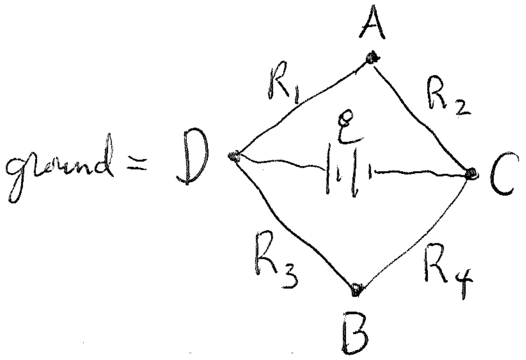
$$\bar{C}^0(X) = \bar{C}^0(\cancel{Y}) \xrightarrow{\delta_Y} C^1(Y) = C^1(X) + \{V_E\}$$

$$\overset{\in}{\varphi} \longmapsto \delta_Y \varphi = \delta_X \varphi + \varphi(A) - \varphi(B)$$

$\text{Power}(\varphi) = \varphi\, {}^t\delta_X^t R^{-1} \delta_X \varphi^t$, same as before. What happens next is that the constraint $V_E = \varphi(A) - \varphi(B)$ is an extra Kirchhoff voltage constraint due to the extra loop in the augmented graph. So it seems that you again have the problem of finding the critical point for the Power subject to the constraint $V_E = \varphi(A) - \varphi(B)$.

What next? You want to link applied emfs at the nodes to applied emfs at the edges.

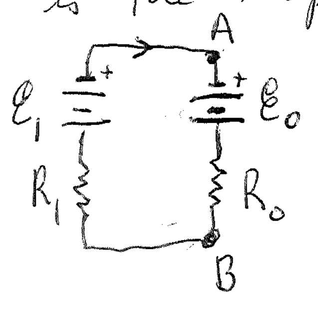
Look at Wheatstone bridge. 2 voltage variables



$\varphi(A), \varphi(B), \varphi(C) = \mathcal{E}$

$$\text{Power} \times 2 = \frac{\varphi(A)^2}{R_1} + \frac{(\overset{\mathcal{E}}{\cancel{\varphi(C)}} - \varphi(A))^2}{R_2} + \frac{\varphi(B)^2}{R_3} + \frac{(\overset{\mathcal{E}}{\cancel{\varphi(C)}} - \varphi(B))^2}{R_4}$$

You want to see the Thevenin result that this circuit with external nodes A, B is equivalent to a pure emf in series with a resistance. If this is true what is the response. Go over this again.



$\mathcal{E}_1 - \mathcal{E}_0 = (R_1 + R_0) I$

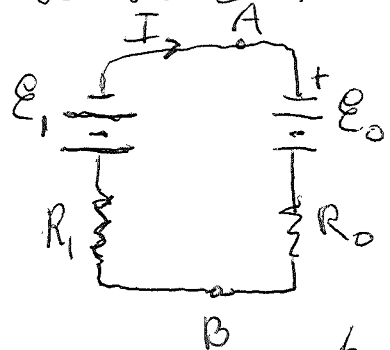
? ?

61 minimize $\dfrac{\varphi(A)^2}{R_1} + \dfrac{(\mathcal{E}-\varphi(A))^2}{R_2}$ | $\dfrac{\varphi(A)}{R_1} + \dfrac{-(\mathcal{E}-\varphi(A))}{R_2} = 0$

$$\varphi(A)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{\mathcal{E}}{R_2} \Longrightarrow \varphi(A) = \frac{\mathcal{E}R_1}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)R_2R_1} = \frac{\mathcal{E}R_1}{R_1+R_2}$$

Sim $\varphi(B) = \dfrac{\mathcal{E}R_3}{R_3+R_4}$ $\quad \underbrace{\varphi(A)-\varphi(B)}_{\mathcal{E}_0} = \left(\dfrac{R_1}{R_1+R_2} - \dfrac{R_3}{R_3+R_4}\right)\mathcal{E}$

Go back to



$$\mathcal{E}_1 - \mathcal{E}_0 = (R_0+R_1)I$$

So, if you adjust $\mathcal{E}_1$ then the current $I=0$ when $\mathcal{E}_1 = \mathcal{E}_0$, the pure emf component of A, B. ~~Next~~ you replace $\mathcal{E}_1, R_1$ by a short circuit. Then $-\mathcal{E}_0 = R_0 I$ gives R_0. Note I negative here.

Back to Thevenin theory. Idea is that the basic object should be an affine subspace of C^1 which is a quadratic space. What do you mean? First example: Connected R-network $\tilde{C}^0(X) \xhookrightarrow{\delta} C^1(X)$ together with a fix edge potential "in series with" the edge resistance. Denote the edge potential by V_a, then the state space is the coset $\delta\tilde{C}^0(X) + V_a = \{\delta\varphi + V_a \mid \varphi \in \tilde{C}^0(X)\}$. You want the critical point of the power $\frac{1}{2}V^tR^{-1}V$.

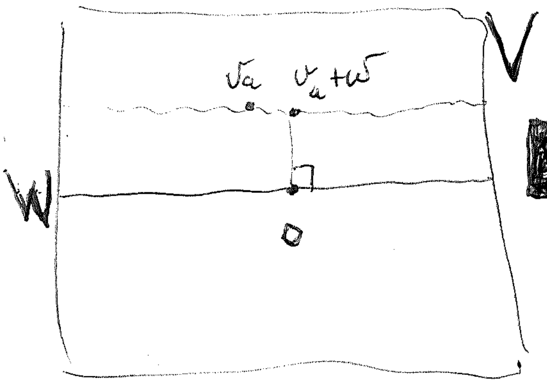
Look at this problem using Lagrange multipliers. Change notation. Given $W \subset V$ and a pos. quad form $\frac{1}{2}v^tR^{-1}v$ on V, a vector $v_a \in V$.

τ1 Repeat the data: Given $W \subset V$ real f.d.v.s., a pos. quad form $\frac{1}{2} v^t R^{-1} v$ on V, and a element v_a of V. Problem: Minimize the quad. form on the coset, affine hyper plane $v_a + W$. The straightforward method is to ~~differentiate~~:

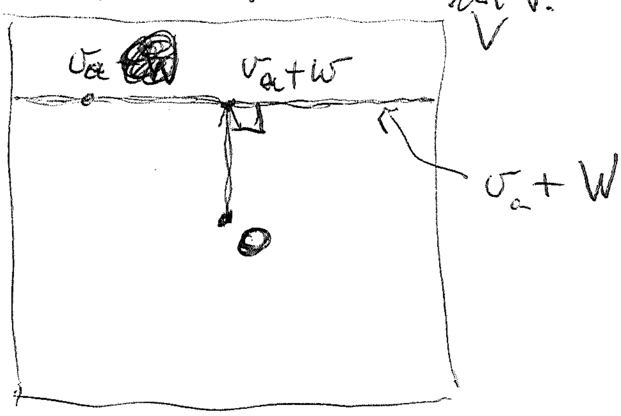
$$\frac{1}{2}\,\delta\Big/\left((v_a+w)^t R^{-1}(v_a+w)\right) = \frac{1}{2}\left\{\delta w^t R^{-1}(v_a+w) + (v_a+w)^t R^{-1}\delta w\right\}$$

$$= \delta w^t R^{-1}(v_a+w)$$

and set $=0$, which means that $v_a + w$ is $\perp$ W for the symmbilinear form R^{-1} on V.



better is:



Next try Lagrange multipliers

$f(v_a)=1$
$f(W)=0.$

$$\frac{1}{2} v^t R^{-1} v \qquad (v - v_a \in W)$$

To keep things simple let $V = \mathbb{R}v_a + W$

$$F = \frac{1}{2} v^t R^{-1} v + \lambda(1 - f^t v)$$

$$0 = \frac{\partial F}{\partial v} \Rightarrow R^{-1} v = \lambda f^t$$

$f \in V^*$

$\mathbb{R} \xrightarrow{f} V^*$

$\mathbb{R} \xleftarrow{f^t} V$

Critical value $\quad \frac{1}{2} v^t R^{-1} v = \lambda f^t v$

$F = \frac{1}{2} v^t R^{-1} v$ $\qquad$ duality is all confused.

$$R^{-1} : V \xrightarrow{\sim} V^* \xrightarrow{v^t} \mathbb{R}$$

v1 power $P(v) = \frac{1}{2} v^t R^{-1} v$, $\mathbb{R} \xleftarrow{v^t} V^* \xleftarrow{R^{-1}} V \xleftarrow{v} \mathbb{R}$

Constraint $f^t v = 1$ $\quad \mathbb{R} \xleftarrow{f^t} V \xleftarrow{v} \mathbb{R}$ $\quad V^* \xleftarrow{f} \mathbb{R}$, $V \xrightarrow{f^t} \mathbb{R}$

$$F = \frac{1}{2} v^t R^{-1} v + \lambda(1 - f^t v)$$

$$0 = \frac{\partial F}{\partial v} \Rightarrow R^{-1} v = \lambda f$$

$$0 = \frac{\partial F}{\partial \lambda} \Rightarrow \mu = f^t v$$

critical point

$$v_c = \lambda R f$$

$$f^t v_c = \mu$$

critical value

critical value is $\frac{1}{2} v_c^t R^{-1} v_c = \frac{1}{2} v_c^t \lambda f = \frac{\lambda}{2}$?

back to $\frac{1}{2}(v_a + w)^t R^{-1}(v_a + w) = P$ (power)

$$\frac{\delta P}{\delta w} = (\delta w)^t R^{-1}(v_a + w_c) = 0$$

$$P = \frac{1}{2} v_a^t R^{-1}(v_a + w_c)$$

Notation confusing!

simplest case $\quad Ax + By + C = 0.$

Constraint

quad form $\frac{1}{2}(x^2 + y^2)$

$$F = \frac{1}{2}(x^2 + y^2) - \lambda(Ax + By + C)$$

$$\frac{\partial F}{\partial x} = x - \lambda A = 0$$

$$\frac{\partial F}{\partial y} = y - \lambda B = 0 \qquad y - \lambda B = 0$$

$$\frac{\partial F}{\partial \lambda} = Ax + By + C = 0 \qquad \lambda A^2 + By + C = 0$$

$$\boxed{\lambda A^2 + \lambda B^2 + C = 0}$$

$$\lambda = \frac{-C}{A^2 + B^2}, \quad x = \frac{-AC}{A^2 + B^2}, \quad y = \frac{-BC}{A^2 + B^2} \qquad F = \frac{1}{2} C^2$$

p1 A pos def matrix, to minimize $\frac{1}{2}x^tAx$ subject to a linear constraint $y^tx = c$, y, c fixed both $\neq 0$.

$$F = \frac{1}{2}x^tAx + \lambda(c - y^tx)$$

$$0 = \frac{\delta F}{\delta x} \Rightarrow Ax = \lambda y, \quad 0 = \frac{\partial F}{\partial \lambda} \Rightarrow c = y^tx.$$

If A is $n \times n$ then $Ax = \lambda y$ is n-equations | x, y $n+1$ unknowns
$c = y^tx$ is 1 equation |

Can you solve: $x = \lambda A^{-1}y$, $c = \lambda y^tA^{-1}y$. Now eliminate λ:

$$\boxed{\lambda = \frac{c}{y^tA^{-1}y}} \qquad \boxed{x = \frac{c}{y^tA^{-1}y}A^{-1}y}$$

$$F = \frac{1}{2}y^tA^{-1}\frac{c}{y^tA^{-1}y}A\frac{c}{y^tA^{-1}y}A^{-1}y = \frac{1}{2}\frac{c^2}{y^tA^{-1}y}$$

$P(v) = \frac{1}{2}v^tR^{-1}v$, $\qquad \mathbb{R} \xleftarrow{v^t} V^* \xleftarrow{R^{-1}} V \xleftarrow{v} \mathbb{R}$

constraint $f^tv = \mu$ $\qquad \mathbb{R} \xleftarrow{f^t} V \xleftarrow{v} \mathbb{R}$, μ

$$F = \frac{1}{2}v^tR^{-1}v + \lambda(\mu - f^tv)$$

$$0 = \frac{\delta F}{\delta v} \Rightarrow R^{-1}v = \lambda f, \quad 0 = \frac{\delta F}{\partial \lambda} \Rightarrow \mu = f^tv$$

$\boxed{v = \lambda Rf}$ $\qquad \mu = \lambda f^tRf$ $\qquad \boxed{\lambda = \frac{\mu}{f^tRf}}$

$\partial_i(x^tAx) = \sum_j a_{ij}x_j + \sum x_j a_{ji}$

$\boxed{v = \frac{\mu}{f^tRf}Rf}$ so you've eliminated λ.

There seems to be 3 interpretations of a linear functional on V, namely:

1) an element $f \in V^*$, 2) a map $f: V \longrightarrow \mathbb{R}$,

3) the transpose map $f^t: \mathbb{R} \longrightarrow V^*$.

But you have only 2 notations, namely, f, f^t

similarly 1) v an element of V, 2) a map $\mathbb{R} \longrightarrow V$

3) a map $V^* \longrightarrow \mathbb{R}$.

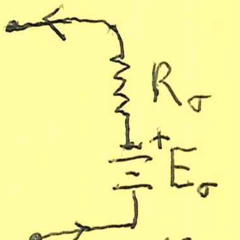
χ1 back to Thevenin theory. Start with a v.s C^1 equipped with pos. quad form – this is the space of edge voltages with the power quadratic form.

Note that there is no subspace of C^1 specified so that you can form a coset. ~~Already before~~ Suppose there's only one edge. Then you have the

the edge voltage $V = V_E + V_R$

Let's consider a connected R-network with e edges, v nodes, l loops: $v - 1 + l = e$.

$\overset{v-1}{C^0} \xhookrightarrow{\delta} \overset{e}{C^1} \longrightarrow \overset{l}{H^1}$, In Thevenin theory it is useful to assume each edge consists of a resistance and a pure emf in series. So for each oriented edge σ one has an R_σ (usually > 0), an internal emf $E_\sigma \in \mathbb{R}$, a voltage drop V_σ, and a current $I_\sigma \in \mathbb{R}$. Ohm's Law



says $V_\sigma = E_\sigma - R_\sigma I_\sigma$. $\quad V + RI = E$ fixed

One has e equations in 2e variables (V, I), so the Kirchhoff Laws give the remaining e equations. So far you use the phase space picture with 2e variables.

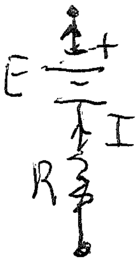
You want to ~~after V+RI=E fixed~~ minimize the power subject to the constraint $V + RI = E$ where E and R are fixed. The power probably is VI, so let $F = VI + \lambda(E - V - RI)$

$$\frac{\partial F}{\partial V} = I - \lambda \qquad \frac{\partial F}{\partial I} = V - \lambda R \qquad \frac{\partial F}{\partial \lambda} = E - V - RI$$

$$\lambda = I = \frac{V}{R} \qquad E = V + RI = \lambda R + R\lambda = 2\lambda R$$

$$\lambda = \frac{E}{2R} \qquad \frac{E}{2R} = I = \frac{V}{R} \quad ??$$

1 Thevenin theory. Consider an R-network connected, where each edge is a resistance in series with a pure emf. State space is $C^1 \oplus C_1$, better a state of the network is a pair (V, I), where $V \in C^1$ gives the voltage drops along the edges, and where $I \in C_1$ gives the currents through the edges.



$\underbrace{V = -RI + E}_{\text{The equation}}$ E = the fixed emfs for the edges. The equation together with the Kirchhoff constraints: $V \in \bar{C}^0$, $I \in H_1$ should have a unique solution (V, I) for every E:

$$\begin{tikzcd}
V \in \bar{C}^0 \arrow[r, hook, "\delta"] & C^1 \arrow[r, two heads] & H^1 \\
 & & \\
\bar{C}_0 & C_1 \arrow[l, two heads] \arrow[uu, "R"] & H_1 \ni I \arrow[l, hook']
\end{tikzcd}$$

$R : H_1 \longrightarrow C^1$ is complementary to δ since $R : H_1 \xrightarrow{\sim} H^1$ is an isom. by positivity of R.

Next you want to eliminate I. What does this mean? V and I are the components of E for an orthogonal splitting of C^1. So it seems meaningless to eliminate I.

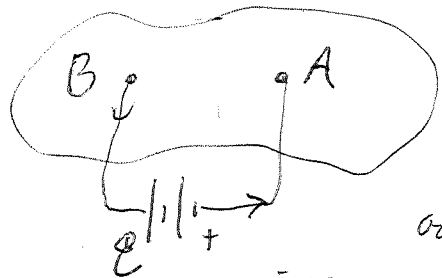
To go any further you probably need to consider when $\bar{C}^0$ is replaced by a subquotient of C^1.

Continue with Thevenin theory. Yesterday you looked at a connected graph in which each edge is a resistance in series with a pure emf. View these internal emfs as a fixed $E \in C^1$, an inhomogeneous term to be added to the circuit equations: $V + RI = E$, $V \in \bar{C}^0$, $I \in H_1$.

w1 Try to understand Thevenin theory in the case of an R-network with internal emfs, where you have ~~an applied~~ a DC voltage source with internal resistance $\neq 0$ applied between nodes A, B.

$$\begin{array}{ccc} \varphi \in C^0 & \xrightarrow{\delta} & C^1 \\ \downarrow & & \\ \varphi(A)-\varphi(B) & & \end{array}$$

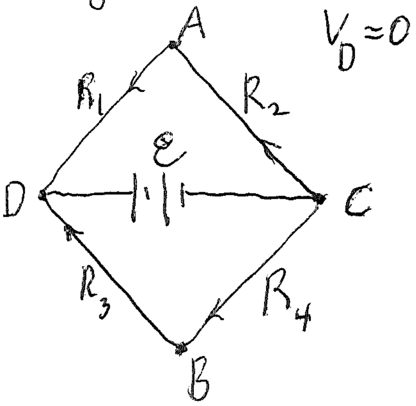
You are beginning to understand what to do: you have to combine what you know about an applied voltage between nodes A, B together with the internal emfs in the edges. Maybe you should review your study of



What happens here is that you relax the Kirchhoff current conditions to allow a current going into A and out of B. Also you restrict the potential difference $\varphi(A)-\varphi(B)$ to be equal to $\mathcal{E}$. What did you learn? It should be the same as attaching a new edges between A, B containing the emf $\mathcal{E}$.

α2

Wheatstone + Thevenin. It seems that you have learned something about Thevenin theory, maybe enough to do the Wheatstone bridge. Let's begin with the internal emf



$V_D = 0$

$$\frac{1}{2}\frac{V_A^2}{R_1} + \frac{1}{2}\frac{(\mathcal{E}-V_A)^2}{R_2} + \frac{1}{2}\frac{V_B^2}{R_3} + \frac{1}{2}\frac{(\mathcal{E}-V_B)^2}{R_4}$$

minimize wrt V_A

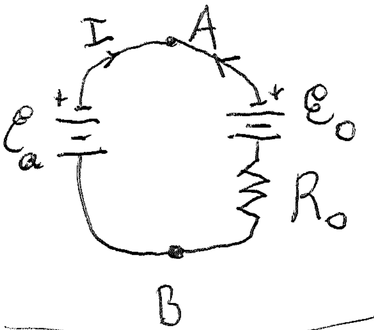
$$\frac{V_A}{R_1} + \frac{V_A - \mathcal{E}}{R_2} = 0, \quad \frac{\mathcal{E}}{R_2} = V_A\left(\frac{R_1+R_2}{R_1R_2}\right)$$

$V_A = \mathcal{E}\dfrac{R_1}{R_1+R_2}$. Similarly $V_B = \mathcal{E}\dfrac{R_3}{R_3+R_4}$. Next let $\mathcal{E}_0 = V_A - V_B$, the output from the [redacted] A, B circuit

$$R_0 = \frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4},$$

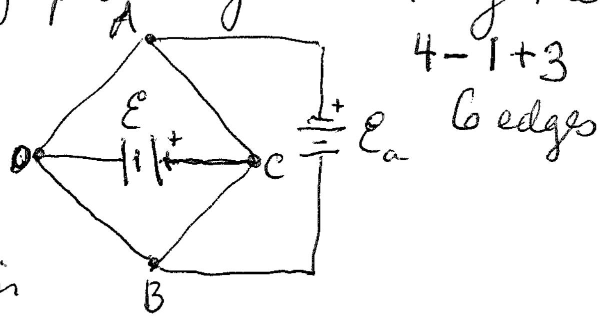
the internal resistance of the AB circuit where $\mathcal{E} = \mathcal{E}_{int}$ has been set to 0.

Suppose now you apply an emf [redacted] $\mathcal{E}_a$ going in at A and out of B.



$$\mathcal{E}_a = \mathcal{E}_0 + R_0 I$$

What sort of equations do you get from the graph augmented by the two emf edges:



4 − 1 + 3

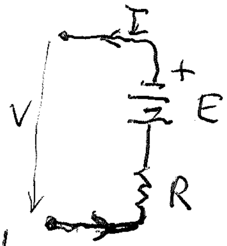
6 edges

You need to get straight the cochain spaces $\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$. $\bar{C}^0 = \{(V_A, V_C, V_B$ [redacted] $, V_D = 0)\}$ has dim = 3. C^1 has 6 components corresponding to the 2 elt subsets of the nodes. $\mathcal{E}$ and $\mathcal{E}_a$ are fixed values for [redacted] the voltage drops for AB and CD. You [redacted] treat them together as an inhomogeneous term in C^1.

B2

The equations to solve are $V = -RI + E$

better might be $V + RI = E$:



except you need to include the Kirchhoff conditions $V \in \bar{C}^0$, $I \in H_1$

Review: Given a conn graph where each edge consists of a pure emf in series with a resistance ($R \geq 0$ and can be zero, $E \in \mathbb{R}$ up to an orientation of the edge). Then one has cochains and chains

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\delta"] & C^1 \arrow[r, two heads] & H^1 \\
\bar{C}_0 & C_1 \arrow[u, "R"] \arrow[l, two heads, "\partial = \delta^t"'] & H_1 \arrow[l, hook']
\end{tikzcd}$$

Assume $R > 0$ to simplify. [redacted] On C^1 you have the power quadratic form $\frac{1}{2} V R^{-1} V$ which yields an orthogonal splitting of C^1 ~~into~~ $\bar{C}^0$ and the [redacted] orthogonal complement given by $R : H_1 \hookrightarrow C^1$. In other words, the [redacted] space of edge potentials splits into the "conservative" ones, the node potentials, and the edge potentials associated to the closed currents:

$$C^1 = \delta \bar{C}^0 \oplus RH_1.$$

[redacted] This is the basic linear splitting.

So far you haven't edge emf's, denote this $E \in C^1$. The splitting amounts to writing

$$E = V + RI$$

(E minus the voltage drop through the resistor = the voltage drop through the edge).

Abstract the Thevenin situation, more precisely, treating an applied voltage between two nodes A, B by means of the augmented graph. Consider

$$\begin{tikzcd} \varphi \in \bar{C}^0(X) \arrow[r, hook] \arrow[d] & C^1(Y) \\ \varphi(A)-\varphi(B) \in \mathbb{R} & \end{tikzcd}$$

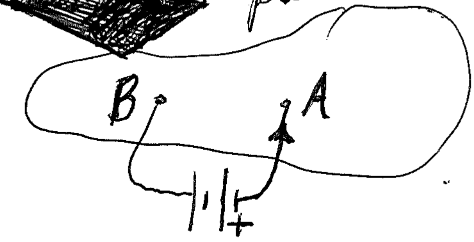
So you have a simple situation.

$$\begin{tikzcd} \bar{C}^0(X) \arrow[r, hook, "\delta"] \arrow[d, "{[A]-[B]}"'] & C^1(X) \\ \mathbb{R} & \end{tikzcd}$$

Start with $C^1(X)$ a Euclidean space, $\bar{C}^0(X)$ a subspace $[A]-[B] \in \bar{C}^0(X)^* = \bar{C}_0(X)$. You want to replace this by $\bar{C}^0(X) \to C^1(X) \oplus \mathbb{R}$. One problem is that there is no norm on $\mathbb{R}$. But this problem might be worthwhile to study. One version.

$$\begin{tikzcd} \mathbb{R} & \bar{C}^0 \arrow[l, "{([A]-[B])^t}"'] \arrow[r, hook] \arrow[d] & C^1 \arrow[d, "R^{-1}"] \\ \mathbb{R} \arrow[r, "{[A]-[B]}"] & \bar{C}_0 & C_1 \arrow[l] \end{tikzcd}$$

It looks like the $\mathbb{R}$ has a positive power



This positive power is related to the internal resistance of the Thevenin equivalent of the edge A, B.

Form the augmented circuit

$$\bar{C}^0(X) \xhookrightarrow{(\delta_X,\ [A]-[B])} C^1(X) \oplus \mathbb{R} = C^1(Y)$$

You have an element E of $C^1(Y)$ giving the emfs of the edges.

One thing you are not clear about, which you should study now is the case of a short exact sequence $0 \to \bar{C}^0 \to C^1 \to H^1 \to 0$

equipped with a quadratic form on C^1 which is degenerate, but is nondegenerate when restricted to $\bar{C}^\circ$. You guess that there's an induced quadratic form on H^1 which includes the degeneracy on C^1.

Review the calculation where you've chosen a complement Y to the subspace $X = \bar{C}^\circ$.

$$\begin{pmatrix} x \\ y \end{pmatrix}^t \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

stationary pt for δx $\quad ax + by = 0,$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -a^{-1}b \\ 1 \end{pmatrix} y$$

$$y^t \begin{pmatrix} -b^* a^{-1} & 1 \end{pmatrix} \begin{pmatrix} a & b \\ b^* & d \end{pmatrix} \begin{pmatrix} -a^{-1}b \\ 1 \end{pmatrix} y = y^t \begin{pmatrix} -b^* a^{-1} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -b^* a^{-1} b + d \end{pmatrix} y$$

$= y^t (d - b^* a^{-1} b) y.$ So the change of variable to make is

$$\begin{pmatrix} 1 & 0 \\ -b^* a^{-1} & 1 \end{pmatrix} \begin{pmatrix} a & b \\ b^* & d \end{pmatrix} \begin{pmatrix} 1 & -a^{-1}b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -b^* a^{-1} & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ b^* & -b^* a^{-1} b + d \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & d - b^* a^{-1} b \end{pmatrix}$$

i.e. $$\begin{pmatrix} x \\ y \end{pmatrix}^t \begin{pmatrix} a & b \\ b^* & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + a^{-1} b y \\ y \end{pmatrix}^t \begin{pmatrix} a & 0 \\ 0 & d - b^* a^{-1} b \end{pmatrix} \begin{pmatrix} x + a^{-1} b y \\ y \end{pmatrix}$$

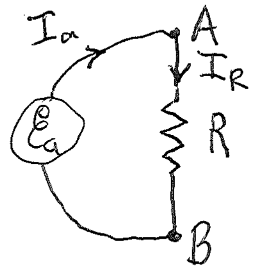
So it becomes clear now that the quadratic form on the quotient space H^1 can be nondegenerate even though the form d on your complement Y is degenerate.

ε 2 Consider the augmented graph ~~situation~~

$$\begin{tikzcd}
\tilde{C}^0(X) \arrow[r, tail] & C^1(X) \arrow[r, two heads] & H^1(X) \\
\tilde{C}^0(Y) \arrow[u, "\wr"] \arrow[r, tail] \arrow[ur, "\delta_X"] \arrow[dr, "{[A]-[B]}"'] & C^1(Y) \arrow[u] \arrow[r, two heads] & H^1(Y) \arrow[u, two heads] \\
 & \mathbb{R} \arrow[u, hook] & \mathbb{R} \arrow[u, hook]
\end{tikzcd}$$

You have the power quadratic form on $C^1(Y)$ which is >0 on all edges except the adjoined one where it's zero.

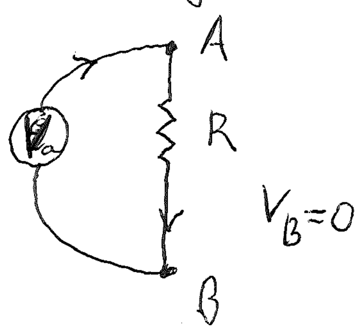
Consider the simplest cases in order to sort out the sign problems. Consider a connected R-network with an applied emf $\mathcal{E}_a$ between 2 nodes A, B.



$$\begin{array}{ccccc}
\tilde{C}^0 & \xrightarrow{\delta} & C^1 & \longrightarrow & H^1 \\
\varphi & \longmapsto & \begin{pmatrix} \varphi(A)-\varphi(B) \\ \varphi(B)-\varphi(A) \end{pmatrix} = \begin{pmatrix} V_R \\ V_a \end{pmatrix} & \longmapsto & V_R + V_a = 0
\end{array}$$

$$I_R - I_a \xleftarrow{(1\ \ -1)} \begin{pmatrix} I_R \\ I_a \end{pmatrix} = \begin{pmatrix} I \\ I \end{pmatrix} \xleftarrow{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} I \in H_1$$

Start again



$$\begin{array}{ccccc}
\tilde{C}^0 & \longrightarrow & C^1 & \longrightarrow & H^1 \\
V_A & \xmapsto{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} & \begin{pmatrix} V_A \\ -V_A \end{pmatrix} = \begin{pmatrix} V_R \\ V_a \end{pmatrix} & \xmapsto{(1\ \ 1)} & V_R + V_a \\
 & & \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} \uparrow & & \\
I_R - I_a & \xleftarrow{(1\ \ -1)} & \begin{pmatrix} I_R \\ I_a \end{pmatrix} = \begin{pmatrix} I \\ I \end{pmatrix} & \xleftarrow{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} & I \\
 & & & & \cap \\
\bar{C}_0 & \twoheadleftarrow & C_1 & \hookleftarrow & H_1
\end{array}$$

equations

$V_R + V_a = 0$

$I_R = I_a$

$V_R = R I_R$

$\begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix}$ is wrong; it says $V_a = 0$.

??

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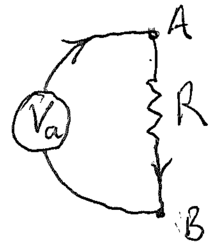


$\cancel{I_a = I_R}$ $\cancel{V_B = 0}$ $\cancel{V_R = V_A}$

$\cancel{R I_R = V_A}$ $\cancel{V_B = V_B + V_A}$

$\cancel{I_a = I_R = \frac{1}{R} V_A = -\frac{1}{R} V_a}$

two edges, 4 variables V_R, I_R, V_a, I_a

[redacted] Kirchhoff: $V_a = V_B - V_A$, $V_R = V_A - V_B$

voltage condition	$V_a + V_R = 0$
current condition	$I_a = I_R$ [redacted]
Ohm	$V_R =$ [redacted] $R I_R$
V_a fixed	

Solution for given V_a is: $V_a = -V_R = -R I_R = -R I_a$.

$I_a = -\frac{V_a}{R}$, $V_a I_a = -\frac{V_a^2}{R}$

$$\check{C}^0 \xrightarrow{\delta} C^1 \longrightarrow H^1$$

$$V_A \xmapsto{\begin{pmatrix}-1\\1\end{pmatrix}} \begin{pmatrix}-V_A\\V_A\end{pmatrix} = \begin{pmatrix}V_a\\V_R\end{pmatrix} \xrightarrow{(1\ \ 1)} V_a + V_R = 0$$

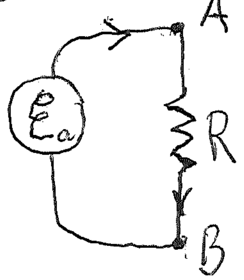
$$0 \xleftarrow{(1\ \ \cdot)} \begin{pmatrix}I_a\\I_R\end{pmatrix} = \begin{pmatrix}I\\I\end{pmatrix} \xleftarrow{\begin{pmatrix}1\\1\end{pmatrix}} I$$

$$\check{C}_0 \longleftarrow C_1 \longleftarrow H_1$$

What you have written here are [redacted] homogeneous linear [redacted] maps between vector spaces. You need to put V_a in as an [redacted] inhomogeneous "forcing" term.

$\eta 2$

$\eta 2$



Now let's correct the preceding page. You have a network with 2 edges labelled a, R so you have 4 variables V_a, I_a, V_R, I_R describing the state of the edges. There are 2 Kirchhoff constraints $V_a + V_R = 0$, $I_a = I_R$, [redacted] and Ohm's Law $V_R = R I_R$. This leaves one [redacted] degree of freedom which hopefully will be handled by putting [redacted] V_a, the voltage drop for the a edge, equal to the applied voltage $\mathcal{E}_a$.

Review:

$$\bar{C}^0 \xrightarrow{\begin{pmatrix}1\\T\end{pmatrix}} C^1 \xrightarrow{(-T\ \ 1)} H^1$$

$$V_C \longmapsto^{\begin{pmatrix}1\\T\end{pmatrix}} \begin{pmatrix}V_C\\V_L\end{pmatrix} \longmapsto^{(-T\ \ 1)} 0$$

$T\hat{V}_C = \hat{V}_L$

$\hat{I}_C + T^*\hat{I}_L = 0$

$$0 \longleftarrow \begin{pmatrix}I_C\\I_L\end{pmatrix} \longleftarrow I_L$$

$$\bar{C}_0 \xleftarrow{(1\ \ T^*)} C_1 \xleftarrow{\begin{pmatrix}-T^*\\1\end{pmatrix}} H_1$$

$\hat{I}_C = \hat{\dot{V}}_C = s\hat{V}_C - V_C(0)$

$\hat{V}_L = \hat{\dot{I}}_L = s\hat{I}_L - I_L(0)$

$$\boxed{\begin{aligned} s\hat{V}_C + T^*\hat{I}_L &= V_C(0) \\ -T\hat{V}_C + s\hat{I}_L &= I_L(0) \end{aligned}}$$

$$\bar{C}^0 \rightarrowtail C^1 \twoheadrightarrow H^1$$

$$\underset{\in \bar{C}^0}{V_R} \longmapsto^{\begin{pmatrix}-1\\1\end{pmatrix}} \begin{pmatrix}-V_R\\V_R\end{pmatrix} = \begin{pmatrix}V_a\\V_R\end{pmatrix} \longmapsto^{(1\ \ 1)} 0$$

$$0 \xleftarrow{(1\ -1)} \begin{pmatrix}I_a\\I_R\end{pmatrix} = \begin{pmatrix}I\\I\end{pmatrix} \longleftarrow I$$

$$\bar{C}_0 \twoheadleftarrow C_1 \xleftarrow{\begin{pmatrix}1\\1\end{pmatrix}} \hat{H}_1$$

Can you write $\begin{pmatrix}\mathcal{E}_a\\0\end{pmatrix} \in C^1$ as $\begin{pmatrix}-V_R\\V_R\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & R\end{pmatrix}\begin{pmatrix}I\\I\end{pmatrix}$?

$-V_R = \mathcal{E}_a$

$V_R + RI = 0$

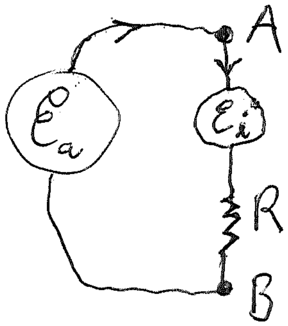
Yes, the solution is unique. Note that the composition

$$\underset{\in H_1}{I} \longmapsto \underset{C_1}{\begin{pmatrix}I\\I\end{pmatrix}} \longmapsto \underset{C^1}{\begin{pmatrix}0\\RI\end{pmatrix}} \longmapsto \underset{\in H^1}{RI}$$

is nondegenerate.

It seems that you can't handle the resistance as a symmetric operator. You probably need an appropriate correspondence. Let's put an ε resistance on the "applied" edge. The power is $\frac{1}{2}\frac{V_R^2}{R} + \frac{1}{2}\frac{V_a^2}{\varepsilon}$. Restrict to $\begin{pmatrix} V_R \\ V_a \end{pmatrix} = \begin{pmatrix} V_A \\ -V_A \end{pmatrix}$ to get $\frac{1}{2}\left(\frac{1}{R}+\frac{1}{\varepsilon}\right)V_A^2$?

You need more examples. Consider the circuit:



Same variables V_a, I_a, V_R, I_R

Kirchhoff: $\begin{cases} I_a = I_R \\ V_a + V_R = 0 \end{cases}$, $\quad V_a = V_B - V_A$, $\quad V_R = V_A - V_B$

You next introduce the inhomogneeous term $\begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_i \end{pmatrix} \in C^1$, and try to split this into a node potential $\begin{pmatrix} V_a \\ V_R \end{pmatrix} \in \delta \bar{C}^0$ plus the edge voltage drop $\begin{pmatrix} 0 & 0 \\ 0 & R \end{pmatrix}\begin{pmatrix} I \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ RI \end{pmatrix}$ corresponding to a loop current in H_1.

This is still confusing: you would like to know whether you can work in the voltage picture. This should be true I think. Discuss why: One has the short exact sequence

$$\bar{C}^0 \overset{\delta}{\hookrightarrow} C^1 \twoheadrightarrow H_1$$

and the quadratic form?

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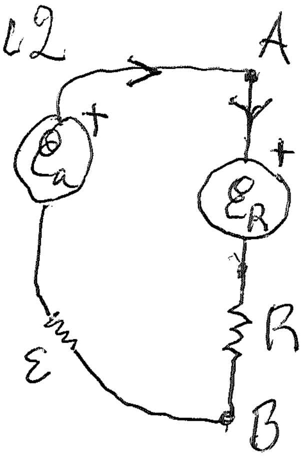


$$-\mathcal{E}_R - RI_R - \varepsilon I_a + \mathcal{E}_a = 0$$

$$\boxed{\mathcal{E}_a - \mathcal{E}_R = (R+\varepsilon) I}$$ signs correct

Now back to the inhomogeneous equations. Let's start again from the edge variables, or coordinates V_R, V_a, I_R, I_a subject to the homogeneous linear conditions:

$$V_R + V_a = 0, \quad I_R = I_a, \quad V_R = RI_R \, ?; \quad V_a = -\varepsilon I_a$$

What are V_R, V_a?

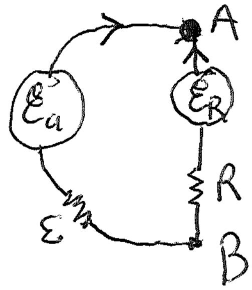
$$V_R = V_A - V_B = -\mathcal{E}_R - RI_R$$

$$V_a = V_B - V_A = -\varepsilon I_a + \mathcal{E}_a$$

Still confused. You need to straighten out the conventions on the applied emfs. Basically you want for each oriented edge $B \overset{R}{\text{—}\!\!\!\wedge\!\!\wedge\!\!\wedge\!\!\rightarrow} (\mathcal{E}) \text{—} A$ | $V_B - RI + \mathcal{E} = V_A$

$$\boxed{V + RI = \mathcal{E}}$$

V_R, V_a, I_R, I_a $\quad V_R = V_a, \; I_R = -I_a$



$$V_R = V_B - V_A = -RI_R + \mathcal{E}_R$$

$$V_a = V_B - V_A = -\varepsilon I_a + \mathcal{E}_a$$

$$V_R + RI_R = \mathcal{E}_R$$

$$V_a + \varepsilon I_a = \mathcal{E}_a$$

$$V_a - RI_a = \mathcal{E}_R$$

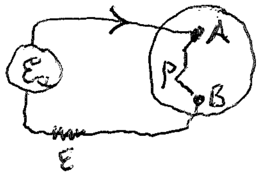
$$\begin{pmatrix}\mathcal{E}_a\\ \mathcal{E}_R\end{pmatrix} = \begin{pmatrix}1 & \varepsilon\\ 1 & -R\end{pmatrix}\begin{pmatrix}V_a\\ I_a\end{pmatrix}$$

$$\boxed{\begin{pmatrix}V_a\\ I_a\end{pmatrix} = \frac{1}{R+\varepsilon}\begin{pmatrix}R & \varepsilon\\ 1 & -1\end{pmatrix}\begin{pmatrix}\mathcal{E}_a\\ \mathcal{E}_R\end{pmatrix}}$$

Let $\varepsilon \to 0$ to get

$$\begin{pmatrix}V_a\\ I_a\end{pmatrix} = \frac{1}{R}\begin{pmatrix}R & 0\\ 1 & -1\end{pmatrix}\begin{pmatrix}\mathcal{E}_a\\ \mathcal{E}_R\end{pmatrix} = \begin{pmatrix}\mathcal{E}_a\\ \frac{\mathcal{E}_a - \mathcal{E}_R}{R}\end{pmatrix}, \text{ if } \mathcal{E}_R = 0: \begin{pmatrix}V_a\\ I_a\end{pmatrix} = \begin{pmatrix}\mathcal{E}_a\\ \frac{\mathcal{E}_a}{R}\end{pmatrix}$$

K2



$\partial e_a = [B] - [A]$

$$\bar C^0(X) \xrightarrow{\delta_y = \begin{pmatrix}\delta_a\\ \delta_x\end{pmatrix}} \begin{pmatrix}\mathbb R\\ C^1(X)\end{pmatrix} \xrightarrow{\begin{pmatrix} e_a^* & p^*\\ 0 & in^*\end{pmatrix}} \begin{pmatrix}\mathbb R\\ H^1(X)\end{pmatrix}$$

$$\bar C_0(X) \xleftarrow{(\partial_a\ \partial_x)=\partial_y} \begin{pmatrix}\mathbb R e_a\\ C_1(X)\end{pmatrix} \xleftarrow{\begin{pmatrix} e_a & 0\\ p & in\end{pmatrix}} \begin{pmatrix}\mathbb R\\ H_1(X)\end{pmatrix}$$

You want to understand the process of attaching an edge to two nodes. Begin with the s. exact seq

$$\begin{tikzcd} \bar C^0(X) \arrow[r, hook, "\delta_X"] \arrow[d, equal] & C^1(X) \arrow[r, two heads] & H^1(X) \\ \bar C^0(Y) \arrow[r, "\delta = \begin{pmatrix}\delta_X\\ \ \end{pmatrix}"] & \begin{pmatrix}C^1(X)\\ \mathbb R\end{pmatrix} & \begin{pmatrix}H^1(X)\\ \mathbb R\end{pmatrix} = H^1(Y) \end{tikzcd}$$

Make this more like vector spaces
Begin with

$$\begin{tikzcd} V' \arrow[r, hook, "\delta"] \arrow[d, "\lambda"] & V \arrow[r, two heads] \arrow[d] & V'' \\ \mathbb R \arrow[r] & \begin{pmatrix}V\\ \mathbb R\end{pmatrix} & \end{tikzcd}$$

$$V' \xrightarrow{\begin{pmatrix}\delta\\ \lambda\end{pmatrix}} \begin{pmatrix}V\\ \mathbb R\end{pmatrix} \longrightarrow \begin{pmatrix}V''\\ \mathbb R\end{pmatrix}$$

Why is the cokernel of $\begin{pmatrix}\delta\\ \lambda\end{pmatrix}$ isom to $\begin{pmatrix}V''\\ \mathbb R\end{pmatrix}$

λ2 Given $V' \overset{\delta}{\rightarrowtail} V \twoheadrightarrow V''$ s. ex. seq.
and $\lambda : V' \to \mathbb{R}$, form the cokernel of $\begin{pmatrix}\delta\\ \lambda\end{pmatrix}$

$$\begin{tikzcd}
 & \mathbb{R} \arrow[r, equal] \arrow[d, "\begin{pmatrix}0\\1\end{pmatrix}"] & \mathbb{R} \arrow[d] \\
V' \arrow[r, tail, "\begin{pmatrix}\delta\\ \lambda\end{pmatrix}"] \arrow[d, equal] & \begin{pmatrix}V\\ \mathbb{R}\end{pmatrix} \arrow[r, two heads] \arrow[d, "(1\ 0)"'] & W \arrow[d] \\
V' \arrow[r, tail, "\delta"] & V \arrow[r, two heads] & V''
\end{tikzcd}$$

If $\lambda = 0$, then

[redacted] $W \simeq \begin{pmatrix}V\\ \mathbb{R}\end{pmatrix} \Big/ \begin{pmatrix}V'\\ 0\end{pmatrix}$
$\cong \begin{pmatrix}V''\\ \mathbb{R}\end{pmatrix}$

To construct a canonical isom $W \simeq \begin{pmatrix}V''\\ \mathbb{R}\end{pmatrix}$

$$\begin{tikzcd}
V' \arrow[r, tail, "\delta"] \arrow[d, "\lambda"] & V \\
\mathbb{R} &
\end{tikzcd}$$

Note that we get a canonical extension

$$\mathbb{R} \rightarrowtail W \twoheadrightarrow V''$$

so it should be clear that you can make a choice to get a retraction of W onto $\mathbb{R}$.

You want a linear functional on W.

W is the ~~co~~fibred ~~co~~product, so you need to extend λ' to λ

$$\begin{tikzcd}
V' \arrow[r, tail, "\delta"] \arrow[d, "\lambda'"'] & V \arrow[d] \arrow[dl, "\lambda"'] \\
\mathbb{R} \arrow[r, tail] & W
\end{tikzcd}
\qquad
\begin{tikzcd}
V' \arrow[r, "\delta"] \arrow[d, "\lambda'"'] & V \arrow[d] \\
\mathbb{R} \arrow[r] & W
\end{tikzcd}$$

$$\begin{tikzcd}
V' \arrow[r, "\delta"] \arrow[d, "\lambda'"'] & V \arrow[d] \\
\mathbb{R} \arrow[r, equal] & \mathbb{R}
\end{tikzcd}$$

$\mu 2$ Review construction:

$$\begin{tikzcd} V' \arrow[r, tail] \arrow[d] & V \arrow[r, two heads] \arrow[d] & V'' \arrow[d, phantom, "?"] \\ X \arrow[r] & X \amalg_{V'} V & \end{tikzcd}$$

Start with

$$V' \rightarrowtail \begin{pmatrix} X \\ X \end{pmatrix} \twoheadrightarrow X \amalg_{V'} V$$

$$V' \rightarrowtail V \twoheadrightarrow V''$$

Given $V' \overset{\delta}{\rightarrowtail} V \twoheadrightarrow V''$ s.e.s.

~~and a map $f: V' \to X$, form~~ given $f: V' \twoheadrightarrow X$

form $W = \mathrm{Coker}\left\{V' \xrightarrow{(\delta,-f)} V \times X\right\} = X \amalg_{V'} V$

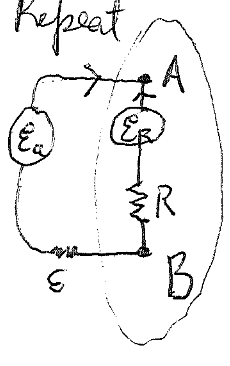
$$\begin{tikzcd} V' \arrow[r, "\delta"] \arrow[d, "f"'] & V \arrow[r, two heads] \arrow[d] & V'' \arrow[d, equal] \\ X \arrow[r, hook] & W \arrow[r, two heads] & V'' \end{tikzcd}$$

You want to compare the original to the aug graph. Since we do cochains, the aug cochains go onto the original cochains

$$\begin{tikzcd} & X \arrow[r, equal] \arrow[d, "\binom{1}{0}"'] & X \arrow[d] \\ V' \arrow[r, tail, "\binom{-f}{\delta}"] \arrow[d, equal] & \begin{pmatrix} X \\ V \end{pmatrix} \arrow[r, two heads] \arrow[d, "(0\ 1)"'] & W \arrow[d] \\ V' \arrow[r, tail] & V \arrow[r, two heads] & V'' \end{tikzcd}$$

$$\begin{tikzcd} & V \arrow[d] \\ X \arrow[r] & W \end{tikzcd}$$

Repeat



$V_a = V_R = V_B - V_A$

$I_a + I_R = 0$

$V_R = -RI_R + \mathcal{E}_R$

$V_a = -\varepsilon I_a + \mathcal{E}_a$

$V_R + RI_R = \mathcal{E}_R$

$$\boxed{\begin{aligned} V_a + \varepsilon I_a &= \mathcal{E}_a \\ V_a - RI_a &= \mathcal{E}_R \end{aligned}}$$

$$\boxed{\begin{aligned} V_a &= \frac{R}{R+\varepsilon}\mathcal{E}_a + \frac{\varepsilon}{R+\varepsilon}\mathcal{E}_R \\ I_a &= \frac{\mathcal{E}_a - \mathcal{E}_R}{R+\varepsilon} \end{aligned}}$$

$$\begin{pmatrix} V_a \\ I_a \end{pmatrix} = \frac{+1}{R+\varepsilon}\begin{pmatrix} +R & +\varepsilon \\ +1 & -1 \end{pmatrix}\begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_R \end{pmatrix}$$

Now you want to attempt:

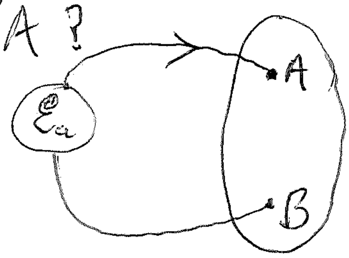
$$\tilde{C}^0(X) \xrightarrow{\binom{\delta}{[B]-[A]}} C^1(Y) = \begin{pmatrix} C^1(X) \\ \mathbb{R} \end{pmatrix} \longrightarrow \begin{pmatrix} H^1(X) \\ \mathbb{R} \end{pmatrix}$$

$[B]-[A] = \partial e_a$. Your variables are $V_X \in C^1(X)$, $\begin{pmatrix} V_X \\ V_a \end{pmatrix} \in C^1(Y)$, also

$I_X \in C_1(X)$ $\begin{pmatrix} I_X \\ I_a \end{pmatrix} \in C_1(Y)$

IDEA: You want the Green's function which is harmonic except for the node current $[B]-[A]$.

V2 Green's function idea: the harmonic potential with a singularity. Begin with a connected R-network X and 2 different nodes A, B. You want to attach an emf edge to force current into A and out from B. From the viewpoint of X one is relaxing the Kirchhoff node current condition and at the same time fixing the voltage drop V_a from B to A?



How do you propose to clarify this situation? A key point should be that the node potential φ_X is restricted by $\varphi(A)-\varphi(B) = \mathcal{E}_a$, but it should be harmonic outside these nodes. Harmonic should involve $\partial R_X^{-1}\delta\varphi_X = 0$.

$$\begin{tikzcd}
\mathbb{R} & \bar{C}^0(X) \arrow[l, "\langle A-B|"'] \arrow[r, leftrightarrow, "\delta"] \arrow[d, "\partial R^{-1}\delta"] & C^1(X) \arrow[d, "R^{-1}"] \\
\mathbb{R} \arrow[r, "{[A]-[B]}"'] & \bar{C}_0(X) & C_1(X) \arrow[l, "\partial"]
\end{tikzcd}$$

$\varphi(A)-\varphi(B) \leftarrow\!\shortmid\ \varphi$

Apparently the diagram on the left can be written

$$\bar{C}^0(X) \xrightarrow{\binom{\delta}{\langle A-B|}} \begin{pmatrix} C^1(X) \\ \mathbb{R}\end{pmatrix} \xrightarrow{\begin{pmatrix} R & 0 \\ 0 & \varepsilon\end{pmatrix}^{-1}} \begin{pmatrix} C_1(X) \\ \mathbb{R}\end{pmatrix} \xrightarrow{(\partial \ \ [A]-[B])} \bar{C}_0(X)$$

so the power form on $\bar{C}^0(X)$ seems to be:

$$\bar{C}^0(X) \xrightarrow{\begin{pmatrix} \partial R^{-1}\delta & 0 \\ 0 & |A-B\rangle \varepsilon^{-1}\langle A-B| \end{pmatrix}} \bar{C}_0(X)$$

Somehow you have to handle the problem that the external node resistance, i.e. the relation between V_a and I_a is given by the induced quadratic form on the subquotient space $\mathbb{R}$ of $\bar{C}^0(X)$.

Consider:

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, leftrightarrow, "\delta"] \arrow[d, "\gamma"] & C^1 \\
\mathbb{R}
\end{tikzcd} \quad \text{vs.} \quad \bar{C}^0 \xrightarrow{\binom{\delta}{\gamma}} \begin{pmatrix} C^1 \\ \mathbb{R}\end{pmatrix}$$

where C^1 is equipped with a pos quad form Q

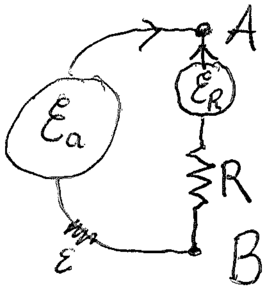
§2 In the first case you ~~restrict~~ Q to $\bar{C}^0$ then push forward via γ. What does this mean? It should mean that you [redacted] look for the stationary point on each $\gamma^{-1}(r)$, coset of [redacted] $\operatorname{Ker}\gamma$, i.e. you ~~restrict~~ the quadratic form to $(\operatorname{Ker}\gamma)^{\perp}$. What about the second?

So the basic situation seems to be that you have $\bar{C}^0 \xrightarrow{\binom{\delta}{\gamma}} \binom{C^1}{V}$, you want to scale the quadratic form ~~appropriately~~

~~Look at~~ Consider again

$$\begin{array}{ccc} \bar{C}^0 & \xrightarrow{\delta} & C^1 \\ \downarrow{\scriptstyle\gamma} & & \\ \mathbb{R} & & \end{array} \quad \text{vs.} \quad \bar{C}^0 \xrightarrow{\binom{\delta}{\gamma}} \binom{C^1}{\mathbb{R}}$$

Other example



V_R, V_a, I_R, I_a

$V_R = V_a, \quad I_R + I_a = 0$

$V_B - RI_R + \mathcal{E}_R = V_A$

$V_B - \varepsilon I_R + \mathcal{E}_a = V_A$

$V_R = V_B - V_A$

$-RI_R + \mathcal{E}_R = V_R$

$V_R + RI_R = \mathcal{E}_R$

$$\boxed{\begin{aligned} V_a + \varepsilon I_a &= \mathcal{E}_a \\ V_a - RI_a &= \mathcal{E}_R \end{aligned}}$$

$$\begin{pmatrix} V_a \\ I_a \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon \\ 1 & -R \end{pmatrix} \begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_R \end{pmatrix}$$

$$= \frac{1}{+R+\varepsilon} \begin{pmatrix} +R & +\varepsilon \\ +1 & -1 \end{pmatrix} \begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_R \end{pmatrix}$$

$$= \frac{1}{R} \begin{pmatrix} R & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_R \end{pmatrix} = \begin{pmatrix} \mathcal{E}_a \\ \frac{\mathcal{E}_a - \mathcal{E}_R}{R} \end{pmatrix}$$

Somehow you would like to get the induced quadratic form as the quotient as a suitable limit. You have a space ~~[illegible]~~ $\bar{C}^0$ with a Q, and a subspace $\operatorname{Ker}\gamma \subset \bar{C}^0$, so you can split orthogonally:

$$\bar{C}^0 = \operatorname{Ker}\gamma \oplus (\operatorname{Ker}\gamma)^{\perp}$$

~~[illegible]~~ You now restrict Q to $(\operatorname{Ker}\gamma)^{\perp}$, whence it descends to $\operatorname{Im}\gamma$

$$\varphi\, \partial R^{-1} \delta \varphi$$

$$\varphi\, \gamma^t t \gamma \varphi \qquad \text{if you let } t \nearrow \infty \text{, then}$$

~~[illegible]~~ you can adjust φ to approach a point in $\operatorname{Ker}\gamma$. ~~[illegible]~~

This may work.

Repeat:

$$\begin{array}{ccc} \bar{C}^0 & \xrightarrow{\delta} & C^1 \\ {\scriptstyle\gamma}\downarrow & & \\ \mathbb{R} & & \end{array} \qquad \text{vs.} \qquad \bar{C}^0 \xrightarrow{\binom{\delta}{\gamma}} \begin{pmatrix} C^1 \\ \mathbb{R} \end{pmatrix}$$

Focus upon the problem. You want to handle an applied voltage on the nodes, call this an external applied voltage, ~~via an the augmented circuit~~ by adjoining edges, then using internal applied voltages

π2

Go back to the problem about handling an "external" applied voltage as an "internal" applied voltage. External means the voltage source is connected between 2 nodes, internal means each edge of the graph contains a voltage source in series with a resistance.

How to proceed? You start with a network X, connected, with 2 nodes A, B specified. You attach an edge to these nodes and get an "augmented" network Y. The chains and cochain spaces for X and Y are related by short exact sequences

$$\begin{tikzcd}
\tilde{C}^0(X) \arrow[r, hook, "\delta_X"] & C^1(X) \arrow[r, two heads] & H^1(X) \\
\tilde{C}^0(Y) \arrow[u, "\wr"] \arrow[r, hook, "\delta_Y"] & C^1(Y) \arrow[u] \arrow[r, two heads] & H^1(Y) \arrow[u] \\
 & \mathbb{R} \arrow[u, hook] \arrow[r, "\sim"] & \mathbb{R} \arrow[u, hook]
\end{tikzcd}$$

$$\begin{tikzcd}
 & \mathbb{R} \arrow[d, hook] & \mathbb{R} \arrow[l, "\sim"'] \arrow[d, hook] \\
\tilde{C}_0(Y) \arrow[d, "\wr"] & C_1(Y) \arrow[l, two heads, "\partial_X"'] \arrow[d] & H_1(Y) \arrow[l, hook'] \arrow[d] \\
\tilde{C}_0(X) & C_1(X) \arrow[l, two heads, "\partial_Y"'] & H_1(X) \arrow[l, hook']
\end{tikzcd}$$

These diagrams don't seem to help much.

δ^2

The voltage (cochain) picture should simplify to a short exact sequence

$$\bar{C}^0(X) \xrightarrow{\binom{\partial_X}{f}} \begin{pmatrix} C^1(X) \\ \mathbb{R} \end{pmatrix} \longrightarrow \begin{pmatrix} H^1(X) \\ \mathbb{R} \end{pmatrix}$$

You are on the wrong track. What's important is how to relate how you handle ~~an~~ external voltage source, better: how you determine the node current response to the external voltage; this must be linked to the augmented graph.

Consider

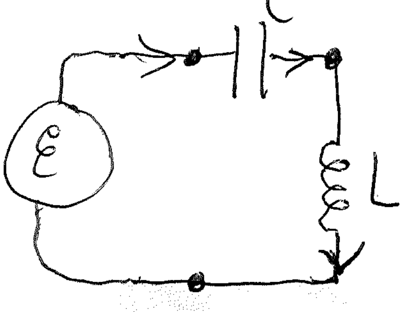
$$\mathbb{R} \overset{f}{\twoheadleftarrow} \bar{C}^0(X) \overset{\delta}{\hookrightarrow} C^1(X).$$

You want to take the power quadratic form on $C^1(X)$, ~~restrict~~ it to $\bar{C}^0(X)$, then push forward the form to $\mathbb{R}$. The push-forward form should be the power form on $\bar{C}^0(X)$ restricted to $(\mathrm{Ker} f)^{\perp}$ descended to $\mathbb{R}$ via the canon.

isom: $\mathrm{Ker}(f)^{\perp} \xrightarrow{f} \mathbb{R}$

$$\bar{C}^0(X) = \mathrm{Ker}(f) \oplus \mathrm{Ker}(f)^{\perp}$$

Go back, review examples.



$V_C, V_L,$

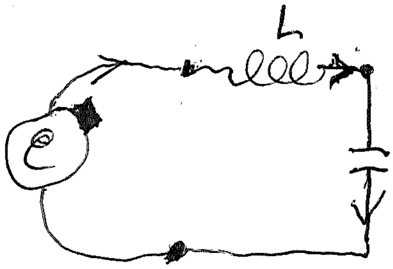
$CsV_C = I$

$LsI = V_L$

$\mathcal{E} + V_C + V_L =$

$\mathcal{E} + \left(\frac{1}{Cs} + Ls\right) I = 0$

σ2



$$-\mathcal{E} + \underbrace{LsI}_{V_L} + \underbrace{\frac{1}{Cs}I}_{V_C} = 0$$

Keep $\mathcal{E}$ fixed, look for stationary point for power $\frac{1}{2}\frac{1}{Ls}V_L^2 + \frac{1}{2}CsV_C^2$

$$\frac{1}{2}\frac{1}{Ls}V_L^2 + \frac{1}{2}Cs(-\mathcal{E}+V_L)^2$$

$$\frac{1}{Ls}V_L + Cs(-\mathcal{E}+V_L) = 0$$

$$\left(\frac{1}{Ls}+Cs\right)V_L - Cs\mathcal{E} = 0$$

$$V_L = +\frac{Cs}{\frac{1}{Ls}+Cs}\mathcal{E} = +\frac{LCs^2}{1+LCs^2}\mathcal{E}$$

$$V_C = +\mathcal{E} - V_L = +\mathcal{E} - \frac{LCs^2}{1+LCs^2}\mathcal{E}$$

$$= \mathcal{E}\left(+1 - \frac{LCs^2}{1+LCs^2}\right) = \frac{+1}{1+LCs^2}\mathcal{E}$$

~~Same except for the sign of $\mathcal{E}$.~~

Next take the augmented graph. What are you after? Variables $V_a, I_a, V_C, I_C, V_L, I_L$

Kirchhoff $I_a = I_L = I_C$ call this I

$$V_a + V_L + V_C = 0 \qquad V_L = LsI, \quad V_C = \frac{1}{Cs}I$$

$$V_a = \varepsilon I - \mathcal{E}$$

τ2 $\quad \varepsilon I + LsI + \frac{1}{Cs} I = \mathcal{E}$ $\qquad V_L = LsI$

$$I = \frac{\mathcal{E}}{\varepsilon + Ls + \frac{1}{Cs}} \quad \text{etc.}$$

You want next the variational method using voltage variables and power quadratic form

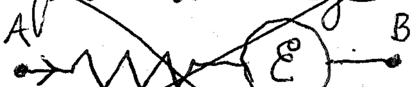
$$\frac{1}{2} \frac{V_a^2}{\varepsilon} + \frac{1}{2} \frac{1}{Ls} V_L^2 + \frac{1}{2} Cs V_C^2$$

$$V_a + V_L + V_C = 0$$

Something is wrong because $\mathcal{E}$ doesn't appear. Probably you have not gotten the right power in the a edge. $\quad V_a I_a = (\varepsilon I_a - \mathcal{E}) I_a$??

There should be a Lagrangian (?) yielding the inhomogeneous Ohm's Law for an edge:

$$V + RI = \mathcal{E}.$$



$V_A - RI + \mathcal{E} = V_B$

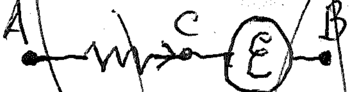
In other words, when you use Thevenin's idea of R + emf edge, there should be some underlying variational principle.

$V_A - V_B = RI - \mathcal{E}$

$V_A - V_B = -RI + \mathcal{E}$

MESS

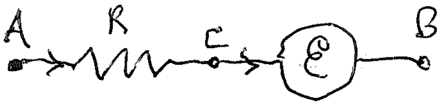
$$V_a = V_A - V_B = \underbrace{V_A - V_C}_{RI} + \underbrace{V_C - V_B}_{-\mathcal{E}}$$



$V_A + RI = V_C \qquad V_C - \mathcal{E} = V_B$

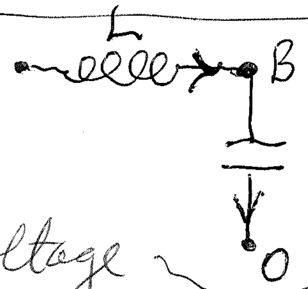
$V_A - RI = -\mathcal{E} + V_B \qquad V_a = V_A - V_B = +RI - \mathcal{E}$

$$\boxed{V + RI = \mathcal{E}}$$

u2 

$$V_A - RI = V_C\,,\quad V_C + \mathcal{E} = V_B$$

$$\left.\begin{aligned} V_A - V_C &= RI \\ V_C - V_B &= -\mathcal{E} \end{aligned}\right\} \Rightarrow \begin{aligned} V_A - V_B &= RI - \mathcal{E} \\ V_B - V_A &= -RI + \mathcal{E} \end{aligned}$$

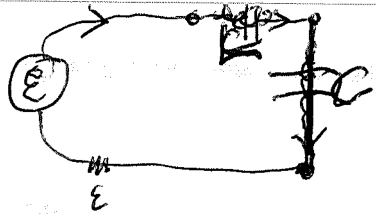
Study 

2 edges 3 nodes 0 loops. You want to attach an applied voltage between O and A. How to proceed? Look at

$$\begin{array}{ccc} \bar{C}^0(X) & \xhookrightarrow{\delta} & C^1(X) \\ \downarrow{\scriptstyle \gamma} & & \\ \mathbb{R} & & \end{array}$$

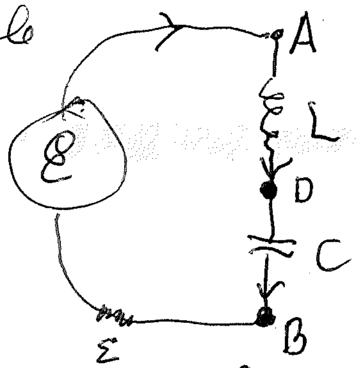
You want to take the power form on $C^1(X)$ restrict via δ and push forward via γ. Meaning of push forward: Split $\bar{C}^0(X) = \operatorname{Ker}\gamma \oplus (\operatorname{Ker}\gamma)^\perp$ for the quadratic form. $\gamma : (\operatorname{Ker}\gamma)^\perp \xrightarrow{\sim} \mathbb{R}$ and you descend the quadratic form by restriction to $(\operatorname{Ker}\gamma)^\perp$ followed by this isom. Alternative

$$\begin{array}{ccccc} \mathbb{R} & \overset{\gamma}{\twoheadleftarrow} & \bar{C}^0(X) & \xhookrightarrow{\delta} & C^1(X) \\ & & \Big\downarrow{\scriptstyle \delta^t R^{-1}\delta} & & \Big\downarrow{\scriptstyle R^{-1}} \\ \mathbb{R} & \xrightarrow{\gamma^t} & \bar{C}_0(X) & \overset{\partial = \delta^t}{\twoheadleftarrow} & C_1(X) \end{array}$$

Example: 

3 nodes
3 edges
1 loop

Example



The problem, ~~aim~~ is to eliminate the current variables, and to reach a situation involving voltage variables and a quadratic form.

Start with the full system $V_a, V_C, V_L, I_a, I_c, I_L$

Kirchhoff [redacted]. $I_a = I_L = I_C$

$$V_B - I_a\varepsilon + \mathcal{E} - LsI_L - \frac{1}{Cs}I_C = V_B$$

$$\mathcal{E} = \left(\varepsilon + Ls + \frac{1}{Cs}\right)I.$$

Maybe you should do Ohm.

$$V_B - \varepsilon I + \mathcal{E} = V_A \qquad \underline{V_a} = V_B - V_A = -\mathcal{E} + \varepsilon I$$

$$V_A - LsI - \frac{1}{Cs}I = V_B \qquad V_a = V_B - V_A = -\frac{1}{Cs}I - LsI$$

You now should know all the variables

$$\mathcal{E} = \left(\varepsilon + Ls + \frac{1}{Cs}\right)I.$$ [redacted]

$$V_L = LsI = \frac{Ls}{\varepsilon + Ls + \frac{1}{Cs}} I$$

$$V_C = \frac{1}{Cs} I = \frac{1/Cs}{\varepsilon + Ls + 1/Cs} I$$

$$V_a = -\mathcal{E} + \varepsilon I$$

as $\varepsilon \to 0$

$$V_L = \frac{CLs^2}{CLs^2+1} I$$

$$V_C = \frac{1}{CLs^2+1} I$$

But it's not very clear.

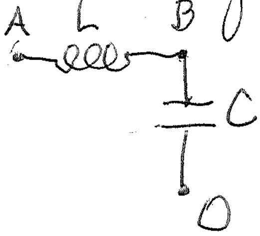
[redacted] X2

$$\begin{tikzcd} V \arrow[d] \\ \tilde{C}^0(X) \arrow[d, "\gamma"'] \\ \mathbb{R} \end{tikzcd}$$

[redacted]

$$\begin{tikzcd} \mathbb{R} & \tilde{C}^0(X) \arrow[l, two heads, "\gamma"'] \arrow[r, hook, "\delta"] \arrow[d, "\partial \tilde{R}^{-1}\delta"] & C^1(X) \arrow[d, "R^{-1}"] \\ \mathbb{R} \arrow[r, hook, "\gamma^t"] & \tilde{C}_0(X) & C_1(X) \arrow[l, two heads, "\partial"'] \end{tikzcd}$$

You should be [redacted] able to deal with this on the quadratic form level.



Because this is a tree one has

$$\begin{array}{ccc} \tilde{C}^0 & \xrightarrow[\sim]{\delta} & C^1 \\ \parallel & & \parallel \\ \left\{\begin{pmatrix} V_A \\ V_B \end{pmatrix}\right\} & & \left\{\begin{pmatrix} V_L \\ V_C \end{pmatrix}\right\} \end{array}$$

$$\begin{pmatrix} V_L \\ V_C \end{pmatrix} = \delta \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix}$$

Power

$$\frac{1}{2} \begin{pmatrix} V_L \\ V_C \end{pmatrix}^t \begin{pmatrix} (Ls)^{-1} & 0 \\ 0 & Cs \end{pmatrix} \begin{pmatrix} V_L \\ V_C \end{pmatrix} = \frac{1}{2} \frac{1}{Ls} V_L^2 + \frac{1}{2} Cs\, V_C^2$$

$$= \frac{1}{2} \frac{1}{Ls} (V_A - V_B)^2 + \frac{1}{2} Cs\, V_B^2$$

Let $\gamma \begin{pmatrix} V_A \\ V_B \end{pmatrix} = V_A$ $\qquad$ $\operatorname{Ker} \gamma = \left\{ \begin{pmatrix} 0 \\ V_B \end{pmatrix} \right\}$

Critical point $\quad \frac{1}{Ls}(V_A - V_B)(-1) + Cs\, V_B = 0$

$$-\frac{1}{Ls} V_A + \left(\frac{1}{Ls} + Cs\right) V_B = 0$$

$$V_A = (1 + LCs^2)\, V_B$$

[redacted]

$\psi 2 \quad V_A - V_B = LCs^2 V_B$

Critical Value $\quad \frac{1}{2}\frac{1}{Ls}\left(LCs^2 V_B\right)^2 + \frac{1}{2}Cs\, V_B^2$

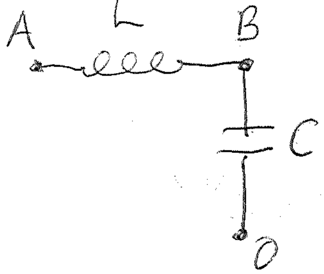
$$= \frac{1}{2}\left\{ \frac{L^2C^2s^4}{Ls} V_B^2 + \frac{LCs^2}{Ls} V_B^2 \right\}$$

$$= \frac{1}{2}\left\{ LC^2s^3 + Cs \right\} V_B^2$$

$$= \frac{1}{2}\left(1 + LCs^2\right) Cs\, V_B^2 \qquad \Big| \quad V_B = \frac{V_A}{1+LCs^2}$$

$$= \frac{1}{2}\frac{Cs}{1+LCs^2} V_A^2 = \frac{1}{2}\frac{1}{\frac{1}{Cs}+Ls} V_A^2 \qquad \text{so it works.}$$

Review this calculation



$$\bar{C}^0 \xrightarrow[\sim]{\delta} C^1$$

$$\begin{pmatrix} 1 & -1 \\ & 1 \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix} \qquad \begin{pmatrix} V_L \\ V_C \end{pmatrix}$$

$V_L = V_A - V_B$

$V_C = V_B$

power $\quad \frac{1}{2}\frac{1}{Ls}\left(V_A - V_B\right)^2 + \frac{1}{2}Cs\, V_B^2$

you want to push forward via $\quad \bar{C}^0 \longrightarrow \mathbb{R}$

$$\begin{pmatrix} V_A \\ V_B \end{pmatrix} \longmapsto V_A$$

stationary point

$$\frac{1}{Ls}\left(V_A - V_B\right)(-1) + Cs V_B = 0$$

$$-\frac{1}{Ls}V_A + \left(\frac{1}{Ls} + Cs\right)V_B = 0 \qquad \boxed{V_A = (1 + LCs^2)\, V_B}$$

$$V_A - V_B = LCs^2 V_B$$

Q2 Critical Value

$$\frac{1}{2}\left(\frac{1}{Ls}\right)\left(LCs^2V_B\right)^2 + \frac{1}{2}CsV_B^2$$

$$= \frac{1}{2}\left\{\frac{L^2C^2s^4}{Ls} + Cs\right\}V_B^2$$ ✓

$$= \frac{1}{2}\frac{1}{Ls}\left((LCs^2)^2 + (LCs)^2\right)V_B^2 = \frac{1}{2}Cs\left(1+LCs^2\right)V_B^2$$

$$= \frac{1}{2}Cs\cancel{\left(1+LCs^2\right)}\frac{1}{\left(1+LCs^2\right)}V_A^2$$

$$= \frac{1}{2}\frac{1}{\frac{1}{Cs}+Ls}V_A^2$$

too messy to be useful.

$$\bar{C}^0 = \left\{\begin{pmatrix}V_A\\V_B\end{pmatrix}\right\}$$

$$\frac{1}{2}\left\{\frac{1}{Ls}\left(V_A^2 - 2V_AV_B + V_B^2\right) + CsV_B^2\right\}$$

$$\begin{pmatrix}\frac{1}{Ls} & -\frac{1}{Ls}\\ -\frac{1}{Ls} & \frac{1}{Ls}+Cs\end{pmatrix}$$

Recall quadratic form on $X \oplus Y$

$$\begin{pmatrix}x\\y\end{pmatrix}^t\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \qquad X^\perp = \left\{\begin{pmatrix}x\\y\end{pmatrix} \,\middle|\, ax+by=0\right\} = \begin{pmatrix}-a^{-1}b\\1\end{pmatrix}Y$$

$$\begin{pmatrix}1 & 0\\ -b^ta^{-1} & 1\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1 & -a^{-1}b\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ -b^ta^{-1} & 1\end{pmatrix}\begin{pmatrix}a & 0\\ c & -ca^{-1}b+d\end{pmatrix}$$

$$= \begin{pmatrix}a & 0\\ 0 & d-ca^{-1}b\end{pmatrix}$$