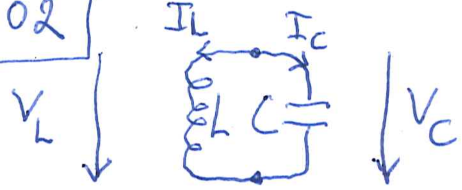
α $\boxed{\text{4 Sept 02}}$



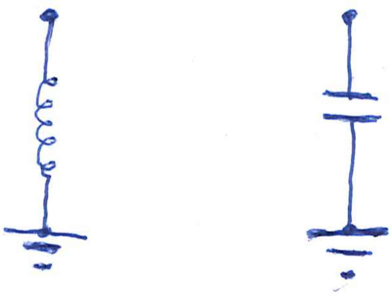
$I_L + I_C = 0$, $V_L = V_C$

$V_L = L\partial_t I_L$, $I_C = C\partial_t V_C$

You have 4 variables to describe a state of the two edges; same as ~~a chain~~ pair consisting of a 1-chain and a 1-cochain.

Idea: Is there a way to handle the constraints using Lagrange multipliers?

$LsI = V$ Each edge has 2 nodes, 2 edges.



Let's review what you have learned. Consider an LC network with e edges, more precisely $p = e_L$ inductance edges and $q = e_C$ capacitance edges. Each edge σ has two real variables associated (V_σ, I_σ). The product $V_\sigma I_\sigma$ is the power passing through the edge. Dynamics: L edge $L_j \partial_t I_j = V_j$ $1 \le j \le e_L$. C edge $C_j \partial_t V_j = I_j$ for $e_L < j \le e_L + e_C = e$. You should have said that V_σ, I_σ are real functions of t.

L edge: $1 \le j \le e_L$ $\quad \partial_t(V_j I_j) = (\partial_t V_j) I_j + V_j \partial_t I_j = \frac{1}{L_j} V_j^2$

C edge: $e_L < j \le e$ $\quad \partial_t(V_j I_j) = V_j(\partial_t I_j) + \frac{1}{C_j} I_j^2$

$(\partial_t V_j) I_j$

β

It seems there is something you overlooked involving power. Look at the simple situation

L edge $\quad L\partial_t I = V,$

$$L\partial_t I = V$$

$$L I \partial_t I = IV$$

$$\parallel$$

$$\partial_t\left(\tfrac{1}{2}LI^2\right)$$

So L edge $\quad IV = IL\partial_t I = \partial_t\left(\tfrac{1}{2}LI^2\right)$

C $\quad$ " $\quad IV =$

$$= C\dot{V}V = \partial_t\left(\tfrac{1}{2}CV^2\right)$$

$$\boxed{\sum_{\sigma\ L\text{-type}} I_\sigma V_\sigma = \partial_t\left\{\tfrac{1}{2}\sum_\sigma L_\sigma I_\sigma^2\right\}}$$

$$\boxed{\sum_{\tau\ C\text{-type}} I_\tau V_\tau = \partial_t\left\{\tfrac{1}{2}\sum_\tau C_\tau V_\tau^2\right\}}$$

s picture $\quad$ L type $\quad LsI = V \qquad LsI^2 = VI$

C type $\quad CsV = I \qquad CsV^2 = IV$

$\cancel{\partial_t(VI)}$

8 closed LC network

$$\begin{tikzcd} \bar C^0 \arrow[r, tail] & C^1 \arrow[r, two heads] & H^1 \\ \bar C_0 & C_1 \arrow[l, two heads] \arrow[u, "Z_s"] & H_1 \arrow[l, tail] \end{tikzcd}$$

states $(I, V) \in H_1 \times \bar C^0$

dim $l + v - 1 = e$

The problem is how to handle constrained motion.

$$Z_s = Ls \oplus (Cs)^{-1}$$

Look at an edge σ where the motion is

$$V_\sigma = L_\sigma \dot I_\sigma \qquad \text{L-type}$$

$$I_\tau = C_\tau \dot V_\tau \qquad \text{C-type.}$$

So the motion on an edge is the same as motion of a particle with constant velocity. ~~Can you think of~~

It looks like you have a particle moving in an e dimensional space with constant velocity $\ddot x = 0$.

Time to calculate the oscillator

$$C^1 = \{(V_C, V_L)\} \supset \bar C^0 = \{(V, V)\}$$

$$C_1 = \{(I_C, I_L)\} \supset H_1 = \{(I, -I)\}.$$

$$V_L = L \dot I_L \qquad V = L(-I)^{\cdot} = -L\dot I$$

$$I_C = C \dot V_C \qquad I = C \dot V$$

So now you see the idea which is to choose generalized independent coordinates to parametrize the states specified by the constraints.

Choose bases for $\bar C^0$ and H_1, together get bases for states of the network.

B closed LC network given:

$$\bar{C}^0 \rightarrowtail C^1 \twoheadrightarrow H^1$$

$$\bar{C}_0 \twoheadleftarrow C_1 \leftarrowtail H_1$$

space of states $\bar{C}^0 \times H_1$. You want to prove that there is a unique flow (1-parameter group e^{tX}) on the space of states ~~such~~ satisfying

$$\dot{I}_\sigma = L_\sigma^{-1} V_\sigma \qquad \text{if } \sigma \text{ L-type}$$

$$\dot{V}_\tau = C_\tau^{-1} I_\tau \qquad \text{if } \tau \text{ C-type}$$

The problem seems to be that you don't know

$$\ddot{I}_\sigma = L_\sigma^{-1} \dot{V}_\sigma$$

What is the situation? The problem is to establish a time evolution, flow e^{tX} on the space of states $\bar{C}^0 \times H_1$, which satisfies

$$\dot{I}_\sigma = L_\sigma^{-1} V_\sigma \qquad \sigma \text{ type L}$$

$$\dot{V}_\tau = C_\tau^{-1} I_\tau \qquad \tau \text{ type C}$$

$$V_\tau = \phi_{d_0\tau} - \phi_{d_1\tau}$$

So V_τ is easily expressed in terms of the natural coordinates on the space of states.

What about I_σ?

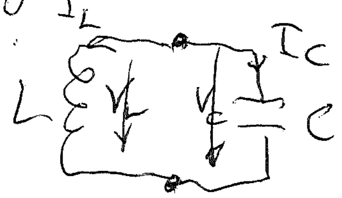
γ Question: Given a (closed) LC network, is there a flow e^{tX} on the state space?

State space = $\bar{C}^0 \oplus H_1$ is the subspace of $C^1 \oplus C_1$ defined by Kirchhoff's 2nd + 1st laws. State ~~space~~ S ~~has~~ dim e. You also have e derivative conditions.

$$L_\sigma \dot{I}_\sigma = V_\sigma \qquad \sigma \text{ L-type.}$$

$$C_\tau \dot{V}_\tau = I_\tau \qquad \tau \text{ C-type}$$

Go back to



$$L\dot{I}_L = V_L$$

$$C\dot{V}_C = I_C$$

You have here 4 unknowns V_L, I_L, V_C, I_C

2 DE's 2 Kirchhoff conditions $I_L + I_C = 0$

$V_L = V_C$

Ask about projection onto the state space

$$\bar{C}^0 \rightarrowtail C^1 \twoheadrightarrow H^1$$

$$\bar{C}_0 \twoheadleftarrow C_1 \leftarrowtail H_1$$

αβγδ εζηθ ικλμ νξοπ ρστυ φχψω

ε

$$V_L = Ls\, I_L \qquad I_C = Cs\, V_C$$

constraint $\quad V_L = V_C \;,\; I_L + I_C = 0.$

$$\begin{pmatrix} 1 & -Ls & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -Cs & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_L \\ I_L \\ V_C \\ I_c \end{pmatrix} = 0$$

$$\begin{vmatrix} 0 & -1 & 0 \\ 0 & -Cs & 1 \\ 1 & 0 & 1 \end{vmatrix} - \begin{vmatrix} -Ls & 0 & 0 \\ 0 & \cancel{-Cs} & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$(-1) - (-Ls)(-Cs) = -1 - LsCs$$

$$V_L = Ls\, I_L \quad \Bigg| \quad \begin{pmatrix} 1 & -Ls \\ & 1 \end{pmatrix} \begin{pmatrix} V_L \\ I_L \end{pmatrix}$$

$$s\, I_L = L^{-1} V_L \qquad \begin{pmatrix} s & -L^{-1} \\ & \end{pmatrix} \begin{pmatrix} I_L \\ V_L \end{pmatrix} = 0$$

$$s\, V_C = C^{-1} I_C \qquad \begin{pmatrix} s & -C^{-1} \\ & \end{pmatrix} \begin{pmatrix} V_C \\ I_C \end{pmatrix} = 0$$

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$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & -Ls & 0 & 0 \\ 0 & 0 & -Cs & 1 \end{pmatrix} \begin{pmatrix} \hat{V}_L \\ \hat{I}_L \\ \hat{V}_C \\ \hat{I}_C \end{pmatrix} = 0$$

$$\hat{V}_L = Ls\,\hat{I}_L$$
$$\hat{I}_C = Cs\,\hat{V}_C$$

$$\det \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & -Ls & 0 & 0 \\ 0 & 0 & -Cs & 1 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -Ls & 1 & 0 \\ 0 & 0 & -Cs & 1 \end{pmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ -Ls & 1 & 0 \\ 0 & -Cs & 1 \end{vmatrix} = \begin{vmatrix} 1 & Cs & 0 \\ -Ls & 1 & 0 \\ 0 & -Cs & 1 \end{vmatrix}$$

$$1 + LCs^2 = \begin{vmatrix} 1 & Cs & 0 \\ -Ls & 1 & 0 \\ 0 & -Cs & 1 \end{vmatrix}$$

What can you do in the general case?

Idea, philosophy. Given a (closed) LC network you get $2e$ variables: V_σ, I_σ $\forall$ edge σ

e constraints Kirchhoff 1 $\partial I = 0$, dim. $= v-1$

" 2 V conservative, dim $= l$

e DE's $\dot{I}_\sigma = L_\sigma^{-1} V_\sigma$ if σ type L

$\dot{V}_\sigma = C_\sigma^{-1} I_\sigma$ ——— C

The main problem is to explain how these e DE's yield a flow on the e dim'l space of states, i.e. the $\begin{pmatrix} V \\ I \end{pmatrix}$ satisfying the constraints

Question: Can you do this in the s picture?

θ Begin with $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ the space of $\begin{pmatrix} V \\ I \end{pmatrix}$ of edge potentials and edge currents. Pass to state space of $\begin{pmatrix} V \\ I \end{pmatrix} \in \begin{pmatrix} \vec{C}^0 \\ H_1 \end{pmatrix}$ space of conservative edge potentials and ~~tope~~ loop currents. You have some sort of flow (partial flow maybe) on $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$. Make precise.

Look at V_L, V_C, I_L, I_C and

$$V_L = L\partial_t I_L \qquad \boxed{\dot{I}_L = L^{-1}V_L}$$

$$I_C = C\partial_t V_C \qquad \boxed{\dot{V}_C = C^{-1}I_C}$$

Is $\dot{I}_L = L^{-1}V_L$? Take 1-edge, coords V_L, I_L. What is the meaning of the conditions $\dot{I}_L = L^{-1}V_L$?

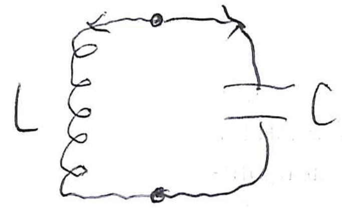
$$I_L(t) = \int_{-\infty}^{t} L^{-1} V_L(t')\,dt'$$

Constraint, connection

What you missed: $e^{t\begin{pmatrix}0&1\\0&0\end{pmatrix}} = \begin{pmatrix}1&t\\&1\end{pmatrix}$

So in $\begin{matrix}C^1\\ \oplus \\ C_1\end{matrix}$ you put this flow

then you compress to the [redacted] state space



coords $\begin{matrix}V_L & V_C\\ I_L & I_C\end{matrix}$

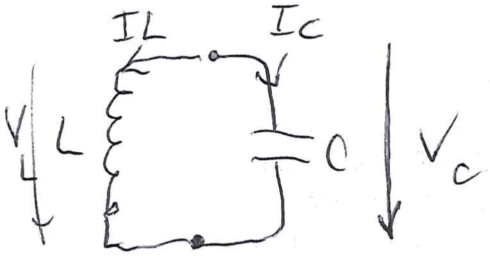
$\dot V_C = C^{-1} I_C$

$\dot I_L = L^{-1} V_L$

solution [redacted]

[redacted] Look at

$V(t) = L \frac{d}{dt} I(t)$



variables $\begin{matrix}V_L & V_C\\ I_L & I_C\end{matrix}$

$\partial_t I_L = L^{-1} V_L$

$\partial_t V_C = C^{-1} I_C$

constraints

$V_L = V_C$

$I_L + I_C = 0$

To understand the theoretical problem. You have a vector space S with an endomorphism X.

case of a tree?

$\tilde C^0 = C^1$

$\tilde C_0 = C_1$

$\boxed{\text{states } \tilde C^0 = C^1 \quad \text{no loop currents}}$

K

You looked at the case of a tree, ~~where~~ which turns out to be a degenerate case.

You want the flow on the state space which is $\bar{C}^0 \cong C^1$ since $H_1 = 0$ for a tree.

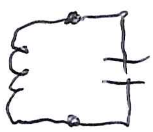
Note ~~that~~ there is only the 0 current in the ~~state~~ space. Look at what happens on an edge:

σ L type: $\dot{I}_\sigma = L^{-1} V_\sigma$, $I_\sigma = 0 \Longrightarrow V_\sigma = 0$

σ C type: $\dot{V}_\sigma = C^{-1} I_\sigma$, $I_\sigma = 0 \Longrightarrow V_\sigma$ constant.

Conclude that a state of the network is given by arbitrary voltage drops for the capacitance edges and 0 voltage drops for the inductance edge, the edge currents all being zero. Any state remains constant in time.

General case: To extend what you did for



First look at the number of variables and the number of equations. You begin with $C^1 \oplus C_1$ the space of edge potential drops and edge currents, get variables (V_σ, I_σ) for each edge, total dim $= 2e$. Next impose Kirchhoff 1 which is vanishing of ∂I in C_0, a total of $v-1$ conditions, and Kirchhoff 2 which says V is conservative, lies in $\bar{C}^0 = \mathrm{Ker}(C^1 \to H^1)$, giving l conditions. Total Kirchhoff conditions: $v-1+l = e$.

Next come dynamics equations

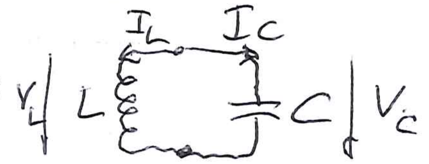
$$\dot{I}_\sigma = L_\sigma^{-1} V_\sigma \qquad \text{L-type edge}$$

$$\dot{V}_\sigma = C_\sigma^{-1} I_\sigma \qquad \text{C-type} \quad \text{"}$$

total e dynamic conditions. So have same number $2e$ of variables and conditions.

λ You can also start with the state space having dim $= v-1+l = e$ and the e dynamic equations

The case of a tree shows there can be problems when you impose the dynamical conditions. Recall $\dot{I}_\sigma = L_\sigma^{-1} V_\sigma$ forces $V_\sigma = 0$ on an inductive edge.

Example 

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -L^{-1} & s & 0 \\ -C^{-1} & 0 & 0 & s \end{bmatrix} \begin{bmatrix} I_C \\ V_L \\ I_L \\ V_C \end{bmatrix} = 0$$

$$sI_L = L^{-1}V_L \qquad sV_C = C^{-1}I_C$$

Idea: I_C, V_L are free variables, unlike I_L and V_C which are part of the symbol

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Consider an LC network, let $\mathcal{S}$ be the state space $\mathcal{S} = \begin{pmatrix} \tilde{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$. Important variables for time evolution on $\mathcal{S}$ are I_σ for σ of L-type and V_τ for τ of C-type. Let's assume these variables form a set of coordinates on $\mathcal{S}$. The differential conditions

$$\dot{I}_\sigma = L_\sigma^{-1} V_\sigma \quad , \quad \dot{V}_\tau = C_\tau^{-1} I_\tau$$

should specify a unique tangent vector field on $\mathcal{S}$. Thus you should have a well defined time evolution on $\mathcal{S}$ arising from the one on the edges and the constraints.

μ Consider a connected ~~closed~~ LC network

A state of ~~this~~ such a network is a pair $(I, V) \in C_1 \times C^1$ satisfying

K1: $\partial I = 0$ (I is a loop current) dim l

K2: $V \in \delta C^0$ (V ~~is~~ conservative) $v-1$

These states form a vector space $\mathcal{S}$ of dim $l + v - 1 = e$.

You want to construct a unique linear flow on $\mathcal{S}$ satisfying

$$V_\sigma = L_\sigma \partial_t I_\sigma \quad \text{for each } \sigma \text{ of } L \text{ type}$$

$$I_\sigma = C_\sigma \partial_t V_\sigma \quad \text{———— } C \text{ —.}$$

There are e equations here.

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point to be checked: coefficient s^e in the determinant is $\neq 0$.

There seems to be something you can say about I & V separately. Condition you want is for the important edge variables $\left[\begin{matrix} I & \text{for } L \text{ type} \\ V & C \end{matrix}\right.$ ~~should~~ to be independent on $\mathcal{S}$.

Suppose you have all L edges.

✓ to understand the flow, assuming the dominant ~~edge~~ variables $\left|\begin{array}{ll} I & L\text{ type} \\ V & C\text{ type}\end{array}\right.$ are independent on S. Now S has codim e in $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$, the independent conditions are given by K1 : $\underbrace{\partial I = 0}_{v-1 \text{ conditions}}$ and K2 : $\underbrace{V \text{ conserv.}}_{l \text{ conditions}}$ 0

$$\overset{e}{S} \subset \overset{2e}{T} \longrightarrow \overset{e}{U}$$

$$\begin{pmatrix} V \\ I \end{pmatrix} \longmapsto \begin{cases} I_\sigma & \text{if L-type} \\ V_\tau & \text{if C-type} \end{cases}$$

Let's try the following on the I-side.
You have e currents subject to $\partial I = 0$ ie $v-1$ conditions leaving l independent loop currents.

You want "symbol" currents $I_1, \dots, I_m$ means there are k-DE's

$$\dot{I}_j = L_j^{-1} V_j \qquad 1 \le j \le m$$

and e-m others:

$$\dot{V}_j = C_j^{-1} I_j \qquad m+1 \le j \le e$$

$$\begin{array}{cccc|ccc|ccc|ccc}
I_1 & & & I_m & V_{m+1} & \cdots & V_e & V_1 & & V_m & I_{m+1} & & I_e \\
\hline
s & & & & & & & -L_1^{-1} & & & & & \\
 & s & & & & 0 & & & \ddots & & & 0 & \\
 & & & s & & & & & & & & & \\
\hline
 & & & & s & & & & & & -C_{m+1}^{-1} & & \\
 & & & & & \ddots & & & 0 & & & \ddots & \\
 & & & & & & s & & & & & &
\end{array}$$

§

Given a connected LC network, assume that the symbol edge variables: $\begin{cases} I_\sigma & \sigma \text{ L-type} \\ V_\sigma & \sigma \text{ C-type} \end{cases}$

are linearly independent on the space $\mathcal{S} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ of states (whence the symbol edge variables) form a basis for $\mathcal{S}^*$. To see what this assumption implies

Let $I_1, \ldots, I_m$ be the edge currents for the L type edges, let $V_{m+1}, \ldots, V_e$ be the edge voltage drops for the C type edges. Then $I_1, \ldots, I_m, V_{m+1}, \ldots, V_e$ are the symbol (edge) variables and they form a complete independent system of coords on $\mathcal{S}$. Let $V_1, \ldots, V_m$ be the voltage drop corresp to $I_1, \ldots, I_m$ resp. Similarly let $I_{m+1}, \ldots, I_e$ be the edge currents corresp to $V_{m+1}, \ldots, V_e$. Recall these $2e$ variables are subject to the 2 Kirchhoff rules, total e equations, and also the dynamical eqns.

$$sI_j = L_j^{-1} V_j, \quad 1 \le j \le m \qquad sV_j = C_j^{-1} I_j, \quad m < j \le e$$

Picture

$$\begin{array}{c} \begin{array}{cccc} I_1 \cdots I_m & V_{m+1} \cdots V_e & V_1 \cdots V_m & I_{m+1} \cdots I_e \end{array} \\ \left[\begin{array}{cc|cc} \begin{matrix} s & & \\ & \ddots & \\ & & s \end{matrix} & 0 & \begin{matrix} -L_1^{-1} & & \\ & \ddots & \\ & & -L_m^{-1} \end{matrix} & 0 \\ \hline 0 & \begin{matrix} s & & \\ & \ddots & \\ & & s \end{matrix} & 0 & \begin{matrix} -C_{m+1}^{-1} & & \\ & \ddots & \\ & & -C_e^{-1} \end{matrix} \end{array}\right] \end{array}$$

You assume that the symbol variables $I_1, \ldots, I_m, V_{m+1}, \ldots, V_e$ are independent of the Kirchhoff relations.

o

[redacted] Consider a connected LC network. Find when time evolution is well defined, Need determinant$^{\text{size}}$ $e \times e$ $\neq 0$. This says dominant edge variables are independent on the space of states $S = \bar{C}^0 \oplus H_1$. This probably means that the

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Consider a connected LC network. Assume that the dynamical equations for the edges determine a flow on the space of states. More precisely the "dominant" edge variables $I_{L_1}, \dots, I_{L_q}$ for an L-edge

$V_{C_1}, \dots, V_{C_p}$ for a C-edge

form a complete independent set of cordinates on the state space $S = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \longrightarrow \begin{pmatrix} C^1_C \\ C_{1L} \end{pmatrix}$

$C^1_C = \{(V_{C_1}, \dots, V_{C_p})\}$ $\quad C_{1,L} = \{(I_{L_1}, \dots, I_{L_q})\}$

~~Whats happening? take any graph, conn~~ ~~You should be able to~~

Start again connected LC network. e edges yield $2e$ dim'l $\begin{matrix} C^1 \\ \oplus \\ C_1 \end{matrix}$

Introduce $C^1_L \oplus C^1_C = C^1$.

$\begin{matrix} C^1 \\ \oplus \\ C_1 \end{matrix} = \begin{pmatrix} C^1_L \oplus C^1_C \\ C_{1L} \oplus C_{1C} \end{pmatrix}$

dominant variables are ~~[illegible]~~

$V_C \in C^1_C$

$I_L \in C_{1L}$

$S = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 = C^1_L \oplus C^1_C \\ C_1 = C_{1L} \oplus C_{1C} \end{pmatrix} \to \begin{pmatrix} C^1_C \\ C_{1L} \end{pmatrix}$

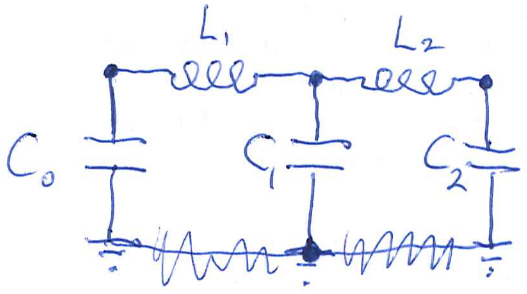
π

Assumption $\mathcal{S} \xrightarrow{\sim} \begin{pmatrix} C^1_C \\ C_{1L} \end{pmatrix}$ $\qquad \bar{C}^0 \xrightarrow{\sim} C^1_C$

$H_1 \xrightarrow{\sim} C_{1L}$

What does this isom mean?

[redacted] This should probably work well for ladder networks. Check this.



$v-1 = 3$
$l = 2$
$e = 5$

Repeat: Discuss the case you understand
• the generic case
- the most nondegen.

Assume that the dominant edge variables form a complete independent coordinate system.

$$I_{L_n} = I_n \qquad V_{C_n} = V_n.$$

$$-V_0 = C_0 s I_0$$
$$V_0 - V_1 = L_1 s I_1$$
$$I_1 - I_2 = C_1 s V_1$$
$$V_1 - V_2 = L_2 s I_2$$

$$\mathcal{S} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1_C \oplus C^1_L \\ C_{1L} \oplus C_{1C} \end{pmatrix}$$

Assumption $\bar{C}^0 \xrightarrow{\sim} C^1_C$
$H_1 \xrightarrow{\sim} C_{1L}$

Question: Geometric interpretation for $\underset{v-1}{\bar{C}^0} \xrightarrow{\sim} C^1_C$
or for $\underset{l}{H_1} \xrightarrow{\sim} C_{1L}$

number of

You are trying to understand ~~the~~ constrained motion, examples arising from LC ~~circuits~~ networks.
~~[illegible]~~ You start with a system of "particles" moving with constant velocity. Then the constraints arise somehow from forces perpendicular to the motion.
Example motion of a charged particle in a magnetic field

There are problems with certain graphs

Symplectic method for handling constraints?

Idea: Abstract situation: ~~finite dim'l $\mathbb{R}$ vector space~~

$$\bar{\mathcal{C}}^0 \longrightarrow \mathcal{C}^1_L \oplus \mathcal{C}^1_C \longrightarrow H^1$$

finite dim'l polarized Euclidean space.

Abstract version of an LC network (closed) is a finite dim'l Euclidean space split into orthogonal subspaces $\mathcal{E} = \mathcal{E}_L \oplus \mathcal{E}_C$ together with a subspace $\Gamma \subset \mathcal{E}$ of "constraints".

Basic L.T. picture

$$\Gamma \hookrightarrow \mathcal{E} \longrightarrow \mathcal{H}$$

$$\check{\Gamma} \twoheadleftarrow \check{\mathcal{E}} \longleftarrow \check{\mathcal{H}}$$

Introduce correspondence $Z_s \subset \mathcal{E} \times \check{\mathcal{E}}$

How do you expect to get a flow on $\begin{pmatrix} \Gamma \\ \check{\mathcal{H}} \end{pmatrix}$

Should the flow you want be the quadratic form?

$$\begin{array}{ccccc} \bar{\mathcal{C}}^0 & \hookrightarrow & \mathcal{C}^1 & \twoheadrightarrow & H^1 \\ & & \uparrow{\scriptstyle Z_s} & & \\ \bar{\mathcal{C}}_0 & \twoheadleftarrow & \mathcal{C}_1 & \hookleftarrow & H_1 \end{array}$$

$$\begin{pmatrix} \bar{\mathcal{C}}^0 \\ H_1 \end{pmatrix} \longrightarrow \mathcal{C}^1 = \mathcal{C}^1_L \oplus \mathcal{C}^1_C$$

σ

~~What really happens?~~

Look at $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \supset Z_s$?

You want to understand the theoretical problem of time evolution on the state space. Today's idea is that this "motion" ~~[illegible]~~ arises from a quadratic form depending on s.

Discuss some ideas! Dilations of the response (impedance) function on the nodes should reconstruct the impedance of the edges.

There are things you don't understand like power ~~energy~~

Let's start with a connected LC network

$$\bar{C}^0 \xrightarrow{\delta} C^1 \longrightarrow H^1$$

$$\bar{C}_0 \xleftarrow{\partial} C_1 \longleftrightarrow H_1$$

Another ingredient is the ~~[illegible]~~ "polarization"

$$C^1 = C^1_L \oplus C^1_C$$

$$C_1 = C_{1L} \oplus C_{1C}$$

~~[illegible]~~ $\text{Power} = \sum_\sigma V_\sigma I_\sigma$ σ runs over edges

~~Problem. Ideas~~ ~~FLOPPY DISKS~~

time to look at two Lagrangian subspace stuff. start again with LC network.

$$\bar{\mathcal{C}}^0 \hookrightarrow \mathcal{C}^1 \twoheadrightarrow H^1$$

$$\bar{\mathcal{C}}_0 \twoheadleftarrow \mathcal{C}_1 \hookleftarrow H_1$$

You are concerned with time flow on the state space $\begin{pmatrix} \bar{\mathcal{C}}^0 \\ H_1 \end{pmatrix}$, which is a Lagrangian subspace of $\begin{pmatrix} \mathcal{C}^1 \\ \mathcal{C}_1 \end{pmatrix}$. And you have ✓

So what do you want to do
Abstract LC network amounts to a retract of a ~~Euclidean space~~ a direct sum of two Euclidean space

$$\bar{\mathcal{C}}^0 \subset \mathcal{C}^1_L \oplus \mathcal{C}^1_C$$

You have a spectral decomposition for such a situation

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today I ought to review a lot of things especially the Hilb version of retract of super v.s.

$$W \xleftarrow{(\beta_+\ \beta_-)} \begin{pmatrix} V_+ \\ V_- \end{pmatrix} \xleftarrow{\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}} W$$

assume $(\beta_+\ \beta_-) = \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}^*$

$1 = h_+ + h_-$, $h_\pm = \alpha_\pm^* \alpha_\pm$

On W you have $1 = h_+ + h_-$ $V_\pm$ obtained by fact.

$$h_\pm = \alpha_\pm^* \alpha_\pm$$

Now you need the time evolution on the state space S. $S = W \oplus W^{\perp}$
How is this defined?

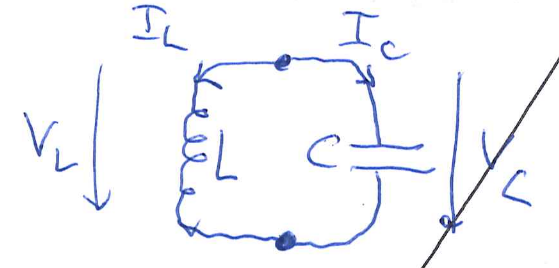
Time evolution involves functions of time equiv. by L.T. functions of s.

Apparently there is a classical mechanics, better statics based upon principle of virtual work – equilibrium is stationary point for energy. This becomes Hamilton's principles that motion is stationary for the action.

How does this translate to your LC situation?

It looks like you must bring in electrical energy

$$\int V(t)\, I(t)\, dt$$



$$L\partial_t I_L = V_L \qquad I_L + I_C = 0$$

$$C\partial_t V_C = I_C \qquad V_L = V_C$$

Let's begin again. Where? Consider an abstract LC network, by which you mean a subspace of a direct sum of two Euclidean spaces.

$$\mathcal{C} \subset \begin{pmatrix} \mathcal{E}_L \\ \mathcal{E}_C \end{pmatrix}$$

You want to construct time evolution on $\mathcal{C} \oplus \mathcal{C}^{\perp}$

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$$\bar{C}^0 \hookrightarrow C^1 \longrightarrow H^1$$

$$L\dot{I}_L = V_L$$
$$C\dot{V}_C = I_C$$

$$\bar{C}_0 \twoheadleftarrow C_1 \longleftrightarrow H_1$$

You have a flow on $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ given by

$$L_j s I_j = V_j \qquad C_k s V_k = I_k$$
$$j = 1, \dots, p \qquad k = 1, \dots, q$$

$$C^0 \hookrightarrow \begin{pmatrix} \mathcal{E}_L \\ \mathcal{E}_C \end{pmatrix} \longrightarrow C^+$$

constraint space

What is the flow on $\begin{pmatrix} \mathcal{E}_L \\ \mathcal{E}_C \end{pmatrix}$?

What does flow mean here?

You need to review symplectic stuff. There should be I think a standard flow on $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ in the case of an LC network. What you do have is a duality pairing between C^1 and C_1, and a symmetric map $Z_s : C_1 \xrightarrow{\sim} C^1$, maybe slightly more: a correspondence

$$\begin{array}{ccc} C^1 = & C^1_L & C^1_C \\ & L_s \big\uparrow s & C_s \big\downarrow s \\ C_1 = & C_{1,L} & C_{1,C} \end{array}$$

To understand flow. Each edge has a ~~partial kind~~ kind of partial flow. Each edge is like a particle moving on a line with 0 acceleration (constant velocity). Meaning of $L\partial_t I = V$. This is incomplete: Only one equation with 2 unknowns.

Question: When you said $\bar{C}^0 \oplus H_1$ is the

✗ Start again on understanding time evolution for an LC ~~network~~. You ~~have~~ have the state space S of dim $= e$, the number of edges; the problem is to find a canonical ~~automorphism~~ linear operator X on S. You want to use the basic "Grassmannian" picture, which yields Euclidean space structure. ~~There is a ...~~ You have two splittings of the same space.

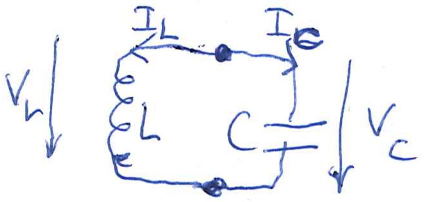
$$\mathcal{C}^1 = \mathcal{C}^1_L \oplus \mathcal{C}^1_C$$

~~dual~~ $$\mathcal{C}^1 = \bar{\mathcal{C}}^0 \oplus (\bar{\mathcal{C}}^0)^\perp$$

The edges of the network

Each edge ———— has certain structure

What is your aim? You want to start with



4 ~~unknowns~~ variables

4 relations. $I_L + I_C = 0$, $V_L = V_C$

$L\dot{I}_L = V_L$, $C\dot{V}_C = I_C$

$$\left\{\begin{pmatrix} V_L & V_C \\ I_L & I_C \end{pmatrix}\right\} \supset \left\{\begin{pmatrix} V_L & V_C \\ \frac{V_L}{Ls} & C s V_C \end{pmatrix}\right\}$$

The thing to do is to look for exponential solutions i.e. where time evolution is e^{st}, but do this in the abstract ~~data~~ setting where

Ψ How to proceed? Begin where?

$$\mathcal{C} \subset \begin{pmatrix} \mathcal{E}_L \\ \mathcal{E}_C \end{pmatrix} \qquad \text{voltage side of} \qquad \tilde{\mathcal{C}}^0 \subset \mathcal{C}^1$$

double. Need $\sqrt{L}, \sqrt{C}$. Do you have any feeling for motion on $\mathcal{C}^1 = \mathcal{C}^1_L \oplus \mathcal{C}^1_C$. Natural inner product

$$A(V_L) \overset{=}{\longmapsto} \frac{1}{Ls} V_L^2 = I_L V_L$$

$$A(V_C) \longmapsto I_C V_C = C s V_C^2$$

so this our quadratic form on $\mathcal{C}^1$. Next want to examine $\tilde{\mathcal{C}}^0 \subset \mathcal{C}^1$.

All this is very puzzling

$$\dot{\xi}_i = -\frac{\partial}{\partial x_i} \frac{k}{2}(x_1 - x_2)^2$$

Idea. Motion of two particles

$$T = \tfrac{1}{2} m_1 x_1^2 + \tfrac{1}{2} m_2 x_2^2 \qquad V = \tfrac{1}{2} k (x_1 - x_2)^2$$

$$H = \frac{\xi_1^2}{2m_1} + \frac{\xi_2^2}{2m_2} + \tfrac{1}{2} k (x_1 - x_2)^2$$

$$\dot{x}_i = \frac{\partial H}{\partial \xi_i} = \frac{\xi_i}{m_i} \qquad \begin{aligned} \dot{\xi}_1 &= -k(x_1 - x_2) \\ \dot{\xi}_2 &= +k(x_1 - x_2) \end{aligned}$$

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state space for free motion (no applied sources), you seemed to believe that the Kirchhoff Laws ~~[illegible]~~ plus the dynamical equations furnished enough conditions. This seems to be true when the dominant variables are linearly independent in $\mathcal{C} \oplus H_1$, which is not always true.

Change notion of dominant?

allow S S^{-1} same importance.

X

ω Can you do the LC oscillator motion in a symplectic framework? You want to do a symplectic reduction. The idea for the symplectic reduction is:

$$\bar{\mathcal{C}}^0 \hookrightarrow \mathcal{C}^1 \twoheadrightarrow H^1 \qquad ?$$

$$\bar{\mathcal{C}}_0 \twoheadleftarrow \mathcal{C}_1 \hookleftarrow H_1 \qquad ?$$

$\begin{pmatrix} \mathcal{C}^1 \\ \mathcal{C}_1 \end{pmatrix}$ is canonically symplectic, $\begin{pmatrix} \bar{\mathcal{C}}^0 \\ 0 \end{pmatrix}$ is isotropic

$\begin{pmatrix} \bar{\mathcal{C}}^0 \\ 0 \end{pmatrix}^{\perp} = \begin{pmatrix} \mathcal{C}^1 \\ H_1 \end{pmatrix}$, the symplectic quotient

is $\begin{pmatrix} \mathcal{C}^1 \\ H_1 \end{pmatrix} \Big/ \begin{pmatrix} \bar{\mathcal{C}}^0 \\ 0 \end{pmatrix} = \begin{pmatrix} H^1 \\ H_1 \end{pmatrix}$

The other ingredient in this game is the impedance which is a correspondence $\Gamma_s \subset \begin{pmatrix} \mathcal{C}^1 \\ \mathcal{C}_1 \end{pmatrix}$, a Lagrangian subspace

L-type edge $\qquad \begin{pmatrix} V_\lambda \\ I_\lambda \end{pmatrix} = \begin{pmatrix} L_\lambda s \\ 1 \end{pmatrix} I_\lambda$

C-type edge $\qquad \begin{pmatrix} V_\gamma \\ I_\gamma \end{pmatrix} = \begin{pmatrix} 1 \\ C_\gamma s \end{pmatrix} V_\gamma$

Γ_s isomorphism $s \neq 0, \infty$.