a'

Symplectic picture. Try to handle the 'good' case. Here's the problem: You want a flow on $\mathcal{K} = \bar{C}^0 \oplus H_1$. Check $\mathcal{K}$ is a Lagrangian subspace of $C^1 \oplus C_1$. Clear.

$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

$$H_1 \hookrightarrow C_1^{*} \twoheadrightarrow \bar{C}_0$$

Next comes the dynamical part. In terms of edges you have

$L_\sigma s I_\sigma = V_\sigma$ σ type L

$C_\tau s V_\tau = I_\tau$ τ type C

$\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ splits into real 2 planes.

$$\begin{pmatrix} C^1_\sigma \\ C_{1,\sigma} \end{pmatrix} = \left\{ \begin{pmatrix} V_\sigma \\ I_\sigma \end{pmatrix} \right\}$$ for each edge σ.

$\Gamma_s \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ splits into $\Gamma_{s,\sigma} \subset \begin{pmatrix} C^1_\sigma \\ C_{1,\sigma} \end{pmatrix}$

where $$\Gamma_{s,\sigma} = \left\{ \begin{pmatrix} L_\sigma s I_\sigma \\ I_\sigma \end{pmatrix} \right\}$$ σ type L

$$= \left\{ \begin{pmatrix} V_\sigma \\ C_\sigma s V_\sigma \end{pmatrix} \right\}$$ σ type C

b'

You want to see Γ_s as a Lagrangian subspace of $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$. It should be true that $\Gamma_s \cap \mathcal{K}$ is the space of solutions with exponential behavior e^{st}.

$$\Gamma_s \cap \mathcal{K} = \{ X \in \mathcal{K} \mid e^{st} X \in \mathcal{K} \ \forall t \} \ ??$$

$$\mathcal{K} = \left\{ \begin{pmatrix} U \\ J \end{pmatrix} \in \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \ \middle| \ \right. ?$$

$$\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \supset \Gamma_s$$

$$\mathcal{K}_s = \left\{ \begin{pmatrix} V \\ I \end{pmatrix} \in \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \ \middle| \ \begin{array}{l} K_1, K_2 \text{ hold} \\ \sigma \text{ type } L \Rightarrow L_\sigma s I_\sigma = V_\sigma \\ \sigma \quad\ \ C \Rightarrow C_\sigma s V_\sigma = I_\sigma \end{array} \right.$$

$\begin{pmatrix} V \\ I \end{pmatrix}$ satisfy $K1, K2$ $\Rightarrow$ $e^{st}\begin{pmatrix} V \\ I \end{pmatrix}$ satisfy $K1, K2$ $\forall t$

$\begin{pmatrix} V \\ I \end{pmatrix}$ sat. σ type L $\Longleftrightarrow$ $V_\sigma(t) = e^{st} V_\sigma$, $I_\sigma(t) = e^{st} I_\sigma$

sat $L_\sigma \partial_t I_\sigma(t) = V_\sigma(t)$

Good viewpoint should be intersection of Lagrangian subspaces. Generic case should have intersection 0. Degree characteristic poly, should occur in the good case. Question: Lagrangian complement for $\mathcal{K}$

C' Line up the Lag subspaces

$$\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \hookrightarrow \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \twoheadrightarrow \begin{pmatrix} H^1 \\ \bar{C}_0 \end{pmatrix}$$

$$\begin{tikzcd} \begin{pmatrix} C^1_L \\ C_{1,C} \end{pmatrix} \arrow[r, hook] & \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \arrow[r, two heads] & \begin{pmatrix} C^1_C \\ C_{1,L} \end{pmatrix} \\ & \mathcal{K} \arrow[u, phantom, "\cup"] \arrow[ur, "\sim"] & \end{tikzcd}$$

so you see that $\mathcal{K}$ and $\begin{pmatrix} C^1_L \\ C_{1,C} \end{pmatrix}$ are complementary

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Review. symplectic algebra

A anti symm non deg $\qquad A : V \xrightarrow{\sim} V^*$

H symm. $\qquad H : V \longrightarrow V^*$

$$d\phi = \iota_X \omega$$

Given $A : V \longrightarrow V^*$ $\qquad {}^tA : \underbrace{V^{**}}_{V} \longrightarrow V^*$ $\qquad$ Assume ${}^tA = -A$

$H : V \longrightarrow V^*$ $\qquad {}^tH = H.$

$X = A^{-1}H \qquad {}^tX = {}^tH\,{}^t(A^{-1}) = H(-A^{-1})$

What's missing? You want to start with a vector space V equipped with $A : V \xrightarrow{\sim} V^*$ such that ${}^tA = -A$

${}^tA : V^* \longleftarrow V$

d where are you headed? harmonic oscillator structure?

$$W \xrightarrow{A} W^* \qquad W^* \xleftarrow{{}^tA} W$$

Assume $A^{-1}\ \exists$ and ${}^tA = -A$. Given $S: W \to W^*$ ${}^tS = S$. Put $X = A^{-1}S$: $W \xrightarrow{S} W^* \xrightarrow{A^{-1}} W$.

~~Then ${}^tX = {}^tS\,{}^t(A^{-1}) = S(-A^{-1})$~~ Claim X preserves A, S i.e.

$${}^tXA + AX = S(-A)^{-1}A + AA^{-1}S = 0$$

$${}^tXS + SX = -SA^{-1}S + SA^{-1}S = 0$$

Conversely if ${}^tXA + AX = 0$, put $S = AX$

$${}^tS = -{}^tXA = AX = S. \qquad {}^tS = {}^t(AX) = {}^tX({}^tA) = {}^tX(-A)$$

But you need to work with Lagrangian subspaces.

Descent idea? For any V, $\begin{pmatrix} V \\ V^* \end{pmatrix}$ is symplectic in a standard way. $V \mapsto \begin{pmatrix} V \\ V^* \end{pmatrix}$ should be analogous to $A \longmapsto (1+TA[[T]])^\times$ λ-rings

$$W \xrightarrow{\begin{pmatrix} 1 \\ A \end{pmatrix}} \begin{pmatrix} W \\ W^* \end{pmatrix}$$

$${}^t\begin{pmatrix} 1 \\ A \end{pmatrix}\begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ A \end{pmatrix} = \begin{pmatrix} 1 & {}^tA \end{pmatrix}\begin{pmatrix} +A \\ -1 \end{pmatrix} = +A - {}^tA$$

$${}^t\begin{pmatrix} X \\ A \end{pmatrix}\begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} X \\ A \end{pmatrix} = \begin{pmatrix} {}^tX & {}^tA \end{pmatrix}\begin{pmatrix} +A \\ -X \end{pmatrix} = +{}^tXA - {}^tAX = +({}^tXA + AX)$$

c

Now where do you start? Let's try the descent angles. Given a v.s. V you get a symplectic vector space $W = \begin{pmatrix} V \\ V^* \end{pmatrix}$ with the skew-form ${}^t\begin{pmatrix} v_1 \\ \lambda_1 \end{pmatrix}\begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_2 \\ \lambda_2 \end{pmatrix} = ?$

$$\mathbb{C} \xrightarrow{v} V \quad \text{yields} \quad V^* \xrightarrow{{}^t v} \mathbb{C}$$

$${}^t\begin{pmatrix} v_1 \\ {}^t v_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_1' \\ {}^t v_2' \end{pmatrix} = \begin{pmatrix} {}^t v_1 & v_2 \end{pmatrix}\begin{pmatrix} {}^t v_2' \\ -v_1' \end{pmatrix} \ ?$$

Go back to

$${}^t\begin{pmatrix} v_1 \\ \lambda_1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_2 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} {}^t v_1 & {}^t \lambda_1 \end{pmatrix}\begin{pmatrix} \lambda_2 \\ -v_2 \end{pmatrix} \ ?$$

$$W = \begin{pmatrix} V \\ V^* \end{pmatrix} \longrightarrow \begin{pmatrix} V \\ V^* \end{pmatrix}^* = \begin{pmatrix} V^* & V \end{pmatrix}.$$

$$\begin{pmatrix} v \\ \lambda \end{pmatrix} \longmapsto \left(\begin{pmatrix} \lambda' & v' \end{pmatrix} \longmapsto \quad ? \right.$$

$$\cancel{{}^t\begin{pmatrix} \lambda^* & v^* \end{pmatrix}} \begin{pmatrix} \lambda' \\ v \end{pmatrix}$$

There ~~is~~ seems to be something artificial about row vectors + column vectors, that causes problems.

Consider $W = \begin{pmatrix} V \\ V^* \end{pmatrix}$. Then $W^* \underset{\text{canon isom.}}{\simeq} \begin{pmatrix} V^* \\ V \end{pmatrix}$

f

So what next? Recall the puzzle:

You have motion on $\mathcal{K} = \bar{C}^0 \oplus H_1$

$$\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \hookrightarrow \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$$

assuming that $\bar{C}^0 \hookrightarrow C^1 \to C^1_C$, $H_1 \to C_1 \to C_{1,L}$ are isos.

$$\begin{aligned} V - 1 &= \# C\text{'s} \\ \ell &= \# L\text{'s} \end{aligned}$$

Now how can you get somewhere?

Suppose you go back to

$$\begin{tikzcd} \bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, two heads] & H^1 \\ & \Gamma_s \arrow[u] \arrow[d] & \\ H_1 \arrow[r, hook] & C_1 \arrow[r, two heads] & \bar{C}_0 \end{tikzcd}$$

You want to split Γ_s into the dominant + recessive parts. This should work for $s = 0$.

type C $\quad \Gamma_{s,\sigma} \xrightarrow{\sim} \left\{ \begin{pmatrix} V_\sigma \\ C_\sigma s V_\sigma \end{pmatrix} \middle| V_\sigma \in \mathbb{R} \right\}$

type L $\quad \Gamma_{s,\sigma} \xrightarrow{\sim} \left\{ \begin{pmatrix} L_\sigma s I_\sigma \\ I_\sigma \end{pmatrix} \middle| I_\sigma \in \mathbb{R} \right\}$

So now you should be able to imitate the simple harmonic oscillator.

What to do today? Consider

$$\begin{tikzcd} \bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, phantom, "="] & C^1_C \arrow[r, phantom, "\oplus"] & C^1_L \\ & \Gamma_s \arrow[u] \arrow[d] \arrow[r, phantom, "="] & \Gamma_{s,C} \arrow[u] \arrow[d] \arrow[r, phantom, "\oplus"] & \Gamma_{s,L} \arrow[u] \arrow[d] \\ & C_1 \arrow[r, phantom, "="] & C_{1,C} \arrow[r, phantom, "\oplus"] & C_{1,L} \end{tikzcd}$$

$\bar{C}^0$ graph of $T : C^1_C \to C^1_L$

g

Idea: You have the Lagrangian subspace

$$\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$$

$\uparrow$

phase space Ω

You want to produce a (endom) flow on $\mathcal{K}$. You have a partial flow on phase space [redacted] given by [redacted]

$$\begin{cases} L\dot{I} = V & L \text{ type} \\ C\dot{V} = I & C \text{ type} \end{cases}$$

~~Consider the point and the~~ Suppose given a pt $K \in \mathcal{K}$, say $K = \begin{pmatrix} V \\ I \end{pmatrix} \in \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ satisfying the constraints;

which means $\iota_K \omega = 0$, ω symplectic form.

means symplectic pairing of K with $\Omega/\mathcal{K}$ ($\cong \mathcal{K}^*$) is 0.

Assume dominant variables indep on $\mathcal{K}$: the comp map

$$\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \hookrightarrow \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1_C \oplus C^1_L \\ C_{1,C} \oplus C_{1,L} \end{pmatrix} \longrightarrow \begin{pmatrix} C^1_C \\ C_{1,L} \end{pmatrix}$$

is an isom.

Start with symplectic space $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ split into $\begin{pmatrix} C^1_C \\ C_{1,C} \end{pmatrix} \oplus \begin{pmatrix} C^1_L \\ C_{1,L} \end{pmatrix}$. Introduce $K \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$

better a subspace $\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \subset \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$

h

Begin with the phase space associated to the edges together with its splitting into L, C parts:

$$\begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1_C \\ C_{1,C} \end{pmatrix} \oplus \begin{pmatrix} C^1_L \\ C_{1,L} \end{pmatrix}$$

and the Lag subspace given the dynamics

$$\begin{array}{ccc} \begin{pmatrix} C^1_C \\ C_{1,C} \end{pmatrix} & & \begin{pmatrix} C^1_L \\ C_{1,L} \end{pmatrix} \\ \cup & & \cup \\ \Gamma_{C,s} & & \Gamma_{L,s} = \left\{ \begin{pmatrix} Ls I_L \\ I_L \end{pmatrix} \middle| \, I_L \in C_{1,L} \right\} \\ \| & & \\ \left\{ \begin{pmatrix} V_C \\ Cs V_C \end{pmatrix} \middle| \, V_C \in C^1_C \right\} & & \end{array}$$

a natural question is whether $\Gamma_s = \Gamma_{C,s} \oplus \Gamma_{L,s}$ is an orbit for an action of $s \in \mathbb{C}^\times$

Take an elt $\begin{pmatrix} V_C & V_L \\ I_C & I_L \end{pmatrix} \in \begin{pmatrix} C^1_C & C^1_L \\ C_{1,C} & C_{1,L} \end{pmatrix}$

Example

$$\Gamma_s = \left\{ \begin{pmatrix} V_C & Ls I_L \\ Cs V_C & I_L \end{pmatrix} \middle| \begin{array}{c} V_C \in C^1_C \\ I_L \in C_{1,L} \end{array} \right\}$$

This should be a Lagrangian subspace of $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$

probably the graph of a quadratic map $Z_s : C_1 \longrightarrow C^1$

$Z_s I_L = Ls I_L$, $Z_s I_C = \frac{1}{Cs} I_C$

i

$$\begin{pmatrix} V_C \\ V_L \end{pmatrix} = Z_s \begin{pmatrix} I_C \\ I_L \end{pmatrix} = \begin{pmatrix} \frac{1}{Cs} & 0 \\ 0 & Ls \end{pmatrix} \begin{pmatrix} I_C \\ I_L \end{pmatrix}.$$

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different approach!!! You have a Lagrangian subspace of $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ namely $\Gamma_s = \begin{pmatrix} 1 \\ Cs \end{pmatrix} C^1_C \oplus \begin{pmatrix} Ls \\ 1 \end{pmatrix} C_{1,L}$.

Why is it Lagrangian:

$$\Gamma_s = \begin{pmatrix} \frac{1}{Cs} \\ 1 \end{pmatrix} C^1_C \oplus \begin{pmatrix} Ls \\ 1 \end{pmatrix} C_{1,L}$$

is the graph of the map $\begin{pmatrix} I_C \\ I_L \end{pmatrix} \longmapsto \begin{pmatrix} \frac{1}{Cs} & 0 \\ 0 & Ls \end{pmatrix} \begin{pmatrix} I_C \\ I_L \end{pmatrix} = \begin{pmatrix} V_C \\ V_L \end{pmatrix}$ from C_1 to C^1, and $\frac{1}{Cs}$, Ls are symmetric form $C_{1,C} \longrightarrow C^1_C$ and $C_{1,L} \longrightarrow C^1_L$ resp.

~~Sum~~ Summary: Big phase space associated to the edges

$$\begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1_C \\ C_{1,C} \end{pmatrix} \oplus \begin{pmatrix} C^1_L \\ C_{1,L} \end{pmatrix} \ni \begin{pmatrix} V_C & V_L \\ I_C & I_L \end{pmatrix}$$

which is symplectic, together with a splitting into symplectic subspaces associated to L, C type.

Also have Lagrangian subspace depending on s

$$\Gamma_s = \Gamma_{C,s} \oplus \Gamma_{L,s} = \begin{pmatrix} 1 \\ Cs \end{pmatrix} C^1_C \oplus \begin{pmatrix} Ls \\ 1 \end{pmatrix} C_{1,L}$$

7 ~~[crossed out]~~ Next you want "Kirchhoff" constraints, better you want the Kirchhoff space $\mathcal{K}$

~~[crossed out]~~ IDEA: Find appropriate forcing terms, inhomogeneous equation of motion, define resolvent, then the residues of the resolvent should give the free motion. Example ~~[crossed out]~~ from Thevenin's thm. where each edge has an ~~[crossed out]~~ emf in series with L or C.

Discuss how to handle Kirchhoff constraints in nonhomogeneous case. Recall your old approach, ~~[crossed out]~~ idea of ~~[crossed out]~~ emf applied at 2 distinct nodes, get linear functional on $\tilde{C}^0$, you minimize the power over the ~~[crossed out]~~ corresponding ~~[crossed out]~~ affine hyperplane ~~[crossed out]~~ of $\tilde{C}^0$.

Puzzle persistant. Why phase space $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$? ~~[crossed out]~~ It seems that ~~[crossed out]~~ the essential data is a polarized Euclidean space $C^1 = C^1_C \oplus C^1_L$ and the subspace $\tilde{C}^0 \subset C^1$. This is the position picture, the time flow should normally follow a 2nd order DE on position space, i.e. follow a 1st order DE on position + momentum space. But the phase space for free motion is $\begin{pmatrix} \tilde{C}^0 \\ H_1 \end{pmatrix}$ and the two parts have different dims, namely $v-1$ and l.

k

central problem $\mathcal{K} = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix}$. Given $\begin{pmatrix} V \\ I \end{pmatrix} \in \mathcal{K}$

define $\begin{pmatrix} \dot{V} \\ \dot{I} \end{pmatrix} \in \mathcal{K}$. This should be straightforward

namely $\begin{pmatrix} V(t) \\ I(t) \end{pmatrix}$? Given $\begin{pmatrix} V_C \\ I_L \end{pmatrix} \in \mathcal{K}$

You want to identify: $\mathcal{K} = \begin{pmatrix} C^1_C \\ C_{1,L} \end{pmatrix} \subset \begin{pmatrix} C^1_C & C^1_L \\ C_{1,C} & C_{1,L} \end{pmatrix}$

What's important is the conditions $C\,\delta V_C = I_C\,\delta t$

$\dot{V}_C = C^{-1} I_C$ $\qquad$ $L\,\delta I_L = V_L\,\delta t$

$\dot{I}_L = L^{-1} V_L$

$$\begin{pmatrix} V_C \\ I_L \end{pmatrix} + \varepsilon \begin{pmatrix} \dot{V}_C \\ \dot{I}_L \end{pmatrix} \longleftrightarrow \begin{pmatrix} V_C & \varepsilon C^{-1} I_C \\ \varepsilon L^{-1} V_L & I_L \end{pmatrix} \in \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$$

You want $I_C \in C_{1,C}$ expressed in terms of I_L?

and $V_L \in C^1_L$ ———— V_C

$\bar{C}^0 \hookrightarrow \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ $\qquad$ You've forgotten the idea that $C^1 = C^1_C \oplus C^1_L$

and that $\bar{C}^0$ projects isomorphically onto C^1_C

l

$$\dot{x} = Ax + Bu \quad , \quad y = Cx + Du$$

$$H(s) = C \frac{1}{sI - A} B + D$$ transfer function

x set of internal state variables
u input, y output

Begin with polarized Euclidean space. Consider

$$C^1 = C^1_C \oplus C^1_L = \{(V_C, V_L)\}$$

C^1_C = 1-cochains based on the C edges.

$$C_1 = C_{1,C} \oplus C_{1,L} = \{(I_C, I_L)\}$$

$C_{1,C}$ = space of 1-chains based in the C edges

power pairing $V_C \cdot I_C$ on C^1_C

~~Impedances~~ Constraints in good case

Let $\bar{C}^0 \subset C^1 = \cancel{C^1_C \oplus C^1_L}$ be the graph

$$\begin{pmatrix} 1 \\ T \end{pmatrix} C^1_C \subset \begin{pmatrix} C^1_C \\ C^1_L \end{pmatrix}$$ of $T : C^1_C \to C^1_L$

Then should have

$$\begin{tikzcd}
 & C^1_L \arrow[d] \arrow[dr, "\sim"] & \\
\bar{C}^0 \arrow[r, hook] \arrow[dr, "\sim"'] & C^1 \arrow[r, two heads] \arrow[d] & H^1 \\
 & C^1_C &
\end{tikzcd}$$

$$\begin{pmatrix} -T & 1 \end{pmatrix} \begin{pmatrix} 1 \\ T \end{pmatrix} = \begin{matrix} -T \\ +T \\ = 0 \end{matrix}$$

Better

$$\boxed{C^1_C \xhookrightarrow{\begin{pmatrix} 1 \\ T \end{pmatrix}} \begin{pmatrix} C^1_C \\ C^1_L \end{pmatrix} \xrightarrow{\begin{pmatrix} -T & 1 \end{pmatrix}} C^1_L}$$

Concrete version of $\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$

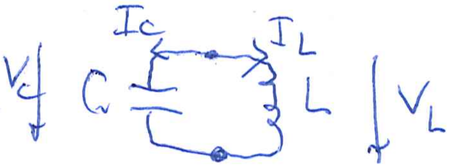
You still need the dynamics.

$$\bar{C}^0 \longrightarrow \begin{pmatrix} C^1_C \\ C^1_L \end{pmatrix} \longrightarrow H^1$$

$$H_1 \longrightarrow \begin{pmatrix} C_{1,C} \\ C_{1,L} \end{pmatrix} \longrightarrow \bar{C}_0$$

You expect to have an endo of $\begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix}$

Simple case



$$\{(V_C = V_L)\} \hookrightarrow \left\{\begin{pmatrix} V_C \\ V_L \end{pmatrix}\right\} \to \left\{\begin{pmatrix} I_C \\ I_L \end{pmatrix}\right\}$$

$CsV_C = I_C$

$LsI_L = V_L$

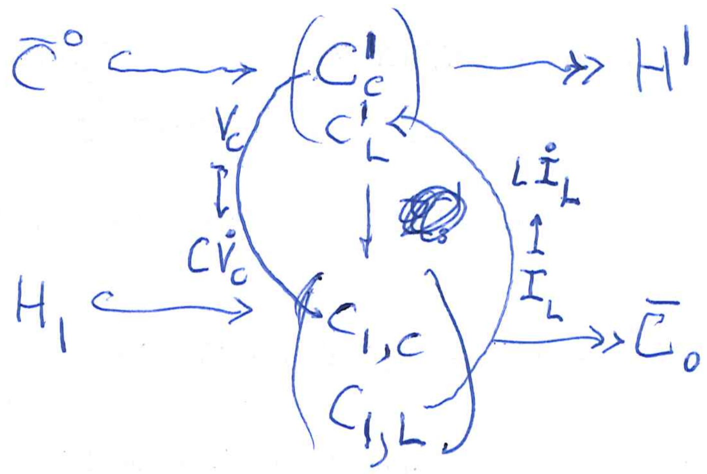
$$\{I_C + I_L = 0\} \qquad \left\{\begin{pmatrix} I_C \\ I_L \end{pmatrix}\right\} = \left\{\begin{pmatrix} CsV_C \\ \frac{1}{Ls}V_L \end{pmatrix}\right\} \longmapsto 0$$

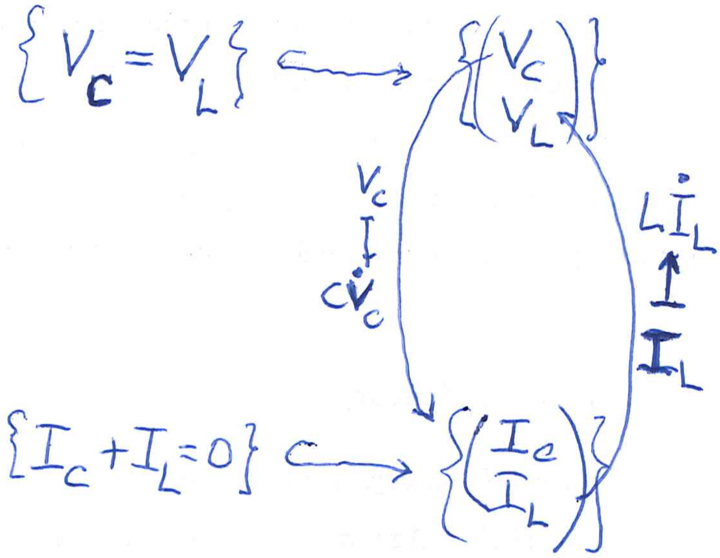
forces $CsV_C + \frac{1}{Ls}V_L = 0$

or $\left(Cs + \frac{1}{Ls}\right)V_C = 0$

n To get an operator

on $K = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix}$ for the simple LC oscillator





You want to find a natural flow on $K = \begin{pmatrix} \{V_C = V_L\} \\ \{I_C + I_L = 0\} \end{pmatrix}$

The idea is to use the derivatives of the dominant variables which are given: $\{V_C = V_L\}^{\cdot} = \frac{1}{C} I_C$

$\{-I_C = I_L\}^{\cdot} = \frac{1}{L} V_L$

$$\dot{V}_C = \frac{1}{C} I_C = -\frac{1}{C} I_L$$

$$\dot{I}_L = \frac{1}{L} V_L = \frac{1}{L} V_C$$

0

Review

$\tilde{C}^0 \hookrightarrow C^1$

$H_1 \hookrightarrow C_1$

The idea to pursue, intersecting 2 Lagrangian subspaces, normally the intersection is zero, the 2 subspaces are transversal, but when the dynamics is given by Γ_s the splitting should have singularities, residues?

$$C^1_C \xrightarrow{\begin{pmatrix}1\\T\end{pmatrix}} \begin{pmatrix} C^1_C \\ C^1_L\end{pmatrix} \xrightarrow{(-T\ \ 1)} C^1_L$$

$$C^1_L \xrightarrow{\begin{pmatrix}-T^*\\1\end{pmatrix}} \begin{pmatrix} C_{1,C} \\ C_{1,L}\end{pmatrix} \xrightarrow{(1\ \ T^*)} C^1_C$$

$$\{V_C\} \xrightarrow{\begin{pmatrix}1\\T\end{pmatrix}} \left\{\begin{pmatrix} V_C \\ V_L\end{pmatrix}\right\}$$

$V_C \downarrow C\dot{V}_C = I_C$ $\qquad$ $V_L = L\dot{I}_L \uparrow I_L$

$$\{I_L\} \xrightarrow{(-T^*\ \ 1)} \left\{\begin{pmatrix} I_C \\ I_L\end{pmatrix}\right\}$$

$$V_C \mapsto \begin{pmatrix} V_C \\ TV_C = V_L\end{pmatrix}$$

$C\dot{V}_C$ $\qquad$ $L\dot{I}_L$

$$\begin{pmatrix} I_C = -T^* I_L \\ I_L\end{pmatrix}$$

equations are V_C, I_L dominant $\qquad$ $C\dot{V}_C = I_C$

$$V_L = TV_C, \quad I_C = -T^* I_L, \quad L\dot{I}_L = V_L$$

$$\boxed{\begin{aligned} \dot{V}_C &= C^{-1} I_C = -C^{-1}T^* I_L \\ \dot{I}_L &= L^{-1} V_L = L^{-1} T V_C \end{aligned}}$$

equations of motion, lead to

$$\ddot{V}_C = -C^{-1}T^*L^{-1}TV_C$$

$$\ddot{V}_C + (C^{-1}T^*L^{-1}T)V_C = 0$$

P situation: You have symplectic space $\Omega = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ and two Lagrangian subspaces $K = \begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix}$ and Γ_s. Consider those s such that $K \cap \Gamma_s = 0$, in which case $\Omega = K \oplus \Gamma_s$. What is the meaning of such a splitting?

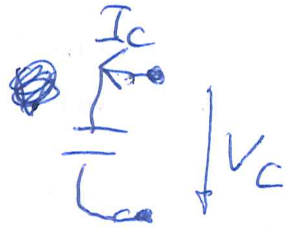
$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

$$H_1 \hookrightarrow C_1 \twoheadrightarrow \bar{C}_0$$

Constant velocity: $L = \frac{m}{2}\dot{x}^2$, $p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$0 = \frac{d}{dt}(p) - \frac{\partial L}{\partial x} = \dot{p}$ so p const and then $x = \dot{x}_0 t + x_0$

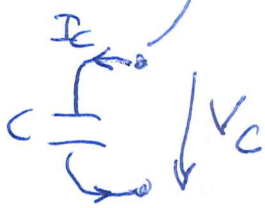
Now find the similar picture for



eqn of motion $C\dot{V}_C = I_C$

$$I_C V_C = C\dot{V}_C V_C = \frac{d}{dt}\left(\frac{1}{2} C V_C^2\right)$$

What's confusing is: state space for



is the set of pairs $\begin{pmatrix} V_C \\ I_C \end{pmatrix}$. Hamiltonian should be $V_C I_C$ which is a quadratic function on state space. Now you need a symplectic form, some multiple of $dV_C\, dI_C$

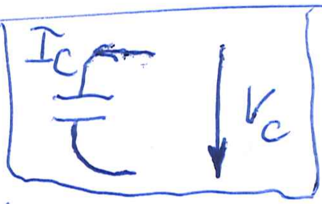
$$d(V_C I_C) = dV_C\, I_C + V_C\, dI_C \overset{?}{=} \iota_X(dV_C\, dI_C)$$

? $\quad L_X(dV_C\,dI_C) = XV_C\,dI_C - dV_C\,XI_C$

$$d(V_C I_C) = V_C\,dI_C + dV_C\,I_C$$

$$\therefore \quad XV_C = V_C \qquad XI_C = -I_C$$

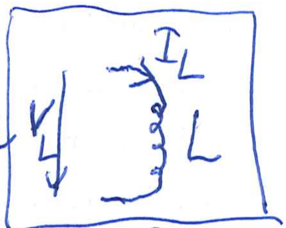
So it seems that $dV_C\,dI_C$ is the wrong symplectic form on the state space.

Try again state space for 

is the set of pairs $\begin{pmatrix} V_C \\ I_C \end{pmatrix}$. Hamiltonian = power $V_C I_C$ which is a quadratic fn. on state space.

partial dynamics $\quad C\dot{V}_C = I_C \implies V_C I_C = C V_C \dot{V}_C = \partial_t\left(\frac{1}{2}C V_C^2\right)$

$\frac{1}{2}CV_C^2$ should be energy

state space for 

is $\left\{\begin{pmatrix} V_L \\ I_L \end{pmatrix}\right\}$, power is $V_L I_L$,

partial dynamics $\quad L\dot{I}_L = V_L \implies V_L I_L = L\dot{I}_L I_L = \partial_t\left(\frac{1}{2}L I_L^2\right)$

$$C\dot{V}_C = I_C \qquad \omega = \frac{dV_C\,dI_C}{V_C I_C}$$

Problem: state space $\left\{\begin{pmatrix} V_C \\ I_C \end{pmatrix}\right\}$, energy $\frac{1}{2}CV_C^2$

time evolution $\quad \dot{V}_C = \frac{1}{C}I_C \quad$ Look for a 1-form on the state space η which would link the energy $\frac{1}{2}CV_C^2$, the partial dyn. $\dot{V}_C = \frac{1}{C}I_C$, $d\eta$

n

$$d\left(\tfrac{1}{2}CV_c^2\right) = CV_c\,dV_c \overset{?}{=} \iota_X \underbrace{d\eta}_{D\,dV_c\,dI_c}$$

where $$X = \underbrace{A}_{\frac{1}{C}I_c}\frac{\partial}{\partial V_c} + B\frac{\partial}{\partial I_c}$$

$$\iota_X(D\,dV_c\,dI_c) = D\tfrac{1}{C}I_c\,dI_c = D\,d\left(\frac{I_c^2}{C\cdot 2}\right)$$

go back to

$C\dot{V}_c = I_c$ $\qquad$ $m\dot{x} = p$

$I_cV_c = C\dot{V}_cV_c = \partial_t\left(\tfrac{1}{2}CV_c^2\right)$ $\qquad$ $px = \partial_t\left(\tfrac{1}{2}mx^2\right)$

$d\eta = D\,dV_c\,dI_c$ $\qquad$ $X = A\frac{\partial}{\partial V_c} + B\frac{\partial}{\partial I_c}$ $\qquad$ $\underset{\overset{\shortparallel}{A}}{XV_c} = \dot{V}_c = \frac{1}{C}I_c$

$$X = \frac{1}{C}I_c\frac{\partial}{\partial V_c} + B\frac{\partial}{\partial I_c}$$

$$\frac{1}{D}\iota_X d\eta = \frac{1}{C}I_c\,dI_c - B\,dV_c$$

$$\frac{1}{D}\iota_X d\eta = \frac{1}{C}I_c\,dI_c - B\,dV_c$$

$$d\left(\tfrac{1}{2}CV_c^2\right) = CV_c\,dV_c \qquad ?$$

Try again. Capacitor $C=1$, $\dot{V} = I$ partial flow

state space $\left\{\begin{pmatrix}V\\I\end{pmatrix}\right\}$ $\qquad$ symplectic form $\omega = D\,dV\,dI$

flow $X = A\partial_V + B\partial_I$

$\iota_X\omega = D(I\,dI - B\,dV)$ $\qquad$ $A = XV = \dot{V} = I$

energy $\tfrac{1}{2}V^2$ $\qquad$ power $\partial_t\left(\tfrac{1}{2}V^2\right) = V\dot{V} = VI$

$$d\left(\tfrac{1}{2}V^2\right) = V\,dV$$

$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

$$H_1 \hookrightarrow C_1 \twoheadrightarrow \bar{C}_0$$

Is there some way you can clarify the symplectic picture? $\mathbb{R}$

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capacitor $C = 1$, state $\begin{pmatrix} V \\ I \end{pmatrix}$, a state moving depending on t satisfies $C\dot{V} = I$.

particle mass m on line: state $\begin{pmatrix} x \\ \dot{x} \end{pmatrix}$

K.E. $\frac{1}{2}m\dot{x}^2$ $\quad L = \frac{1}{2}m\dot{x}^2 \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}(m\dot{x}) = m\ddot{x} = 0$

What's different about the C case? $\quad$ Energy $\frac{1}{2}V^2$?

$V = x$

$$C\dot{V} = I \qquad m\dot{x} = p$$

$$C\dot{V}V = IV \qquad m\dot{x}x$$

$$CV\dot{V}dt = IVdt$$

$$\left[\frac{1}{2}CV^2\right]_{t=a}^{t=b} = \int_a^b IV\,dt$$

Q charge in capacitor $\qquad CV = Q$

$\dot{Q} = I$ the current $\qquad C\dot{V} = I$

inf work done under $Q \mapsto Q + \delta Q$ should be

$V\delta Q$. $\quad$ energy

$\shortparallel$

$C V \delta V$

$$\int_0^V CV\,dV = \frac{1}{2}CV^2.$$

particle mass m no force applied,
configuration space $\left\{\begin{pmatrix} x \\ \dot{x} \end{pmatrix}\right\}$, kinetic energy $\frac{1}{2}m\dot{x}^2$

Lagrange DE $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}(m\dot{x}) =$ $m\ddot{x} = 0$

flow on config space $\begin{pmatrix} x \\ \dot{x} \end{pmatrix} \longmapsto \begin{pmatrix} \dot{x} \\ 0 \end{pmatrix}$?

Start again with capacitor capacitance C
state space $\left\{\begin{pmatrix} V \\ I \end{pmatrix}\right\}$ partial dynamics: $C\dot{V} = I$

power $VI = VC\dot{V} = \partial_t\big(\underbrace{\tfrac{1}{2}CV^2}_{}\big)$

energy of state $\begin{pmatrix} V \\ I \end{pmatrix}$

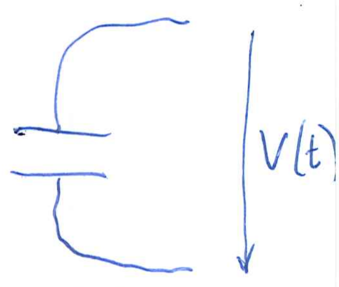
motivation: $CV = Q$ basic prop of capacitor
$C\dot{V} = I$

work done in moving charge δQ to higher V is $V\delta Q$

$$\int_a^b V\,dQ = \int_{x=a}^{x=b} V(x)\,\frac{dQ}{dx}\,dx$$

take $x =$ time t

$$\int_{t=a}^{t=b} V(t)\,\frac{dQ}{dt}\,dt = \int_{t=a}^{t=b} VI\,dt$$



IDEA. Recall the splitting of $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$
into $\mathcal{K} \oplus \Gamma_5$, two Lagrangian subspaces.
Is this related to the construction of a Green's function? Recall Feynmann's Green function. Titchmarsh

Review: Connected graph X, σ edge of X, $Y = X - \mathrm{Int}(\sigma)$ the graph with the same nodes and same edges except that σ has been removed. $Y \hookrightarrow X \twoheadrightarrow X/Y$ leads to $S^1 = [0,1]/\{0,1\}$

$$0 \to H_1 Y \longrightarrow H_1 X \longrightarrow \underset{\mathbb{R}\sigma}{\overset{}{H_1(X/Y)}} \longrightarrow H_0 Y \longrightarrow \underset{\mathbb{R}}{H_0 X} \to 0$$

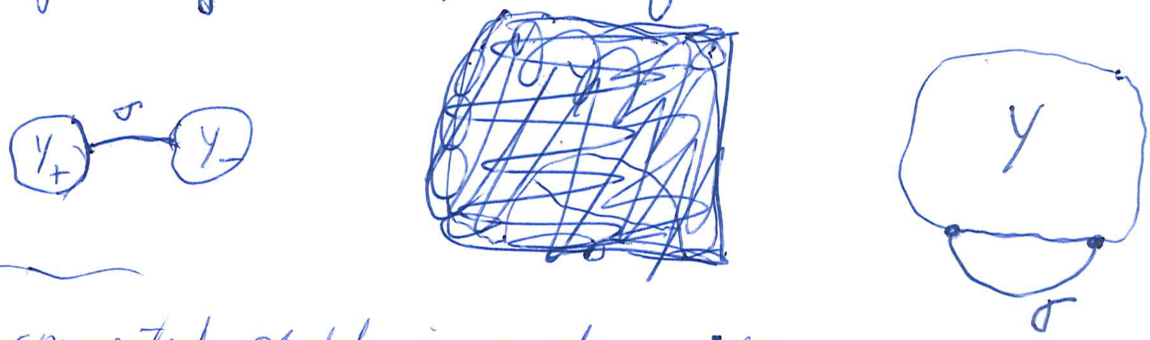
$H_1(X/Y) = \mathbb{R}\sigma$, $H_0 X = \mathbb{R}$ as X is conn.

Case 1: $H_1 Y \xrightarrow{\simeq} H_1 X$ ($\Leftrightarrow$ Y has 2 components joined by σ)

Case 2: $H_1 X / H_1 Y$ 1-diml ($\Leftrightarrow$ Y connected).

get new loop in X by adding a path in Y joining the endpoints of σ

Pictures of X



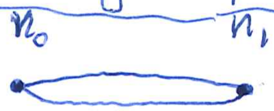
Facts: A connected graph is a tree iff removing any edge disconnects the graph.

~~If X a tree, σ an edge,~~

Define a tree as a graph without

seems to be hard to define tree. Instead consider a maximal set of edges which ~~does not~~ upon removal leaves the graph connected.

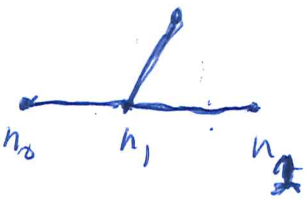
V (apparently no page W)



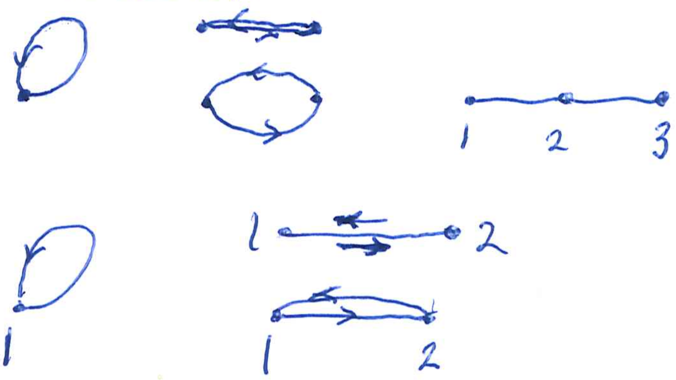
looks like you need the groupoid of loops.

$\gamma_1 \cdot \gamma_2 \cdot \gamma_3$

$n_0 \quad n_1 \quad n_2 \quad n_0$



list possibilities



IDEA: Fix a basepoint $*$ in a tree T, then ~~you have for each node $*$~~ distance from $*$ function, get contraction. Key point should be same sort of nilpotent, upper triangular matrices

Notion of a ~~loop~~ path involves cancellation of inverse edges.

Go back over the structure of $\begin{matrix} C^1 \\ \oplus \\ C_1 \end{matrix}$

Review C-edge, states $\begin{pmatrix} V \\ I \end{pmatrix}$, power $= VI$

time dep state sat $C\dot{V} = I$

freq dep state sats $CsV = I$

The first thing to do is to look at $s \notin i\mathbb{R}$ in general to see if you get the splitting

$$K \oplus \Gamma_s = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} \qquad \Gamma_s = \begin{pmatrix} 1 \\ Cs \end{pmatrix} C^1_C \oplus \begin{pmatrix} Ls \\ 1 \end{pmatrix} C_{1,L}$$

× ~~Example~~ Examine $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ together with the Lagrangian subspace Γ_5. You believe that the essential structure is simply a polarized Hilb space.

$$C^1 = C^1_C \oplus C^1_L$$

$C^1_C = \{V_C\}$ where V_C is a real 1-cochain on the capacitor edges.

$$C_1 = C_{1,C} \oplus C_{1,L}$$

$C_{1,C} = \{I_C\}$ where I_C is a real 1-chain on the capacitor ~~inductor~~ edges.

power pairing $\sum V_C I_C$ between 1-cochains + 1-chains.

Same for inductor edges.

So next look at the dynamics on capacitor edges. states $\begin{pmatrix} V_C \\ I_C \end{pmatrix} \in \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$. given time dep state must have $C\dot{V}_C = I_C$, where C is the diagonal matrix from C^1_C to $C_{1,C}$, whose entry for edge σ is the capacitance C_σ. ∴ it's a positive real ~~diagonal matrix~~ symmetric map $C^1_C \longrightarrow C_{1,C} = (C^1_C)^*$

$$V_C \longmapsto (CV_C = I_C) = (V_C' \longmapsto CV_C V_C')$$

$$V_C \longmapsto (I_C \longmapsto \sum V_C I_C)$$

y

So for a single capacitor
capacitance C

$$C^1 \oplus C_1 = \left\{ \begin{pmatrix} V \\ I \end{pmatrix} \right\} \quad \text{state space}$$

$$\cup \qquad \cup$$

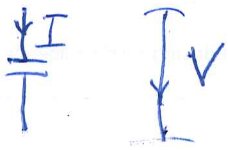
$$\Gamma_s = \left\{ \begin{pmatrix} V \\ CsV \end{pmatrix} \right\} = \begin{pmatrix} 1 \\ Cs \end{pmatrix} C^1_C$$

$x_0 + t x'$

~~Symplectic space of dim 2~~

Digress, examine symplectic stuff. You have a real symplectic plane. Maybe you also have a linear flow (same as a linear op on the plane?)

Review. capacitor / inductor C



state space

$$C^1 \oplus C_1 = \left\{ \begin{pmatrix} V \\ I \end{pmatrix} \right\} \simeq \mathbb{R}^2.$$ [redacted] It may be important to take account of the orientation of the edges. Changing the orientation sends $\begin{pmatrix} V \\ I \end{pmatrix}$ to $\begin{pmatrix} -V \\ -I \end{pmatrix}$. Associated to a state $\begin{pmatrix} V \\ I \end{pmatrix}$ is its power VI, which is indep of orientation. Power gives a duality between C^1 and C_1, whence a symplectic form

$$^t\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 & I_1 \end{pmatrix} \begin{pmatrix} I_2 \\ -V_2 \end{pmatrix} = V_1 I_2 - I_1 V_2$$

and a symmetric form

$$^t\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = V_1 I_2 + I_1 V_2 = \text{polarization of } \begin{pmatrix} V \\ I \end{pmatrix} \mapsto VI.$$

So the state space for a capacitor is real 2 dim'l symplectic. Possible time evolutions are given by elts of $sl_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, a+d=0 \right\}$. These should correspond to quadratic Hamiltonians. Obvious examples $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

What next? ~~States~~ Consider states $\begin{pmatrix} V(t) \\ I(t) \end{pmatrix}$ ~~def~~ depending on t. For the capacitor C you require $C\dot{V} = I$, For the inductor L you require $L\dot{I} = V$. ~~[illegible]~~

A symplectic form $A : V \longrightarrow V^*$ $\quad {}^tA = -A$

$H : V \longrightarrow V^*$ symm, $H = {}^tH$ $\quad$ A nondeg

~~[illegible]~~ better ~~A~~ $V^* \xrightarrow{A} V$

$$V \xrightarrow{H} V^* \xrightarrow{A^{-1}} V \qquad \boxed{X = A^{-1}H}$$

$${}^tXA + AX = {}^t(A^{-1}H)A + A(A^{-1}H) = 0$$

$${}^tH\underbrace{(A^{-1})^t A}_{-A^{-1}}$$

$$H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

$$X = A^{-1}H = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad X = A^{-1}H = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$$