a] goal: to prove that if $F^pX_{\geq k}$ is defined to corresp to $F^p\Omega_{\geq k}$ via the ~~canon~~ basic ident $X = \Omega$, then the basic heq $X \sim Q$ induces $F^pX_{\geq k} \sim F^p\Omega_{\geq k}$.

Consider
$$F^pX^t = \bigoplus t^k F^pX_{\geq k} \subset T' \otimes X$$
$$F^p\Omega^t = \bigoplus t^k F^p\Omega_{\geq k} \subset T' \otimes \Omega$$

Then under the basic ident $X = \Omega$ we have

$$\begin{array}{ccc} F^pX^t & \subset & T' \otimes X \\ \| & & \| \\ F^p\Omega^t & \subset & T' \otimes Q \end{array}$$

Next consider ~~Q~~ $Q^t \subset T' \otimes Q$. ~~This~~ This induces

basic ident. $\Rightarrow$

$$\begin{array}{ccccc} X_T(R_T(Q^t)) & \longrightarrow & X_{T'}(R_{T'}(T' \otimes Q)) & = & T' \otimes X \\ \| & & \| & & \| \\ \Omega_T(Q^t) & \longrightarrow & \Omega_{T'}(T' \otimes Q) & = & T' \otimes \Omega \end{array}$$

horizontal arrows injective.
image of bottom is Ω^t
$\therefore$ image of top is X^t because $X^t = \Omega^t$ under the basic ident.

More generally

$$\begin{array}{ccc} F^p_{I_T(Q^t)} & \longrightarrow & T' \otimes X \\ \| & & \| \\ F^p\Omega_T(Q^t) & \longrightarrow & T' \otimes \Omega \end{array}$$

image of bottom is $F^p\Omega^t$
$\therefore$ image of top is F^pX^t.

b) So we learn that $Q^t \subset T' \otimes Q$ induces

$$\begin{array}{ccccc} X_T(R_T(Q^t)) & \xrightarrow{\sim} & X^t & \subset & T' \otimes X \\ \| & & & & \| \\ \Omega_T(Q^t) & \xrightarrow{\sim} & \Omega^t & \subset & T' \otimes \Omega \end{array}$$

Now consider the basic heq

$$\begin{array}{ccccc} X_T(R_T(Q^t)) & \xrightarrow{\sim} & X^t & \subset & T' \otimes X \\ \wr & & & & \wr \\ \Omega_T(Q^t) & \xrightarrow{\sim} & \Omega^t & \subset & T' \otimes \Omega \end{array}$$

square commutes. Thus tensor basic heq $X \sim \Omega$ induces $X^t \sim \Omega^t$ i.e. basic heq $X \sim Q$ induces $X_{\geq k} \sim \Omega_{\geq k}$. Same for

$$\begin{array}{ccccc} F^P_{I_T}(Q^t) & \xrightarrow{\sim} & F^P X^t & \subset & T' \otimes X \\ \wr & & & & \wr \\ F^P \Omega_T(Q^t) & \xrightarrow{\sim} & F^P \Omega^t & \subset & T' \otimes X \end{array}$$

appears that injectivity is not needed.

9/2 - 0407 Repeat what I worked on yesterday

Claim: If we define $F^P X_{\geq k}$ to corresp to $F^P \Omega_{\geq k}$ under the basic ident. $X \rightleftarrows \Omega$, then the basic heq $X \sim Q$ induces $F^P X_{\geq k} \sim F^P \Omega_{\geq k}$

To prove this via T-method. One has

$$\begin{array}{ccccc} F^P X^t & = & \bigoplus t^k F^P X_{\geq k} & \subset & T' \otimes X \\ \wr & & & & \| \\ F^P \Omega^t & = & \bigoplus t^k F^P \Omega_{\geq k} & \subset & T' \otimes Q \end{array}$$

c| Thus via $1\otimes$ basic ident $X \sim \Omega$ we have $F^P X^t = F^P \Omega^t$.

Next we have incl. homom. $Q^t \subset T\otimes Q$ which induces horizontal arrows

$$
* \left(\begin{array}{ccccccc}
F^P_{I_T(Q^t)} & \subset & X_T(R_T(Q^t)) & \longrightarrow & X_{T'}(R_{T'}(T'\otimes Q)) & = & T'\otimes X \\
\| & & \| & & \| & & \| \\
F^P\Omega_T Q^t & \subset & \Omega_T(Q^t) & \longrightarrow & \Omega_{T'}(T'\otimes Q) & = & T'\otimes \Omega
\end{array}\right.
$$

Image of bottom arrow is $F^P\Omega^t$

$\therefore$ image of top arrow is $F^P X^t$

Repeat. By defn. $F^P X^t = \oplus\, t^k F^P X_{\geqslant k} \subset T'\otimes X$ and $F^P\Omega^t = \oplus\, t^k F^P \Omega_{\geqslant k} \subset T'\otimes\Omega$ ~~coincides~~ agree under the basic ident. $X = \Omega$.

have diagram $*$ middle horizontal arrows induced by

Start again: diag

$$
\begin{array}{ccccc}
X_T(R_T(Q^t)) & \longrightarrow & X_{T'}(R_{T'}(T'\otimes Q)) & = & T'\otimes X \\
\| & & \| & & \| \\
\Omega_T(Q^t) & \longrightarrow & \Omega_{T'}(T'\otimes Q) & = & T'\otimes\Omega
\end{array} \quad *
$$

horizontal arrows induced by $Q^t \subset T'\otimes Q$

image of bottom is Ω^t

$\Rightarrow$ image of top is X^t (by defn of $X_{\geqslant k}$)

have similar diag to $*$ where vertical basic idents are replaced by basic heq.

$\Rightarrow$ basic $X \sim \Omega$ induces $X^t \sim \Omega^t$, i.e. $X_{\geqslant k} \sim \Omega_{\geqslant k}$ for all k.

replace left column by $F^P_{I_T(Q^t)} = F^P\Omega_T(Q^t)$

get images $F^P X^t$ and $F^P\Omega^t$ + $F^P X^t \sim F^P\Omega^t$

d| ~~[illegible]~~ 0444 trace map now

$Q^t \longrightarrow L^t \otimes B$

induce

$$\begin{array}{ccccc} X_T(R_T(Q^t)) & \longrightarrow & X_{L^t}(R_{L^t}(L^t \otimes B)) & = & L^t_\flat \otimes X(RB) \\ \| & & \| & & \| \\ \Omega_T(Q^t) & \longrightarrow & \Omega_{L^t}(L^t \otimes B) & = & L^t_\flat \otimes \Omega B \end{array}$$

$$\begin{array}{ccccc} F^p_{I_T(Q^t)} & \longrightarrow & F^p_{I_{L^t}(L^t \otimes B)} & = & L^t_\flat \otimes F^p_{IB} \\ \| & & & & \| \\ F^p\Omega_T(Q^t) & \longrightarrow & F^p(L^t \otimes \Omega B) & = & L^t_\flat \otimes F\Omega B \end{array}$$

need

$T \qquad L^t \subset T' \otimes L$

$Q^t \qquad L^t \otimes B \subset T' \otimes L \otimes B$

$X_T(R_T(Q^t)) \longrightarrow L^t_\flat \otimes X(RB)$

$T' \otimes X(RQ) \qquad \underline{(T' \otimes L)_\flat \otimes X(RB}$

no good

so you definitely need the ~~injectivity~~ isom. $X_T(R_T(Q^t)) \xrightarrow{\sim} X^t$

---

repeat $Q^t \longrightarrow L^t \otimes B$ induces

$$\begin{array}{ccccc} X^t & = & X_T(R_T Q^t) & \longrightarrow & L^t_\flat \otimes X(RB) \\ F^pX^t & = & F^p_{I_T(Q^t)} & \longrightarrow & L^t_\flat \otimes F^p_{IB} \end{array}$$

$$\begin{array}{ccccc} \Omega^t & = & \Omega_T(Q^t) & \longrightarrow & L^t_\flat \otimes \Omega B \\ F^p\Omega^t & = & F^p\Omega_T(Q^t) & \longrightarrow & L^t_\flat \otimes F^p\Omega B \end{array}$$

e) ~~essentially~~ there is nothing here I think

$$Q^t \longrightarrow L^t \otimes B$$

induces

$$X^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_\natural \otimes X(RB)$$

such that

$$F^p X^t = F^p_{I_T(Q^t)} \longrightarrow F^p_{L^t \otimes IB} X_{L^t}(L^t \otimes B) = L^t_\natural \otimes F^p_{IB}$$

~~So you see~~

Repeat $\quad Q^t \longrightarrow L^t \otimes B \quad$ gives rise to

$$X^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_\natural \otimes X(RB)$$

i.e. to $\underline{X_{\geq k}} \longrightarrow J^k_\# \otimes X(RB)$

I have now reviewed ~~the aspec~~ most aspects of T-theory, but I still can't give a complete outline from the beginning.

Start again. 0845

I'm still on the X version of Nistor construction

Introduce $\quad Q^t = \oplus t^k Q_{\geq k} \subset T' \otimes Q$

graded subalgebra

get $\quad X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$

such that $\quad F^p_{I_T(Q^t)} \longrightarrow T' \otimes F^p_{IQ}$

The image of $\quad X_T(R_T(Q^t)) \quad$ is $\quad X^t = \oplus t^k X_{\geq k}$

———— $\quad F^p_{I_T(Q^t)} \quad$ is $\quad F^p X^t = \oplus t^k F^p X_{\geq k}$

under basic ident. map agrees with

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega Q$$

image is Ω^t for filt.

$\boxed{f}$ have [0830] $Q^t \subset T' \otimes Q$

get $X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(Q^t)) = T' \otimes X$

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega$$

Define $X_{\geq k}$ so that $X^t = \oplus t^k X_{\geq k}$ is the image of former.

<u>Note image of latter is $\Omega^t = \oplus t^k \Omega_{\geq k}$</u>

0937 now I haven't got it correct yet but it's coming:

$$Q^t = \oplus t^k Q_{\geq k} \subset T' \otimes Q$$

graded T-subalgebra

$$* \quad X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$$

Image ~~has~~ form $X^t = \oplus t^k X_{\geq k}$

Image of $F^p_{I_T(Q^t)}$ has the form

$$F^p X^t = \oplus t^k F^p X_{\geq k}$$

where $(F^p X_{\geq k})$ is a decreasing filtration of X.

Next ~~have~~ have basic identification of $*$ with

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega$$

~~where~~ The image of ~~F~~ $\Omega_T(Q^t)$ is

$$\Omega^t = \oplus t^k \Omega_{\geq k}$$

where $\Omega_{\geq k} = \Omega Q_{\geq k}$ before

The image of $F^p \Omega_T(Q^t)$ is $F^p \Omega^t = \oplus t^k F^p(\Omega_{\geq k})$

9] So we get $F^p X_{\geq k} = F^p \Omega_{\geq k}$ via ~~under~~ the basic ident.

once more for keeps.

$$Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q$$

graded T subalg

1) $$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$$

image has form $X^t = \bigoplus t^k X_{\geq k}$

image of $F^p_{I_T(Q^t)}$ has form $F^p X^t = \bigoplus t^k F^p X_{\geq k}$

have $F^p X_{\geq k} = X_{\geq k}$ for $p \leq -1$.

Also have

2) $$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega$$

image ~~has~~ is $\Omega^t = \bigoplus t^k \Omega_{\geq k}$

where $\Omega_{\geq k} = \Omega Q_{\geq k}$ before

image of $F^p \Omega_T(Q^t)$ is $F^p \Omega^t = \bigoplus t^k F^p(\Omega_{\geq k})$

~~Now have basic ident identifies 1), 2) such that $F^p_{I_T(Q^t)} = F^p \Omega_T(Q^t)$. Conclude~~

Now have basic ident between 1), 2).
At the left is the basic ident. $X = \Omega$ tensored with T'.
Identifies $F^p_{I_T(Q^t)}$ with $F^p \Omega_T(Q^t)$.
Conclude via basic identif $X = \Omega$ correspond
$F^p X_{\geq k}$ and $F^p \Omega_{\geq k}$.

But also have basic heq between 1) + 2). Shows $X \sim \Omega$
restricts to give $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$.

h] So now must continue.

at this point we need to understand D.

How do we do the grading

So what next?

You have $Q = \bigoplus Q_n$ grading

$\ni 1 \in Q_0$ and $\ni Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

You want what?

$$t^D : Q \longrightarrow T' \otimes Q \qquad l$$

induces

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

has form t^D where D is derivation extending D.

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

has form t^{LD}

1417 to ~~recover~~ finish off.

go over the remaining points.

grading of Q gives

$$t^D : Q \longrightarrow T' \otimes Q \qquad \text{linear resp. } 1$$

induces alg hom.

$$t^D : RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

induces

$$t^{LD} : X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

$\boxed{i}$ these maps express the ~~two~~ degree operators

these maps determine the induced gradings on RQ and $X(RQ)$ and the degree operators corresponding T module.

at the same time we have extensions

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$$RQ \subset T \otimes RQ \xrightarrow{\sim} R_T(Q^t) \subset T' \otimes RQ$$

$$X(RQ) \subset T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t) \subset T' \otimes X(RQ).$$

At some point I probably want to ~~write~~ set

$$R^t = R_T(Q^t) = (RQ)^t$$

$$I^t = I_T(Q^t) = (IQ)^t$$

I still have to find out what I need. I need ~~to show~~ the fact that

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X^t \subset T \otimes X(RQ)$$

d) go back to outline

we are doing the X analogue of Nistor construction.

1. define $F^p X_{\geq k}$ (decreasing bifiltration of $X = X(RQ)$ by subcomplexes)

such that basic heq $X \sim \Omega$ induces $F^p X_{\geq k} \sim F^p(\Omega^Q_{\geq k}) \quad \forall p, k$

whence we get

$$X_{\geq k} \sim \Theta(\Omega Q_{\geq k})$$

def: $F^p X_{\geq k}$ corresponds to $F^p \Omega_{\geq k}$ via $X \sim \Omega$.

2. define

Check $\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

$\gamma_- F^p X_{\geq 2j} = \gamma_- F^p X_{\geq 2j-1}$

D on RQ

L_D, h_D on $X(RQ)$,

$L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

$h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$.

3. filtered alg hom. $Q \longrightarrow L \otimes B$

$Q_{\geq k} \longrightarrow J^k \otimes B$

induces maps $X_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$

compatible with p filt. $F^p X_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_I B$

such that relative basic heq $X \sim \Omega Q$, $X(RB) \sim \Omega B$

$$\begin{array}{ccc} F^p X_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p_I B \\ \wr & & \wr \\ F^p \Omega_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p \Omega B \end{array}$$

k what remains in the proof? Answer. what remains.

Consider

$$R_T(Q^t) = R^t \subset T' \otimes RQ$$

$$I_T(Q^t) = I^t \subset T' \otimes IQ$$

$$\cancel{X_T(R_T(Q^t)) = X_T(R^t) \subset T' \otimes X}$$

$$\cancel{F^p_{I_T(Q^t)} = F^p_{I^t} \subset T' \otimes F^p_{IQ}}$$

important is that

$$X_T(R_T(Q^t)) = X_T(R^t) = X^t \subset T' \otimes X$$

$$F^p_{I_T(Q^t} = F^p_{I^t} = F^p X^t$$

because then ~~the defn~~ you have

$$\begin{array}{ccc} X_T(R^t) & \subset & T' \otimes X \\ h_D & & 1 \otimes h_D \end{array}$$

$$h_D\left(F^p_{I^t}\right) \subset F^{p-2}_{I^t} \implies (1 \otimes h_D)(F^p X^t) \subset F^{p-2} X^t$$

$$\implies h_D\left(F^p X_{\geq k}\right) \subset F^{p-2} X_{\geq k}.$$

you need to be able to identify

$$X_T(R^t) \quad \text{with} \quad X^t \subset T' \otimes X$$

$$F^p_{I^t} \quad \text{with} \quad (F^p X)^t \subset T' \otimes F^p_{IQ}$$

seems to work.

12] so what comes next? everything makes much sense!

remaining is the injectivity and the consistency.

injectivity:

$$\begin{tikzcd}
Q \arrow[r, phantom, "\subset"] \arrow[rrr, bend left, "t^D"] & T\otimes Q \arrow[r, "\sim"] & Q^t \arrow[r, phantom, "\subset"] & T'\otimes Q \\
RQ \arrow[r, phantom, "\subset"] & T\otimes RQ \arrow[r, "\sim"] & R^t \arrow[r, phantom, "\subset"] & T'\otimes RQ \\
X(RQ) \arrow[r, phantom, "\subset"] & T\otimes X(RQ) \arrow[r, "\sim"] & X^t \arrow[r, phantom, "\subset"] & T'\otimes X(RQ)
\end{tikzcd}$$

you need injectivity.

The point is that if V is graded $V = \bigoplus V_n$ and filtered $V_{\geq k} = \bigoplus_{n\geq k} V_n$ then $V \xrightarrow{t^D} T'\otimes V$ extends to

$$\begin{tikzcd}
T\otimes V \arrow[r, hook] & T'\otimes V \\
T'\otimes V \arrow[u, phantom, "\cap"] \arrow[ur, "\sim"] & t^{k+n}\otimes V_n \\
t^k\otimes V \arrow[ur, mapsto] &
\end{tikzcd}$$

I'm still confused

---

start again 1528

Consider $X_T(R_T(Q^t))$.

$$X_T(R_T(T\otimes Q)) \xrightarrow{\sim} X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q))$$

$$\underbrace{X(RQ) \subset T\otimes X(RQ) \xrightarrow{\sim} X_T(R_T(Q^t)) \subset T'\otimes X(RQ)}_{t^{LD}}$$

m] Let's try final step

$$A \xrightarrow{p+tq} S \otimes B$$

$$A \xrightarrow{\iota} Q \xrightarrow{r} S \otimes B$$

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_* \cancel{(p+tq)_*}"] \arrow[rr, bend left, "(p+tq)_*"] & X(RQ) \arrow[r, "r_*"] \arrow[d, "P_m(L_D)\gamma_-"] & S_\flat \otimes X(RB) \arrow[d, "P_m(t\partial_t)\gamma^t_-"] \\
& X(RQ)_{\geq 2m+1} \arrow[r] \arrow[dr, "l_{2m+1}"'] & S_{\flat, \geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
& & J^{2m+1}_\# \otimes X(RB)
\end{tikzcd}$$

so ~~as~~ I need to establish

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r, "(r_*)_{\geq k}"] \arrow[dr, "l_k"'] & S_{\flat, \geq k} \otimes X(RB) \arrow[d, "ev_1 \otimes 1"] \\
& J^k_\# \otimes X(RB)
\end{tikzcd}$$

commutes. I seem to recall doing this.
First point.

$$\begin{tikzcd}
\mathbb{C} \arrow[r, phantom, "\subset"] \arrow[d, phantom, "\cap"] & S \arrow[d, phantom, "\cap"] \\
T \arrow[r, phantom, "\subset"] & L^t
\end{tikzcd}
\qquad
\begin{tikzcd}
Q \arrow[r, "r"] \arrow[d, "t^D"] & S \otimes B \arrow[d, phantom, "\cap"] \\
Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}
\qquad
\begin{tikzcd}
X(RQ) \arrow[r, "r_*"] \arrow[d, "t^{L_D}"] & S_\flat \otimes X(RB) \arrow[d, phantom, "\cap"] \\
X^t \arrow[r, "l^t"] & L^t_\flat \otimes X(RB)
\end{tikzcd}$$

identifies $l^t : X^t \longrightarrow L^t_\flat \otimes X(RB)$ $\qquad L^t_{\flat c} \otimes X(RB)$
as the T linear extension of $X(RQ) \xrightarrow{r_*} S_\flat \otimes X(RB)$

n] so what, I've been over this ~~without~~ before. How much T-theory required? 1541. Go back and rearrange, and maybe introduce notation.

~~Start off with~~

$$\begin{tikzcd} Q \arrow[r, "h"] \arrow[d, "t^D"] & S \otimes B \arrow[d, phantom, "\cap"] \\ Q^t \arrow[r, "w^t"] & L^t \otimes B \end{tikzcd}$$

define [illegible] assembled from
$Q \xrightarrow{w} L \otimes B$
$Q_{\geq k} \longrightarrow J^k \otimes B$

$$\begin{tikzcd} A \arrow[r, "c"] \arrow[rr, bend left, "p+t_\varepsilon"] & Q \arrow[r, "f"] \arrow[d, "t^D"] & S \otimes B \arrow[d, phantom, "\cap"] \\ & Q^t \arrow[r, "w^t"] & L^t \otimes B \end{tikzcd}$$

$$\begin{tikzcd} X(RA) \arrow[r] & X(RQ) \arrow[r, "f"] \arrow[d, "t^{LD}"] & S_\# \otimes X(RB) \arrow[d, phantom, "\cap"] \\ & X^t \arrow[r, "l^t"] & L^t_\# \otimes X(RB) \end{tikzcd}$$

we know l^t is a T-module map, where multiplication by t^{-1} on $L^t_\#$ is given by the inclusion induced maps

$$J^k_\# \longrightarrow J^{k-1}_\#$$

We know ~~$T \otimes X$~~ t^{LD} extends to T-module isom. $T \otimes X(RQ) \xrightarrow{\sim} X(RQ)^t$. Thus $l^t : X(RQ)^t$ is the T-module extn of

$$X(RQ) \longrightarrow X(S \otimes RB) \longrightarrow S_\# \otimes X(RB) \subset L^t_\# \otimes X(RB)$$

0] What this means is that

$$\bigoplus_k t^k X_{\geq k} = \bigoplus_{n \geq k} t^k X_n$$

now $Q \xrightarrow{} S \otimes B \subset L^t \otimes B$

~~anyway what next~~?

$D \qquad\qquad t\partial_t$

$$X(RQ) \longrightarrow L^t_\natural \otimes X(RB)$$

$L_D \qquad\qquad t\partial_t$

so in particular you get

$$X_n \longrightarrow J^n_\# \otimes X(RB)$$

easy to describe namely

$$X(RQ) \longrightarrow X(R(L^t \otimes B))$$

Am trying to define

$Q \longrightarrow S \otimes B$ lin. map

$D \qquad t\partial_t \otimes 1$ resp 1 + grading

$$X(RQ) \longrightarrow X(R(S \otimes B)) \longrightarrow X_S(R_S(S \otimes B)) = S_\natural \otimes X(RB)$$

$L_D \qquad\qquad t\partial_t \otimes 1$

most of what you must do is notation.

The problem seems to ~~be~~ concern the def. of the trace map. two approach R, Ω graded + filtered.

P 1628 Seems that there are two approaches to the trace map graded + filtered.

graded: you have

$$Q \longrightarrow L \otimes B$$

$$Q_n \longmapsto J^n \otimes B$$

$$Q \longrightarrow S \otimes B$$

linear resp grading and 1.

$D \qquad t\partial_t \otimes 1.$

$$X(RQ) \longrightarrow S_\natural \otimes X(RS)$$

$L_D \qquad t\partial_t \otimes 1.$

filtered: you have

$$Q \longrightarrow L \otimes B$$

$$Q_{\geq k} \longrightarrow J^k \otimes B$$

get hom.

$$Q^t \longrightarrow L^t \otimes B$$

resp grading $\qquad t\partial_t \,.\qquad t\partial_t \otimes 1$

$$X_T(R_+(Q^t)) \longrightarrow L^t_\natural \otimes [redacted]\, X(RB)$$

$X^{t''}$

$t\partial_t \qquad t\partial_t \otimes 1.$

Consistency amounts to

9| 9/3-0517

two approaches to trace map
graded and filtered

graded. $Q \longrightarrow L \otimes B \qquad Q_n \longrightarrow J^n \otimes B$

assemble $Q \xrightarrow{f} S \otimes B$

linear resp 1 and grading

$D \qquad\qquad t\partial_t \otimes 1$

f induces

$$X = X(RQ) \xrightarrow{f_*} S_\natural \otimes X(RB)$$
$$L_D \qquad\qquad t\partial_t \otimes 1$$

whence $X_n \xrightarrow{f_{*,n}} J^n_\# \otimes X(RB)$

filtered $Q \xrightarrow{w} L \otimes B$ hom. fil. algs

$Q_{\geq n} \longrightarrow J^n \otimes B$

assemble $Q^t \xrightarrow{w^t} L^t \otimes B$ hom. grd T-alg.

induces $\ell^t : X^t = X_T(R_T(Q^t)) \longrightarrow L^t_\natural \otimes X(RB)$

~~whence~~ $\cancel{\ell_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)}$

because w^t hom.

$$\ell^t : (F^p X)^t = F^p_{I_T(Q^t)} \longrightarrow L^t_\natural \otimes F^p_{IB}$$

whence $\ell_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$

$$F^p X_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB}$$

consistency

$$\begin{array}{ccc} X(RQ) & \xrightarrow{f_*} & S_\natural \otimes X(RB) \\ {\scriptstyle t^{L_D}} \downarrow & & \cap \\ X^t & \xrightarrow{\ell^t} & L^t_\natural \otimes X(RB) \end{array}$$

√ Can say

$$X(RQ) \xrightarrow{t^{LD}} X^t \xrightarrow{\ell^t} L^t_\flat \otimes X(RB)$$

how do I go about explaining the link.

$$X_{\geq k} = \bigoplus_{n \geq k} X_n \xrightarrow{l_k} J^k_\# \otimes X(RB)$$

Logic: You have

$$\begin{tikzcd}
X \arrow[r, "f_X"] \arrow[d, "t^{LD}"'] & S_\flat \otimes X(RB) \\
X^t \arrow[r, "\ell^t"] & L^t_\flat \otimes X(RB) \arrow[u, phantom, "\cap"]
\end{tikzcd}$$

means that

$$\begin{tikzcd}
X_n \arrow[d, phantom, "\cap"] \arrow[dr, "(f_X)_n"] & \\
X_{\geq n} \arrow[r, "l_n"] & J^n_\# \otimes X(RB)
\end{tikzcd}$$

Thus

$$\begin{tikzcd}
l_k : \bigoplus_{n \geq k} X_n = X_{\geq k} \arrow[rr, "l_k"] & & J^k_\# \otimes X(RB) \\
X_n \subset X_{\geq n} \arrow[u, phantom, "\cup"] \arrow[rr, "l_n"] \arrow[rr, bend right, "f_{n*}"'] & & J^n_\# \otimes X(RB) \arrow[u, "\iota_\# \otimes 1"']
\end{tikzcd}$$

l_k on X_n is given by

$$X_n \xrightarrow{f_{n*}} J^n_\# \otimes X(RB) \xrightarrow{in_\# \otimes 1} J^k_\# \otimes X(RB)$$

clear then that $X_{\geq k} \xrightarrow{(f_X)_{\geq k}} S_{\flat \geq k} \otimes X(RB) \xrightarrow{\omega_1} J^k_\# \otimes X(RB)$ is just l_k.

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now what next

0622 so let's begin review.

Nistor construction

$Q_{\geq k}$ alg. filtr.

$\Omega Q_{\geq k}$ spanned by $x_0 dx_1 \ldots dx_n$ / $\sum \mathrm{ord}(x_i) \geq k$.

$\Omega Q_{\geq k}$ mixed complex

$\iota_k \in HC^0(\Omega Q_{\geq k+1}, \Omega Q_{\geq k})$ inclusion class

Nistor constructs $s_k \in HC^2(\Omega Q_{\geq k}, \Omega Q_{\geq k+1})$

$s_k \iota_k = S$, $\iota_k s_k = S$, s_k inverse up to S for ι_k.

unique mod $\mathrm{Ker}\, S$.

γ on Q, ΩQ, $\Omega Q_{\geq k}$

$\gamma = (-1)^k$ on $\Omega Q_{\geq k} / \Omega Q_{\geq k+1}$

$\gamma_- = \frac{1}{2}(1-\gamma)$

$\gamma_- \Omega_{\geq 2j} = \gamma_- \Omega_{\geq 2j+1}$

replace s_k by $\frac{1}{2}(s_k + \gamma s_k \gamma)$, $[s_k, \gamma] = 0$

get

$s'_{2j-1} \in HC^2(\gamma_- \Omega Q_{\geq 2j-1}, \gamma_- \Omega Q_{\geq 2j+1})$

inverse up to S of the inclusion class the other way.

Define

$Ch^{2m}(\iota, \iota^\gamma) = s'_{2m-1} \cdot s'_{2m-3} \cdots s'_1 \cdot Ch^0(\iota, \iota^\gamma)$

$\in HC^{2m}(\Omega A, \gamma_- \Omega Q_{\geq 2m+1})$

$Ch^0(\iota, \iota^\gamma)$ rep by $\Omega A \xrightarrow{\iota_*} \Omega Q \xrightarrow{\gamma_-} \gamma_- \Omega Q = \gamma_- \Omega Q_{\geq 1}$

t] then trace map

Θ, Θ' give rise to $Q \xrightarrow{w} L \otimes B$

hom. filt. algs $Q_{\geq k} \to J^k \otimes B$

filt DG hom.

$$\Omega Q \longrightarrow L \otimes \Omega B$$

$$\Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

map of mixed complexes

$$\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$$

$$l_k(\Theta, \Theta') \in HC^0(\Omega Q_{\geq k}, J^k_{\#} \otimes \Omega B).$$

Def

$$Ch^{2m}(\Theta, \Theta') \in HC^{2m}(\Omega A, J^{2m+1}_{\#} \otimes \Omega B)$$

$$\| \; l_{2m+1}(\Theta, \Theta') \; Ch^{2m}(\iota, \iota^{\gamma}).$$

X version of Nistor construction

1. define bifiltration of $X = X(RQ)$ by subcxs $F^p X_{\geq k}$ such that the SHEQ $X \sim \Omega$ restricts to give $F^p X_{\geq k} \sim F^p \, \Omega Q_{\geq k}$ $\forall p, k$.

Then $\mathcal{X}_{\geq k} = (X_{\geq k} / F^p X_{\geq k}) \sim \Theta(\Omega Q_{\geq k})$.

insert $\gamma_- \mathcal{X}_{\geq 2g} = \gamma_- \mathcal{X}_{\geq 2g+1}$. Ch^0

2. grading $Q = \bigoplus Q_n$ as v.s. induces gradings in RQ $X(RQ)$.

D on Q, RQ, L_D on $X(RQ)$

canon ϕ for RQ $\quad h_D = h^{\phi}(\iota, D)$.

Claim $\quad L_D - k : F^p X_{\geq k} \to F^{p-2} X_{\geq k+1}$

$\quad h_D : F^p X_{\geq k} \to F^{p-2} X_{\geq k}$

$k \geq 1$ $\quad 1 - \frac{1}{k} L_D$ gives $s_k \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$

ult. get $Ch^{2m}(\iota, \iota^{\gamma}) \in HC^{2m}(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 2m+1})$

u)

3. hom. of filt alg $w: Q \longrightarrow L \otimes B$

$Q_{\geq k} \longrightarrow J^k \otimes B$

induces maps of supercxs.

$$\ell_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

comp. with p filt: $\ell_k(F^p X_{\geq k}) \subset J^k_\# \otimes F^p_{IB}$.

$$\ell_k \in HC^0(\mathcal{X}_{\geq k}, J^k_\# \otimes \mathcal{X}_B).$$

define $Ch^{2m}(\theta, \theta') \in HC^{2m}(\mathcal{X}_A, J^{2m+1}_\# \otimes \mathcal{X}_B)$

clearly agrees with Nistor mod Ker S.

Finally we have the proofs to do and the consistency between | my construction / Nistor const (X-version)

Proofs use standard identification SI and standard homotopy equivalence SHEQ in relative form

$$X_S(R_S A) \underset{\sim}{=} \Omega_S A \otimes_S$$

$$F^p_{I_S A} \qquad F^p(\Omega_S A \otimes_S)$$

To handle gradings and filtrations.

$T' = \mathbb{C}[t, t^{-1}]$

grading $V \longrightarrow T' \otimes V$

section of $\delta_1 : T' \otimes V \longrightarrow V$

image closed under $D_t = t\partial_t$

I'm still in the process of finding what to say

So far have the following

V | 0852

S subalg $\subset A$ have relative constructions

$\Omega_S A$, $R_S A$, $I_S A$, $\Omega_S A \otimes_S$, $X_S(A)$, $X_S(R_S A)$,

$F^p_{I_S A}$, and

SI
$$\left| \begin{array}{lll} X_S(R_S A) & = & \Omega_S A \otimes_S \\ F^p_{I_S A} & = & F^p(\Omega_S A \otimes_S) \end{array} \right. \quad \text{s.t.}$$

SHEQ
$$\left| \begin{array}{lll} X_S(R_S A) & \sim & \Omega_S A \otimes_S \\ F^p_{I_S A} & \sim & F^p(\Omega_S A \otimes_S) \end{array} \right. \quad \text{s.t.}$$

Introduce $T' = \mathbb{C}[t, t^{-1}]$ $\deg(t) = 1$.

$T = \mathbb{C}[t] \subset T'$.

~~grading~~ V vector space $\delta_1 : T' \otimes V \longrightarrow V$ the specialization $t \longmapsto 1$

Given $V = \bigoplus_n V_n$, let D corresp. deg op: $D = n$ on V_n,

Then $t^D : V \longrightarrow T' \otimes V$

(is a section of δ_1
(whose image stable under $D_t \otimes 1$

this construction gives equivalence between gradings and maps with these two properties

Given a dec. filt. $(V_{\geq k})$ of V

$$V^t = \bigoplus_{k \in \mathbb{Z}} t^k V_{\geq k} \subset T' \otimes V$$

V^t graded T-submodule of $T' \otimes V$.

this construction gives equivalence between filtrations and such submodules.

W] When $V_{\geq k} = \bigoplus_{n \geq k} V_n$ is the filtration belonging to the grading

$$V \subset T \otimes V \underset{\substack{\text{isom. of} \\ \text{graded} \\ T\text{-mods.}}}{\xrightarrow{\sim}} V^t \subset T' \otimes V.$$

Apply this to Q

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

This is confusing; it might be better to use the diagram as a summary.

In words, where the filt. arises from grading, then the map t carries ~~maps~~ V into V^t and ~~this is~~ extends to ~~the T-module extension is an~~ an isomorphism of T-modules

$$T \otimes V \xrightarrow{\sim} V^t.$$

~~(3)~~

$$Q \longrightarrow T' \otimes Q \quad \text{induces}$$

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

Somehow you have lost the thread. Work backwards from what you need to prove.

You need to prove the stuff 1. 2. 3.

1. Define $F^p X_{\geq k}$ + show SHEQ $X \sim \Omega$ induces $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$.

Steps. $Q^t \subset T' \otimes Q$ alg hom. induces

$$a_1 : X_T(R_T(Q^t)) \longrightarrow T' \otimes X(RQ)$$

$$a_2 : \Omega_T(Q^t) \longrightarrow T' \otimes \Omega Q$$

~~image of latter is Ω^t~~

~~check $\operatorname{Im}(a_2) \supseteq \Omega^t$~~ ~~$\supseteq a_2 F^p R_T(Q^t)$~~

$\underline{x}\rfloor$ define $\left(\begin{array}{l} X^t = \operatorname{Im} a_1 \\ F^p X^t = a_1\left(F^p_{I_T(Q^t)}\right) \end{array} \right.$

check $\left(\begin{array}{l} \Omega^t = \operatorname{Im} a_2 \\ F^p \Omega^t = a_2\left(F^p \Omega_T(Q^t)\right) \end{array} \right.$

SI for Q^t rel T [redacted] for Q tensored with T'
intertwines a_1, a_2
shows $\left(\begin{array}{l} X^t = \Omega^t \\ F^p X^t = F^p \Omega^t \end{array} \right.$ under SI $X = Q$

SHEQ for Q^t/T and Q $(T'\otimes)$
shows $X^t \sim \Omega^t$ under SHEQ $X \sim Q$
$F^p X^t \sim F^p \Omega^t$

~~2. define D on R_Q, h_D~~

Next consider 2. everything has been defined already. Method of proof is $R^t = R_T(Q^t)$
$I^t = I_T(Q^t)$

$$X_T(R^t) \simeq X^t \subset T' \otimes X$$

$$F^p_{I^t} \simeq F^p X^t \subset T' \otimes F^p_{I_Q}$$

Consider stage 2 1012

have defined L_D, h_D on X and $F^p X_{\geq k}$

~~so~~ need $X_T(R^t) \xrightarrow{\sim} X^t \subset T' \otimes X$

$F^p_{I^t} \xrightarrow{\sim} F^p X^t$

L_D, h_D $\quad 1 \otimes L_D,\ 1 \otimes h_D$

because then you use rel. ~~f~~ version

$$h_D\left(F^p_{I^t}\right) \subset F^{p-2}_{I^t} \implies 1 \otimes h_D\left(F^p X^t\right) \subset F^{p-2} X^t$$

4 For stage 2 we need

$$\boxed{\begin{array}{ll} X_T(R^t) \xrightarrow{\sim} X^t \subset T'\otimes X & \\ F^p_{I^t} \xrightarrow{\sim} F^p X^t & \\ L_D,\ h_D & 1\otimes L_D,\ 1\otimes h_D. \end{array}}$$

For stage 3, the result is ~~the~~

the filt alg hom $\omega : Q \to L\otimes B$
$Q_{\geq k} \dots J^k \dots$

induces supercx maps $\forall k$

$$X_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$$

compatible as k varies

compat with p filtration

comp via SHEQ with

$$\Omega_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$$

~~to deduce from~~ want this to follow from

$\omega^t : Q^t \longrightarrow L^t \otimes B$ induces map

$$X^t \longrightarrow L^t_{\natural} \otimes X(RB)$$

compatible with
- supercomplex structure
- p-filtration, i.e. $\omega^t(F^p X^t) \subset L^t_{\natural} \otimes F^p_{IB}$
- graded T-module structure.

~~...~~ with $\Omega^t \longrightarrow L^t_{\natural} \otimes \Omega B$
via SHEQ.

$\boxed{Z}$ ~~1151~~ 1227

Consistency: my map

$$X(RA) \xrightarrow{(p+tq)_*} S_q \otimes X(RB) \xrightarrow{\mu_m \otimes 1} J^{2m+1}_{\#} \otimes X(RB)$$

X-Nistor map.

$$\begin{tikzcd}
X(RA) \arrow[r] \arrow[rr, bend left, "(p+tq)_*"] & X(RQ) \arrow[r, "f_*"] \arrow[d, "P_m(L_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(D_t)\gamma^t_-"] \arrow[dd, bend left, "\mu_m \otimes 1"] \\
& \gamma_- X_{\geq, 2m+1} \arrow[r, "f_{*, \geq 2m+1}"] \arrow[dr, "\ell_{2m+1}"'] & S_{q, 2m+1} \otimes X(RB) \arrow[d, "\delta_1 \otimes 1"] \\
& & J^{2m+1}_{\#} \otimes X(RB)
\end{tikzcd}$$

~~YES.~~ Go over outline again

$X = X(RQ)$ $\quad\Omega = \Omega Q$

$\Omega_{\geq k}$ spanned by $x_0 dx_1 \cdots dx_n$ $\quad \sum \mathrm{ord}(x_i) \geq k$

$F^p \Omega_{\geq k}$ Hodge filtration

define $F^p X_{\geq k} \subset X$ to corresp to $F^p \Omega_{\geq k}$ via the SI $X = \Omega$.

Lemma 1. SHEQ $X \sim \Omega$ restricts to yield $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$.

define $X_{\geq k} = F^p X_{\geq k}$, $p \leq -1$

have $X_{\geq k} \sim \Omega_{\geq k}$ via SHEQ

define $X_{\geq k} = (X_{\geq k} / F^p X_{\geq k})$

have $X_{\geq k} \sim \theta(\Omega_{\geq k})$.

a

~~[illegible]~~

9/4 - 0611

outline

my construction

$A \rightrightarrows L \otimes B$ cong mod J

$S = \bigoplus_{k \geq 0} t^k J^k$ $\quad S_\natural = \bigoplus_{k \geq 0} t^k J^k_\#$ $\quad$ J-adic traces

$\mu_m : S/K^{m+1} \longrightarrow J^{2m+1}_\#$ trace

$p + tq : A \longrightarrow S \otimes B$ lin resp 1 homom mod $K \otimes B$

$X(RA) \longrightarrow X(S \otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB) \longrightarrow J^{2m+1}_\# \otimes X(RB)$

$F^p_{IA} \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \sum_{i \geq 0} \natural(K^i) \otimes F^{p-2i}_{IB} \longrightarrow J^{2m+1}_\# \otimes F^{p-2m}_{IB}$

get $\mathcal{X}_A \longrightarrow J^{2m+1}_\# \otimes \mathcal{X}_B[2m]$

$ch^{2m}(\theta, \theta') \in HC^{2m}(\mathcal{X}_A, J^{2m+1}_\# \otimes \mathcal{X}_B)$

Nistor construction

$Q = QA$, ι, γ, filtration $(Q_{\geq k})$ comp w. alg str.

$\Omega Q_{\geq k}$ spanned by $x_0 dx_1 \ldots dx_n$ $\quad \sum \mathrm{ord}(x_i) \geq k$

comp with DG alg structure, b, K etc.

$\iota_k \in HC^0(\Omega Q_{\geq k+1}, \Omega Q_{\geq k})$ inclusion class

Nistor constructs $s_k \in HC^2(\Omega Q_{\geq k}, \Omega Q_{\geq k+1})$

$s_k \iota_k = S$, $\iota_k s_k = S$, s_k inverse to ι_k up to S, unique mod $\mathrm{Ker}(S)$

γ on $\Omega Q_{\geq k}$, $\gamma_- = \frac{1}{2}(1-\gamma)$

$\gamma = (-1)^k$ on $\Omega Q_{\geq k} / \Omega Q_{\geq k+1}$, $\cancel{\gamma_- \Omega Q_{\geq k} = 0}$

$\Longrightarrow \gamma_- \Omega Q_{\geq 2j} = \gamma_- \Omega Q_{\geq 2j+1}$

rep s_k by average $\frac{1}{2}(s_k + \gamma s_k \gamma)$ can suppose $[\gamma, s_k] = 0$

(b) get $S'_{2j-1} \in HC^2(\gamma_-\Omega Q_{\geq 2j-1}, \gamma_-\Omega Q_{\geq 2j+1})$
inverse up to S for inclusion.

define $Ch^0(\iota, \iota\gamma) \in HC^0(\Omega A, \gamma_-\Omega Q_{\geq 1})$

$$\Omega A \xrightarrow{\iota} \Omega Q \xrightarrow{\gamma_-} \gamma_-\Omega Q = \gamma_-\Omega Q_{\geq 1}$$

define $Ch^{2m}(\iota, \iota\gamma) \in HC^{2m}(\Omega A, \gamma_-\Omega Q_{\geq 2m+1})$ by

$$Ch^{2m}(\iota, \iota\gamma) = S'_{2m-1} \cdot S'_{2m-3} \cdots S'_1 \cdot Ch^0(\iota, \iota\gamma).$$

finally θ, θ' induce homom. of filtered algs

$$\omega : Q \longrightarrow L \otimes B$$
$$Q_{\geq k} \longrightarrow J^k \otimes B$$

ω induces $\Omega Q \to L \otimes \Omega B$ hom of filt. DG algs.
$\Omega Q_{\geq k} \to J^k \otimes \Omega B$

comp. with $J^k \to J^k_\#$ get maps of mixed complexes

$$\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$

whence $\ell_k(\theta, \theta') \in HC^0(\Omega Q_{\geq k}, J^k_\# \otimes \Omega B)$

Def. $Ch^{2m}(\theta, \theta') = \ell_{2m+1}(\theta, \theta')\, Ch^{2m}(\iota, \iota\gamma)$
$\in HC^{2m}(\Omega A, J^{2m+1}_\# \otimes \Omega B)$

X version Nistor construction

~~$\Omega = \Omega Q$, $X = X(RQ)$~~ Recall

SI: $X(RQ) = \Omega Q$
$F^p_{IQ} = F^p \Omega Q$

SHEQ: $X(RQ) \sim \Omega Q$
$F^p_{IQ} \sim F^p \Omega Q$

~~X version define~~ $F^p \Omega_{\geq k} = F^p(\Omega Q_{\geq k})$
define $F^p X$

(c) X version of Nistor construction

Recall SI : $X(RQ) = \Omega Q$ such that
$F^P_{IQ} = F^P \Omega Q$

SHEQ : $X(RQ) \sim \Omega Q$ restricts to give
$F^P_{IQ} \sim F^P \Omega Q$

(SHEQ ~~consists~~ consists of
$$\begin{tikzcd} X \arrow[r, shift left, "cP"] \arrow[loop below, "h^X"] & \Omega \arrow[l, shift left, "c^{-1}P"] \arrow[loop below, "h^\Omega"] \end{tikzcd}$$
$$P = [\partial^X, h^X] = [\underset{b+B}{\partial^\Omega}, h^\Omega]\ )$$

put $X = X(RQ)$, $\Omega = \Omega Q$
$F^P \Omega_{\geq k} = F^P(\Omega Q_{\geq k})$

define $F^P X_{\geq k} \subset X$ to corresp to $F^P \Omega_{\geq k}$ via SI.

$F^P X_{\geq k}$ subcomplexes

Claim: The SHEQ restricts to give $F^P X_{\geq k} \sim F^P \Omega_{\geq k}$ $\forall p, k$

consequence of : $F^P \Omega_{\geq k}$ stable under d, b, K, etc.

γ commutes with SI, SHEQ.
$\gamma - (-1)^k : F^P \Omega_{\geq k} \longrightarrow F^P \Omega_{\geq k+1}$
$\gamma_- F^P \Omega_{\geq 2j} = \gamma_- F^P \Omega_{\geq 2j+1}$, $\gamma_- F^P X_{\geq 2j} = \gamma_- F^P X_{\geq 2j+1}$

---

put $F^P \Omega_{\geq k} = F^P(\Omega_{\geq k})$
have $F^P \Omega_{\geq k} \subset \Omega_{\geq k} \cap F^P \Omega$, equal in degrees $> p$ but in degree p, $F^P \Omega_{\geq k}$ is $b(\Omega^{p+1}_{\geq k})$ not $\Omega^p_{\geq k} \cap b\Omega^{p+1}$

have $\gamma = (-1)^k$ on $F^P \Omega_{\geq k} / F^P \Omega_{\geq k+1}$.
$\gamma_- F^P \Omega_{\geq 2j} = \gamma_- F^P \Omega_{\geq 2j+1}$

(d) X version of N const.

Recall SI, SHEQ.

Abbreviate $X = X(RQ)$, $\Omega = \Omega Q$, $\Omega_{\geq k} = \Omega Q_{\geq k}$

have $F^p \Omega_{\geq k}$ Hodge filtration of

Claim $\gamma = (-1)^k$ on $F^p\Omega_{\geq k}/F^p\Omega_{\geq k+1}$

clear in degrees $\neq p$

in degree p $\quad b : \Omega^{p+1}_{\geq k}/\Omega^{p+1}_{\geq k+1} \twoheadrightarrow b\Omega^{p+1}_{\geq k}/b\Omega^{p+1}_{\geq k+1}$

Start again

X-version of N const.

Recall SI $\quad X(RQ) = \Omega Q \supset F^p_{IQ} = F^p \Omega Q$

SHEQ $\quad \sim \quad \supset \quad \sim$

Abbreviate: $X = X(RQ)$, $\Omega = \Omega Q$.

have Hodge filt. $F^p\Omega_{\geq k}$ of $\Omega_{\geq k} = \Omega Q_{\geq k}$.

define $F^pX_{\geq k}$ to corresp to $F^p\Omega_{\geq k}$ via SI.

Lemma 1: $F^pX_{\geq k}$ subcomplex of X

The SHEQ $X \sim Q$ induces $F^pX_{\geq k} \sim F^p\Omega_{\geq k}$ $\forall p, k$.

Consequence of fact that $F^p\Omega_{\geq k}$ stable under d, b, κ etc.

have $F^pX_{\geq k} = X_{\geq k}$ for $p \leq -1$, where $X_{\geq k} = \Omega_{\geq k}$ under the SI. Put

$$\mathcal{X}_{\geq k} = (X_{\geq k}/F^pX_{\geq k})$$

Then SHEQ induces $\mathcal{X}_{\geq k} \sim \theta(\Omega_{\geq k})$.

(e) Next have symmetry γ ~~acting~~ on X, Ω preserving the structure discussed above.

have $\gamma = (-1)^k$ on $F^p\Omega_{\geq k}/F^p\Omega_{\geq k+1}$

true in degree p because $F^p\Omega_{\geq k}$ is $b\,\Omega^{p+1}_{\geq k}$ and $\gamma = (-1)$. real reason is

$$b : \Omega^{p+1}_{\geq k}/\Omega^{p+1}_{\geq k+1} \twoheadrightarrow F^p\Omega_{\geq k}/F^p\Omega_{k+1}$$

Start again 8946.

X version of Nistor cons.

Recall SI $X(RQ) = \Omega Q \ni F^p_{IQ} = F^p\Omega Q$

and SHE $X(RQ) \sim \Omega Q \ni F^p_{IQ} \sim F^p\Omega Q$.

~ stands for $X(RQ) \underset{c^{-1}P}{\overset{c^P}{\rightleftarrows}} \Omega Q$, with h^X on $X(RQ)$ and h^Ω on ΩQ

$$\ni [\partial, h^X] = 1-P, \quad [b+B, h^\Omega] = 1-P.$$

Abbreviate: $X = X(RQ)$, $\Omega = \Omega Q$,

and consider Hodge filt $F^p\Omega_{\geq k}$ of $\Omega_{\geq k} = \Omega Q_{\geq k}$.

define $F^pX_{\geq k} \subset X$ [illegible] $\ni F^pX_{\geq k} = F^p\Omega_{\geq k}$ under SI

<u>Lemma 1.</u> $F^pX_{\geq k}$ subcx of X.

The SHEQ restricts to yield $F^pX_{\geq k} \sim F^p\Omega_{\geq k}$ $\forall p, k$.

Consequence of $F^p\Omega_{\geq k}$ closed under d, b, κ, etc.

have symmetry γ of X, Ω ~~compatible with~~ preserving all the structure discussed above.

have general. of ( )

$$\gamma = (-1)^k \quad \text{on} \quad F^p\Omega_{\geq k}/F^p\Omega_{\geq k+1}$$

~~Note that $(F^p\Omega_{\geq k})^p \supseteq b(\Omega^{p+1}_{\geq k})$~~

obvious in degrees $\neq p$ consequence of ( )

in degree p follows from fact that b maps $\Omega^{p+1}_{\geq k}/\Omega^{p+1}_{\geq k+1}$ onto $(F^p\Omega_{\geq k})^p/(F^p\Omega_{\geq k+1})^p$

~~(F) get $\gamma_- F^p\Omega_{\geq 2j} = \gamma_- F^p\Omega_{\geq 2j+1}$~~

~~$\gamma_- F^p X_{\geq 2j} = \gamma_- F^p X_{\geq 2j+1}$~~

~~$\gamma_- \mathcal{X}_{\geq 2j} = \gamma_- \mathcal{X}_{\geq 2j+1}$~~

~~define $Ch^0(\iota, \iota\gamma) \in HC^0(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 1})$~~

$$\cancel{X(RA) \xrightarrow{\iota} X(RA) = X_{\geq 0} \xrightarrow{\gamma_-} \gamma X_{\geq 0} = \gamma_- X_{\geq 1}}$$
$$\cancel{F^p_{IA} \to F^p_{IQ} = F^p X_{\geq 0} \to \gamma F^p X_{\geq 0} = \gamma_- F^p X_{\geq 1}}$$

~~better $\mathcal{X}_A \xrightarrow{\iota} \mathcal{X}_Q = \mathcal{X}_{\geq 0} \xrightarrow{\gamma_+} \gamma\mathcal{X}_{\geq 0} = \gamma_- \mathcal{X}_{\geq 1}$~~

~~Next need X-analogue of δ_k.~~

$Q \xrightarrow{\iota} L\otimes B$

$0 \to A \to \mathbb{C}[\varepsilon]\otimes Q \to \mathbb{C}[\varepsilon]\otimes L\otimes B$, $S\otimes B$

9/5-1434 time to finish!! do I want to

9/6-0541 review

my construction starting from $A \rightrightarrows L\otimes B$ cong mod $J\otimes B$

ending with $X(RA) \to J^{2m+1}_\# \otimes X(RB)$

$F^p_{IA} \to J^{2m+1}_\# \otimes F^{p-2m}_{IB}$

whence $Ch^{2m} \in HC^{2m}(\mathcal{X}_A, J^{2m+1}_\# \otimes \mathcal{X}_B)$

Nistor construction, introduce $Q = QA$, $Q_{\geq k}$, γ,

construct univ. char $Ch^{2m}(\iota, \gamma\iota) \in HC^{2m}(\Omega A, \gamma_- \Omega Q_{\geq 2m+1})$

quasi-hom. θ, θ' induces homo filt. algs

$w: Q \to L\otimes B$

$Q_{\geq k} \to J^k \otimes B$

this induces $\Omega Q_{\geq k} \to J^k_\# \otimes \Omega B$ map mixed complexes

$l_k(\theta,\theta') \in HC^0(\ ,\ )$

get $Ch^{2m}(\theta,\theta') \in HC^{2m}(\Omega A, J^{2m+1}_\# \otimes \Omega B)$.

Next X version of N construction.

Recall SI, SHE

(9) X version of N const.

SI, SHEQ

$F^p\Omega_{\geq k}$, $F^p X_{\geq k}$

<u>Lemma 1.</u> $F^p X_{\geq k}$ subcx of X

SHEQ restricts to yield $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$

• $\gamma = (-1)^k$ on $F^p\Omega_{\geq k}/F^p\Omega_{\geq k+1}$ $\Longrightarrow$ $\gamma_- F^p X_{\geq 2j} = \gamma_- F^p X_{\geq 2j+1}$

D section: Main point to define L_D, h_D on X

~~Show~~ + prove $L_D = [\partial, h_D]$ $\gamma = (-1)^{L_D}$

<u>Lemma 2.</u> $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

h_D p $p-2$

k k

$1 - k^{-1} L_D$ determines s_k , s'_{2j-1} $\rightsquigarrow$ $ch^{2m}(\iota, \sigma\iota)$

~~Lemma 3:~~ define $X_{\geq k} \xrightarrow{l_k} J^k_\# \otimes X(RB)$

to corresp to $\Omega_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$ under SI

L3: l_k map of scxs. resp. p filtration commuting with SHE

get $l_k(\theta, \theta') \in HC^0(X_{\geq k}, J^k_\# \otimes X_B)$.

Consistency. $f = p + tq : A \longrightarrow S \otimes B$

$f' = \sum_n t^n \omega_n : Q \longrightarrow S \otimes B$

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] \arrow[rr, bend left, "f_* = \alpha u_*"] & X(RQ) \arrow[r, "f'_*"] \arrow[d, "P_m(L_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(D_t)\gamma_{t,-}"] \arrow[dd, bend left, "\mu_m \otimes 1"] \\
 & \gamma_- X_{\geq 2m+1} \arrow[r, "f'_{*, \geq 2m+1}"] & S_{q, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
 & & J^{2m+1}_\# \otimes X(RB)
\end{tikzcd}$$

(h) 0658 time to concentrate on the T. theory proofs.

start at the end with trace map.

recall two versions - graded, filtered

graded: have homom. $\omega : Q \longrightarrow L \otimes B$ such that $\omega(Q_n) \subset J^n \otimes B$.

get $f' : Q \longrightarrow S \otimes B$

linear map resp 1 and grading $D \leftrightarrow D_t$

get $f'_* : X = X(RQ) \longrightarrow S_\natural \otimes X(RB)$

respecting supercx st. & grading $L_D \leftrightarrow D_t$.

get $f'_{*,n} : X_n \longrightarrow J^n_\# \otimes X(RB)$

filtered: have filt. alg hom. $Q \xrightarrow{\omega} L \otimes B$, $Q_{\geq k} \to J^k \otimes B$

whence $\omega^t : Q^t \longrightarrow L^t \otimes B$.

induces $X^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_\natural \otimes X(RB)$

such that $F^p X^t = F^p_{I_T(Q^t)}$ maps into $L^t_\natural \otimes F^p_{IB}$

get $l_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$

resp. p filt. + comp as k varies.

0736 : check the steps.

nbif. $X = \Omega$, $X \sim \Omega$ recall

intro $F^p \Omega_{\geq k}$, $F^p X_{\geq k}$

Claim : $F^p X_{\geq k}$ subcomplex of X

SHE restricts to $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$

SHE induces $\mathcal{X}_{\geq k} \sim \Theta(\Omega_{\geq k})$

$\gamma = (-1)^k$ on $F^p \Omega_{\geq k} / F^p \Omega_{\geq k+1} = F^p X_{\geq k} / F^p X_{\geq k+1}$

$\gamma_- F^p X_{2j} = \gamma_- F^p X_{\geq 2j+1}$, $\gamma_- \mathcal{X}_{\geq 2j} = \gamma_- \mathcal{X}_{\geq 2j+1}$

(i) intro D on Q, RQ

" L_D, h_D on $X(RQ)$

$$L_D = [\partial, h_D] \quad , \quad [L_D, h_D] = 0 \quad , \quad \gamma = (-1)^{L_D}$$

Lemma $\left(\begin{array}{llll} L_D - k : & P_k & & {}^{p-2}{}_{k+1} \\ h_D : & P_k & & {}^{p-2}{}_k \end{array} \right.$

~~Let~~ $1 - k^{-1} L_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

defines $s_k \in HC^2(X_{\geq k}, X_{\geq k+1})$

~~since~~ $L_D = [\partial, h_D]$, $h_D : F^p X_{\geq k} \to F^{p-2} X_{\geq k}$

implies s_k inverse ~~up~~ to S for class of inclusion

agrees with Nistor ~~up to~~ mod $\ker(S)$.

$1 - (2j-1)^{-1} L_D : \gamma_- F^p X_{\geq 2j-1} \longrightarrow \gamma_- F^p X_{\geq 2j} = \gamma_- F^p X_{\geq 2j+1}$

defines $S'_{2j-1} \in HC^2(\gamma_- X_{\geq 2j-1}, \gamma_- X_{\geq 2j+1})$

define $Ch^{2m}(\iota, \gamma\iota)$

$$X_A \xrightarrow{\iota_*} X_Q \xrightarrow{\gamma_-} \gamma_- X_{\geq 0} = \gamma_- X_{\geq 1}$$
$$\xrightarrow{1 - L_D} \gamma_- X_{\geq 3} \longrightarrow \cdots \xrightarrow{1 - \frac{1}{2m-1} L_D} \gamma_- X_{\geq 2m+},$$

finally let $\ell_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$

corresp to $\ell_k : \Omega_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$

~~under~~ via SI

L: ℓ_k map of supercomplexes comp. with ~~[illegible]~~ p-filtration

homotopy equivalent to ℓ_k^Ω via S.H.E.

~~Th~~ get $X_{\geq k} \longrightarrow J^k_\# \otimes X_B$ whence $Ch^{2m}(\theta, \theta')$

(d) consistency: look at two ~~defns~~.

$$X(RA) \xrightarrow{f_*} S_q \otimes X(RB) \xrightarrow{\mu_m} J_\#^{2m+1} \otimes X(RB)$$

$$X(RA) \xrightarrow{i_*} X(RQ) \xrightarrow{P_m(L_D)\gamma_-} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{l_{2m+1}} J_\#^{2m+1} \otimes X(RB).$$

$$\begin{tikzcd}
X(RA) \arrow[r, "i_*"] \arrow[rr, bend left, "f_*"] & X(RQ) \arrow[r, "f'_*"] \arrow[d, "P_m(L_D)\gamma_-"'] & S_q \otimes X(RB) \arrow[d, "P_m(D_t)\gamma_{t,-}"] \arrow[dd, bend left=60, "\mu_m"] \\
& \gamma_- X_{\geq 2m+1} \arrow[r, "f'_{*,\geq 2m+1}"] \arrow[dr, "l_{2m+1}"'] & S_{q,\geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_\#^{2m+1} \otimes X(RB)
\end{tikzcd}$$

square commutes as $f' : Q \longrightarrow S \otimes B$ compatible with grading $D \leftrightarrow D_t$.

Last lemma says triangle commutes: Thus ~~we has~~ recall proof.

$$\begin{tikzcd}
Q \arrow[r, "f'"] \arrow[d, "t^D"'] & S \otimes B \\
Q^t \arrow[r, "w^t"] & L^t \otimes B \arrow[u, hook]
\end{tikzcd}$$

$$\begin{tikzcd}
X \arrow[r, "f'_*"] \arrow[d, "t^{L_D}"'] & S_q \otimes X(RB) \\
X^t \arrow[r, "(w^t)_*"] & L^t_q \otimes X(RB) \arrow[u, hook]
\end{tikzcd}$$

(k) ~~[illegible]~~ So what tricky is the defn. of $b_k : X_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$ which is done via Ω picture.

Now I have to sort this out. The idea is that you need two pictures, namely

Ω description of $X^t = \bigoplus t^k X_{\geq k}$

Also you need X^t described via $X \xrightarrow{t^{LD}} X^t$.

$$a \longmapsto a + t\,da \longmapsto (p+tq)a$$

Details: ~~[illegible]~~

$$\begin{tikzcd} A \arrow[r, "\iota"] \arrow[rr, bend left] & Q \arrow[r, "t^D"] & Q^t \arrow[r, "w^t"] & L^t \otimes B \end{tikzcd}$$

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{t^{LD}} X^t \xrightarrow{(w^t)_*} L^t_{\natural} \otimes X(RB).$$

Basically you have

$$\begin{tikzcd} A \arrow[r, "\iota"] & Q \arrow[r] \arrow[d, "t^D"] & S \otimes B \arrow[d, phantom, "\cap"] \\ & Q^t \arrow[r, "w^t"] & L^t \otimes B \end{tikzcd}$$

I have to sort out the details. logic. You have

$$\begin{tikzcd} Q \arrow[r, "f'"] \arrow[d, "t^D"] & S \otimes B \arrow[d, phantom, "\cap"] \\ Q^t \arrow[r, "w^t"] & L^t \otimes B \end{tikzcd} \qquad \begin{tikzcd} X(RQ) \arrow[r] \arrow[d] & X_S(R_S(S \otimes B)) \arrow[d] \\ X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(R_{L^t}(L^t \otimes B)) \end{tikzcd}$$

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d] & S_{\natural} \otimes X(RB) \arrow[d, phantom, "\cap"] \\ X_T(R_T(Q^t)) \arrow[r] & L^t_{\natural} \otimes X(RB) \end{tikzcd}$$

(c) You must use D on Q to define the Nistor character

$$
\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "f'_*"] \arrow[d] & S_\natural \otimes X(RB) \\
& \gamma X_{\geq 2m+1} \arrow[r, "f'_{*, \geq 2m+1}"] & {}
\end{tikzcd}
$$

---

two pictures of trace map first

start with $Q \longrightarrow L \otimes B$ filtered

$Q_{\geq k} \longrightarrow J^k \otimes B$ alg homom.

~~then~~ have induced maps $X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$

$\Omega_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$

compatible with p filt. and SHEQ

our way to handle this is to form

$Q^t \longrightarrow L^t \otimes B$ homom.

have induced maps $X^t = X_T(R_T(Q^t)) \longrightarrow L^t_\natural \otimes X(RB)$

$\Omega^t = \Omega_T(Q^t) \longrightarrow L^t_\natural \otimes \Omega B$

compat. with p filt. and SHEQ

this is picture I need for ~~defining X-Nistor~~ doing X Nistor

Second picture: linear map $Q \xrightarrow[\text{comp. with grading}]{f'} S \otimes B \subset L^t \otimes B$

induces $X \xrightarrow{f'_*} S_\natural \otimes X(RB)$

(m) 9/7-0506 ~~to~~ discuss trace maps.

~~to~~ have filt alg hom $\omega : Q \longrightarrow L \otimes B$

$Q_{\geq k} \longrightarrow J^k \otimes B$

ω induces map for each k

$$l_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

resp. | superc× st.
| p-filtration: $F^p X_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB}$

comp. as k varies

ω also induces map of mix cxs

$$l_k : \Omega_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$

compatible these maps are h with SI, SHE

~~also~~ compatible as k varies

T version: have $\omega^t : Q^t \longrightarrow L^t \otimes B$ hom. of graded T algs.

induces

$$l^t : X^t = X_T(R_T(Q^t)) \longrightarrow L^t_\flat \otimes X(RB)$$

comp. with | superc× st
| p filt. $F^p X^t = F^p_{I_T(Q^t)} \longrightarrow L^t_\flat \otimes F^p_{IB}$
| graded T-modules structure

also induces

$$l^t : \Omega^t = \Omega_T(Q) \longrightarrow L^t_\flat \otimes \Omega B$$

resp | mixed cx str.
| graded T-module structure

two l^t maps comp. with SI and ~~SH~~ s.h.e.

---

so far have used $X^t = X_T(R_T(Q^t))$

$F^p X^t = F^p_{I_T(Q^t)}$

$\Omega^t = \Omega_T(Q^t)$

(n) 0521 next point: the graded approach to trace maps. This time instead of

$w : Q \to L \otimes B$ we use $f' : Q \to S \otimes B \subset L^t \otimes B$

i.e. $f' = w^t t^D$. ~~etc~~

have $f'_* : X = X(RQ) \longrightarrow L^t_\natural \otimes X(RB)$

f' compatible with grading: $D \longleftrightarrow D_t$

f'_* compat. " " $L_D \longleftrightarrow D_t$

$$f'_{*,n} : X_n \longrightarrow J^n_\# \otimes X(RB)$$

Notice | no Ω version here
| no p filtration

content seems to be only that

$$f' : Q \longrightarrow S \otimes B$$

induces $f'_* : X \longrightarrow S_\natural \otimes X(RB)$

$$L_D \qquad t\partial_t \otimes 1$$

yielding ~~$f'_{*,\geq k} : X_{\geq k} \longrightarrow S_{\natural,\geq k} \otimes X(RB)$~~

$$f'_{*,n} : X_n \longrightarrow J^n_\# \otimes X(RB)$$

Cons. claims

$$\begin{tikzcd} X_{\geq k} \arrow[r, "f'_{*,\geq k}"] \arrow[dr, "l_k"'] & S_{\natural,\geq k} \otimes X(RB) \arrow[d, "-\delta_1 \otimes 1."] \\ & J^k_\# \otimes X(RB) \end{tikzcd}$$

to prove this you ~~want to~~ need to use the T theory approach.

$$\begin{tikzcd} X \arrow[r, "f'_*"] \arrow[d, "t^{L_D}"'] & S_\natural \otimes X(RB) \arrow[d, phantom, "\cap"] \\ X^t \arrow[r, "f^t"] & L^t_\natural \otimes X(RB) \end{tikzcd} \qquad \begin{tikzcd} X_k \arrow[r] \arrow[d, phantom, "\cap"] & J^k_\# \otimes X(RB) \arrow[d, phantom, "\|"] \\ X_{\geq k} \arrow[r, "l_k"] & J^k_\# \otimes X(RB) \end{tikzcd}$$

(0) in words what goes on?

$$l_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB) \iff l^t : X^t \longrightarrow L^t_b \otimes X(RB)$$

comp as k varies being T-module map

so what I have are two versions non T and T.

non T defective because $l_{\geq k}$ not defined suitably.

0627 let's concentrate on the other side

wait. the real problem is this.

you have grading X_n and filtration $X_{\geq k}$ defined differently. Use defn of $X_{\geq k}$ arising in T-theory approach: $X^t = \mathrm{Im}\{X_T(R_T(Q^t)) \longrightarrow T' \otimes X\}$

T-theory. $Q^t \subset T' \otimes Q$

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$$

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega$$

grading on Q induces grading on RQ, $X(RQ)$.

as $Q \longrightarrow T' \otimes Q$

$$t^D : RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$t^{LD} : X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ).$$

Stop and concentrate 0703

grading $Q \longrightarrow T' \otimes Q$

induces gr. $X \longrightarrow T' \otimes X$

now $Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$

~~X(RQ)~~

? $X \subset T \otimes X \xrightarrow{\sim} X_T(R_T(Q^t)) \longrightarrow T' \otimes X$

(P) list of topics
what do you really want to prove
the grading map $t^D : Q \longrightarrow T' \otimes Q$
induces the grading map

$$t^{LD} : X(RQ) \longrightarrow T' \otimes X(RQ)$$

fact. of t^D

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

leads to fact of t^{LD}

$$X(RQ) \subset T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T(Q^t)) \longrightarrow T' \otimes X(RQ)$$

---

injectivity. The idea is the isom

$$T \otimes Q \xrightarrow{\sim} Q^t$$

of graded T-modules induces

$$\begin{array}{ccccccc} X(RQ) & \longrightarrow & X_T(R_T(T \otimes Q)) & \xrightarrow{\sim} & X_T(R_T(Q^t)) & \longrightarrow & \cancel{X}_{T'}(R_{T'}(T' \otimes Q)) \\ & & \| & & & & \| \\ X(RQ) & \subset & T \otimes X(RQ) & & & & T' \otimes X(RQ) \end{array}$$

Go over the logic.
points $\quad X(RQ) \longrightarrow X_T(R_T(T \otimes Q)) = T \otimes X(RQ)$
is $\quad \xi \longmapsto 1 \otimes \xi$
next the isom $\quad X_T(R_T(T \otimes Q)) \xrightarrow{\sim} X_T(R_T(Q^t))$
says that $t^D : Q \longrightarrow Q^t$ induces a
map $\quad X \longrightarrow X_T(R_T(Q^t))$
whose extension $\quad T \otimes X \longrightarrow X_T(R_T(Q^t))$ is isom.

The problem is you have a mess of objects.

9

~~in~~ injectivity.

I want to fit things into diagrams.

~~I want~~ have

$$T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T(Q^t)) \longrightarrow T' \otimes X(RQ)$$

~~idea might be~~

~~$t^D : Q \longrightarrow T' \otimes Q$~~

~~induces~~

~~t~~

first do grading, steps.

$$t^D : Q \longrightarrow T' \otimes Q$$

induces a hom.

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

~~corresp to a~~ is form t^D where D is ...

induces

$$X(RQ) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$$

corresp to a grading of $X(RQ)$.

next get

X

---

start again 1311

$$t^D : Q \longrightarrow T' \otimes Q$$

induces $t^{RD} : X(RQ) \longrightarrow T' \otimes X(RQ)$

Now t^D factors

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

n

$Q = \bigoplus_n Q_n$ grading

$Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ assoc. filtration

$Q \subset T \otimes Q \simeq Q^t \subset T' \otimes Q$

What do you say?

Start with V vector space

A grading det.

$V \longrightarrow T' \otimes V \qquad v \longmapsto t^n v, \; v \in V_n$

section of δ_1

image stable under D_t

$Q = \bigoplus_n Q_n \qquad Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

1400 keep on trying. more words.

We have to organize the T-theory business.

~~[illegible]~~ principle to use is that a grading of V equiv. to a section $V \to T' \otimes V$ of δ_1 whose image is homogeneous.

a filtration equiv to a graded T-submodule of $T' \otimes V$

~~wh~~ ~~grading~~

$V_{\geq k} = \bigoplus_{n \geq k} V_n \iff V^t = \operatorname{Im}(T \otimes V \to T' \otimes V)$.

with this dictionary.

start w grading on Q, D

$t^D : Q \longrightarrow T' \otimes Q$

induces $RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$

get grading on RQ, degree of D ! deriv. ext D on RQ

(5) ~~Suppose~~ get

$$X(RQ) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T'\otimes X(RQ)$$

next want filt

1532.

start with grading on Q and assoc. filtration.

$$Q = \bigoplus Q_n$$

get

$$Q \subset T\otimes Q \xrightarrow{\sim} Q^t \subset T'\otimes Q$$

$$X(RQ) \to T\otimes X(RQ) \xrightarrow{\sim} X_T(R_T(Q^t)) \to T'\otimes X(RQ)$$

X

$$\begin{array}{ccccccc} X(RQ) & \to & X_T(R_T(T\otimes Q)) & \xrightarrow{\sim} & X_T R_T(Q^t) & \to & X_{T'}(R_{T'}(T'\otimes Q)) \\ \| & & \| & & & & \| \\ X(RQ) & \subset & T\otimes X(RQ) & \xrightarrow{\sim} & X(RQ)^t & \subset & T'\otimes X(RQ) \end{array}$$

Anyway what next ??? Complete mess.

$$T\otimes Q \xrightarrow{\sim} Q^t \subset T'\otimes Q$$

$$X_T(R_T(T\otimes Q)) \xrightarrow{\sim} X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T'\otimes X(RQ)$$

what claims to make?

Difficult 1643.

I have to focus on the results to be stated, statements.

main result. Consider ~~natural~~ map

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T'\otimes X(RQ)$$

induced by the inclusion $Q^t \subset T'\otimes Q$. Then

a) inj b) image is X^t

(t) 9/8 - 0630

Consider the maps

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T'\otimes X(RQ)$$

induced by the inclusion $Q^t \subset T'\otimes Q$.

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T'\otimes Q) = T'\otimes \Omega Q$$

a) injective

b) images respectively $X^t = \oplus\, t^k X_{\geq k}$, Ω^t

c) image of $F^p_{I_T(Q^t)}$ is $F^pX^t = \oplus\, t^k F^p X_{\geq k}$

this version I know already, this one doesn't involve D.

So my difficulties involve D. & injectivity.

basic idea behind injectivity is

$$T\otimes Q \xrightarrow{\sim} Q^t$$

induces ~~isom~~ isom.

$$T\otimes X(RQ) = X_T(R_T(\overset{T\otimes Q}{\cancel{Q^t}})) \xrightarrow{\sim} X_T(R_T(Q^t))$$

composition $T\otimes X(RQ) \longrightarrow X_T(R_T(Q^t)) \longrightarrow T'\otimes X(RQ)$

0703 keep after the ideas.

three defs of filtration ~~$X_{\geq k}$ spanned by~~

$X_{\geq k}$ spanned by $\rho x_1 \cdots \rho x_n$
$\natural(\rho x_1 \cdots \rho x_n\, d(\rho x_{n+1}))$

$X_{\geq k} = \Omega_{\geq k}$ via the SI.

$$X_{\geq k} = \bigoplus_{n\geq k} X_n$$

Let's concentrate on the grading picture

I want to start with

$$Q = \oplus\, Q_n$$

grading of Q as v.s., $1 \in Q_0$.

(4) and explain the induced grading on $X(RQ)$.
Also the induced filtration.
You have crazy ideas floating around.
It's logical to push through a proof
~~tik~~ from the $X \cdot R$ viewpoint

Start with grading $Q = \oplus Q_n \qquad 1 \in Q_0$

D degree op on Q : $D = n$ on Q_n

have
$$t^D : Q \longrightarrow T' \otimes Q$$

(linear map resp 1.
section of $t \mapsto 1$ specialization
image stable under $D_t = t\partial_t \otimes 1$.
in fact P_t intertwines D on Q.

~~t^D induces a maps~~
~~$RQ \longrightarrow R_{T'}(T' \otimes Q)$~~
~~$X(RQ) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$~~

compatible with structure

spread it out:

t^D induces a homom.
$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$
section of $t \mapsto$ specialization
image stable under D_t since RQ
generated by $\rho x \quad x \in Q$

---

start again 1144. the problem is ~~to~~ to find
assertions for T theory part. Let's work backwards
Consistency of ~~the mod~~ trace map.
assume trace map defined by
$$l^t = (\omega^t)_* : X^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_{\gamma} \otimes X(RB)$$
recall ~~f ...~~

(v)

$$\begin{tikzcd} Q \arrow[r, "f'"] \arrow[d, "t^D"'] & S \otimes B \\ Q^t \arrow[r, "w^t"] & L^t \otimes B \arrow[u, phantom, "\cap"] \end{tikzcd} \Longrightarrow \begin{tikzcd} X \arrow[r, "f'_*"] \arrow[d, "t^\flat"'] & S_\flat \otimes X(RB) \\ X^t \arrow[r, "l^t"] & L^t_\flat \otimes X(RB) \arrow[u, phantom, "\cap"] \end{tikzcd}.$$

$\Longrightarrow$ $l^t : X^t \longrightarrow L^t_\flat \otimes X(RB)$ is the T-linear extension

$$T \otimes X \longrightarrow L^t_\flat \otimes X(RB)$$

of $\quad f'_* : X \longrightarrow L^t_\flat \otimes X(RB)$.

what's the logic? On one hand we have

$$t^{LD} : T \otimes X \xrightarrow{\sim} X^t$$

which expresses $\quad X_{\geq k} = \bigoplus_{n \geq k} X_n$.

On the other hand we ~~have~~ know l^t is a T-module map $X^t \longrightarrow L^t_\flat \otimes X(RB)$ which means

$$\begin{tikzcd} X_{\geq n} \arrow[r, "l_n"] & J^n_\# \otimes X(RB) \arrow[d, "\text{ind. by } J^n \subset J^k"] \\ X_{\geq k} \arrow[u, phantom, "\cap"] \arrow[r, "l_k"] & J^k_\# \otimes X(RB) \end{tikzcd}$$

commutes. Also know $\quad l^t t^{LD} = f'_*$

$$\begin{tikzcd} X_n \arrow[d, hook] \arrow[dr, "f'_{*,n}"] & \\ X_{\geq n} \arrow[r, "l^*_n"] & J^n_\# \otimes X(RB) \end{tikzcd}$$

It follows that,

$$\begin{tikzcd} X_{\geq k} \arrow[r, "f'_{*,\geq k}"] \arrow[dr, "l^*_k"'] & L^t_{\flat, \geq k} \otimes X(RB) \arrow[d, "\delta,"] \\ & J^k_\# \otimes X(RB) \end{tikzcd}$$

W What's important is ~~that~~ maybe

$$\begin{tikzcd}
X_n \arrow[d, hook] \arrow[dr, "f'_{*,n}"] & \\
X_{\geq n} \arrow[r, "l_n"] \arrow[d, hook] & J^n_{\#} \otimes X(RB) \arrow[d] \\
X_{\geq k} \arrow[r, "l_k"] & J^k_{\#} \otimes X(RB)
\end{tikzcd}$$

and this implies

$$\begin{tikzcd}
X_{\geq k} \arrow[r, "f'_{*,\geq k}"] \arrow[dr, "l_k"'] & L^t_{p,\geq k} \otimes X(RB) \arrow[d, "\delta_1"] \\
& J^k_{\#} \otimes X(RB)
\end{tikzcd}$$

commutes.

What points are used?

have $\quad t^{LD} : X \longrightarrow X^t$

$$t^D : Q \longrightarrow Q^t$$

induces $\quad t^{LD} : X \longrightarrow X^t$

~~and the which extends~~

such that T-module extension

$$T \otimes X \xrightarrow{\sim} X^t$$

Let's try different ~~B~~ viewpoint

start with $\quad Q = \bigoplus_n Q_n \qquad L \in Q_0$

define ~~$\mathcal{D}$~~ on Q, $\quad D_t$ on ~~$\otimes$~~ $T' \otimes Q$

(X) start with

$$Q = \bigoplus_n Q_n \qquad Q_{\geq k} = \bigoplus_{n \geq k} Q_n$$

D on Q defd by $D = n$ on Q_n

$$t^D : Q \longrightarrow T' \otimes Q$$

$$Q^t = \bigoplus_k t^k Q_{\geq k} = \bigoplus_{n \geq k} t^k Q_n$$

$$= \bigoplus_n \bigoplus_{i \geq 0} t^{-i} t^n Q_n = T \otimes \bigoplus_n t^n Q_n$$

So what?

map $\quad t^D : Q \longrightarrow T' \otimes Q$

image $\quad t^D Q = \bigoplus_n t^n Q_n$

you probably have ~~on~~ the wrong viewpoint.

$Q = \bigoplus_n Q_n$, $\quad Q_{\geq k} = \bigoplus_{n \geq k} Q_n$, $\quad Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q$

observe

$$\cancel{Q \xrightarrow{t^D} Q^t \subset T' \otimes Q}$$

$$Q \xrightarrow{t^D} Q^t \subset T' \otimes Q \qquad a$$

linear resp 1. $\qquad$ T-subalg.

$$T \otimes X \twoheadrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

$$t^i \otimes x \longmapsto t^{i+|x|} x$$

find

(y) 9/19 -0435

start with grading $\quad Q = \bigoplus Q_n$

~~filt~~ $\quad Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

$$Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q.$$

Q^t is a ~~[illegible]~~ graded T subalg of $T' \otimes Q$

induced map

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$$

~~explain~~

have $\quad t^D : Q \longrightarrow T' \otimes Q$

| lin. recip 1
| section of $t \mapsto 1$ specialization
| image is graded (all homogeneous)
| subspace, closed under $D_t = t\partial_t \otimes 1$.

~~induces [illegible] map~~

~~$X(RQ) \quad X \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$~~

0500 start again with grading ~~[illegible]~~ $Q = \bigoplus Q_n$

No find the assertion you need, such as

$t^D : Q \longrightarrow T' \otimes Q$ induces & $t^{LD} : X \longrightarrow T' \otimes X$

let's start again.

key steps $\quad Q^t \subset T' \otimes Q$ induce

$$X_T(R_T(Q^t)) \longrightarrow X_{T'} \cdots = T' \otimes X$$

$$t^D : Q \longrightarrow T' \otimes Q \quad \text{induces}$$

$$t^{LD} : X \longrightarrow T' \otimes X$$

$$t^D : Q \longrightarrow Q^t \quad \text{induces}$$

$$X \longrightarrow X_T(R_T(Q^t))$$

since the T-module map $T \otimes Q \longrightarrow Q^t$ exist.

$Q \longrightarrow Q^t$ is isom. have

$$T \otimes X \xrightarrow{\sim} X_T(R_T(Q^t)) \qquad T \text{ mod iso}$$

(2) $t^0 : Q \longrightarrow Q^t$ induces

$$X \longrightarrow X_T(R_T(Q^t))$$

since $T \otimes Q \xrightarrow{\sim} Q^t$ T mod ext.

have $T \otimes X = X_T(R_T(T \otimes Q)) \simeq X_T(R_T(Q^t))$

only need surjectivity.

Continue: ~~how to get 18~~

~~grading~~ problem - to find assertions

grading on Q ~~induces~~ determines grading on $X = X(RQ)$.

---

instead of wasting more time let's write everything out. 05-19.

Claim grading on Q determines gradings on RQ and $X = X(RQ)$ comp. with structure, e.g. ~~such that~~ making RQ is a graded algebra

grading on Q equivalent to linear map

1) $$Q \longrightarrow T' \otimes Q$$

section of $t \mapsto 1$ specialization
image graded, closed under $D_t = t\partial_t \otimes 1$

This map induces (as $1 \mapsto 1$) a homom.

2) $$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

alg hom. ✓
section of $t \to 1$ spec. ✓
image graded because RQ generated by ρx with x homogeneous.

$$\begin{array}{ccc} x & \longmapsto & t^{|x|}x \\ \downarrow & & \downarrow \rho \\ \rho x & \longmapsto & t^{|x|}\rho x \end{array}$$

1) also induces

3) $$X(\cancel{RQ}) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X\cancel{RQ}$$

map of s.cxs.
section of spec. $t \mapsto 1$

A: ~~[illegible]~~ (9/9) cont.

image homog. because X spanned by elts $\rho x_1 \cdots \rho x_r$, $\int (\rho x_1 \cdots \rho x_r d(\rho x_{r+1}))$ where $x_i \in Q$ are homog.

This reminds me of yesterday's idea of listing descriptions of the ~~filt.~~ grading

X_n spanned by elements $*$ where x_i homog. and $\sum |x_i| = n$.

$X_n = \mathrm{Ker}(L_D - n)$ where D ! deriv. on RQ extending D on Q

~~The~~ map $t^{L_D} : X \to T' \otimes X$ corresp to the grading is the map induced by $t^D : Q \to T' \otimes Q$

Go on now to the next stage which ~~[illegible]~~ concerns ~~[illegible]~~ filtrations. descriptions

$X_{\geq k} = \bigoplus_{n \geq k} X_n$ $X_{\geq k}$ spanned by elts $*$ where $\sum \mathrm{ord}(x_i) \geq k$.

$X^t =$ ~~[illegible]~~ image of T module map $T \otimes X \longrightarrow T' \otimes X$ ext t^{L_D}

$X^t =$ image of $X_T(R_T(Q^t)) \longrightarrow T' \otimes$ ~~[illegible]~~ X

~~[illegible]~~ Next return to composition

$$T \otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

~~Next~~ Ultimately what happens?

have

$$T \otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

first map is T-module map extending the map induced by ~~[illegible]~~ $t^D : Q \to Q^t$. surjective because Q^t generated by $t^D Q$ as T-module. second map induced by inclusion $Q^t \subset T' \otimes Q$

B ~~in~~ the composition is the T module map extending $t^{LD} : X \longrightarrow T' \otimes X$, and we know it is injective with image X^t for the filtration $X_{\geq k} = \bigoplus_{n \geq k} X_n$.

---

grading. $Q = \bigoplus Q_n$

have $X_T(R_T(Q^t)) \longrightarrow T' \otimes X$

induced by $Q^t \subset T' \otimes Q$.

also have $X \longrightarrow X_T(R_T(Q^t))$

induced by $Q \longrightarrow Q^t$

~~have~~ get $T \otimes X \longrightarrow X_T(R_T(Q^t))$

T module map ext.

have $X \longrightarrow$

table of maps

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$$X \subset T \otimes X \xrightarrow{\sim} X_T(R_T(Q^t)) \subset T' \otimes X$$

~~[illegible]~~ Review: You have

$f' : Q \longrightarrow S \otimes B \subset L^t \otimes B$

$f'_* : X \longrightarrow S_q \otimes X(RB) \subset L^t_q \otimes X(RB)$

respects grading: L_D on X, D_t on $L^t_q \otimes X(RB)$

You have also

$w^t : Q^t \longrightarrow L^t \otimes B$

$l^t : X^t = X_T(R_T(Q^t)) \longrightarrow L^t_q \otimes X(RB)$

C Now have

$$\begin{tikzcd} Q \arrow[r, "t^D"] \arrow[rr, bend right, "f'"] & Q^t \arrow[r, "\omega^t"] & L^t \otimes B \end{tikzcd}$$

and

$$\begin{tikzcd} X \arrow[r, "t^{LD}"] \arrow[rr, bend right, "f'_*"] & X^t \arrow[r, "\ell^t"] & L^t_D \otimes X(RB) \end{tikzcd}$$

The point is that ℓ^t is the unique T-module ~~[illegible]~~ map extending

---

9/10-0520 Outline

my construction
Nistor construction
X version of Nistor construction
consistency

these are mostly done in outline at least
what remains is how to handle the grading:
typical problem: You have defined $X_{\geq k}$ to corresp to $\Omega_{\geq k}$ via the SI. You have also defined a grading $X = \bigoplus X_n$. You need to prove that $X_{\geq k} = \bigoplus_{n \geq k} X_n$. This should follows from $L_D - k : X_{\geq k} \to X_{\geq k+1}$ and the fact that $L_D = n$ on X_n. Why?

$X = X_{\geq 0} = \bigoplus X_n$

Then we know $L_D = 0$

$X_{\geq k}$ is closed under L_D

$L_D X \subset X_{\geq 1}$. So $X_{\geq 1}$ could be any subspace $L_D X \subset X_{\geq 1} \subset X$. So by spectral theory all we know is that

$X_{\geq k} = \bigoplus_{n \geq k} X'_n$ where $X'_n \subset X_n$

D $\quad X_{\geq k}$ is L_D stable so that

$$X_{\geq k} = \bigoplus_n X_n \cap X_{\geq k}$$

eg $\quad X = \bigoplus X_n$

$$X_{\geq 1} = \bigoplus X_n \cap X_{\geq 1}$$

then we have $\quad L_D = 0$ on

$$X/X_{\geq 1} = \bigoplus_n X_n / X_n \cap X_{\geq 1}$$

conclude $\quad X_n = X_n \cap X_{\geq 1}$ for $n \geq 1$.

$$\therefore X_{\geq 1} = X_0 \cap X_{\geq 1} \oplus X_1 \oplus X_2 \oplus \cdots$$

$$X_{\geq 2} = X_0 \cap X_{\geq 2} \oplus X_1 \cap X_{\geq 2} \oplus X_2 \oplus X_3$$

$$(L_D - 1) X_{\geq 1} \subset X_{\geq 2}$$

so you do reach a contradiction because $\bigcap X_{\geq k} = 0$.

But even if this works it is not a good argument. Instead I want to add statements like

$$X_{\geq k} = \bigoplus_{n \geq k} X_n$$

X_n spanned by $\rho x_1 \cdots \rho x_n$, $\natural(\rho x_1 \cdots \rho x_n\, d(\rho x_{n+1}))$

take this as a definition of X_n

E So what? We now must takkle the grading.

It seems that in the ND section you want to ~~[illegible]~~ say the philosophical ~~alternative~~ statements: grading on Q induces grading on ρ description of elements and in the NT section you want to do the precise version without explanations. ~~So it seems that one has yield.~~

Go on with the grading stuff

$$Q \xrightarrow{t^D} Q^t \subset T' \otimes Q$$

$$T \otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

first arrow surjective

composition is t^{LD}, injective.

What's the goal? The goal is

$$T \otimes X \xrightarrow{\sim} X^t$$

i.e. $X_{\geq k} = \bigoplus_{n \geq k} X_n$.

~~What~~ Work backwards, organize the end: trace map and consistency, review

$w : Q \longrightarrow L \otimes B$, $Q_{\geq k} \longrightarrow J^k \otimes B$

$$\boxed{\cancel{f': Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B}}$$

$f' : Q \longrightarrow S \otimes B$ graded $\qquad f' = t^n w : Q_n \longrightarrow t^n J^n \otimes B$

$$\boxed{f'_n = t^n w : Q_n \longrightarrow t^n J^n \otimes B}$$

F

$$\begin{tikzcd} Q \arrow[r, "f'"] \arrow[d, "t^D"'] & S \otimes B \arrow[d, phantom, "\cap"] \\ Q^t \arrow[r, "\omega^t"] & L^t \otimes B \end{tikzcd} \implies \begin{tikzcd} X \arrow[r, "f'_*"] \arrow[d, "t^{LD}"'] & S_\natural \otimes X(RB) \arrow[d, phantom, "\cap"] \\ X^t \arrow[r, "\ell^t"] & L^t_\natural \otimes X(RB) \end{tikzcd}$$

~~[illegible]~~ Since

t^{LD} extends to T-mod. isom $T \otimes X \xrightarrow{\sim} X^t$

ℓ^t is a T-module map.

conclude ~~$\ell_k : X_{\geq k} = \bigoplus_{n \geq k} X_n \longrightarrow J^k_\# \otimes X(RB)$~~

~~[illegible] such that~~

ℓ^t is the T-module map extending f'_*

~~B~~ecause $L^t_\natural$ has ~~[illegible]~~ T action given by $J^n_\# \to J^k_\#$ induced by $J^n \subset J^k$ for $n \geq k$.

Thus $\ell_k : X_{\geq k} = \bigoplus_{n \geq k} X_n \longrightarrow J^k_\# \otimes X(RB)$

is the map whose restriction to X_n is

$$X_n \xrightarrow{f'_{*,n}} J^n_\# \otimes X(RB) \longrightarrow J^k_\# \otimes X(RB)$$

latter induced by $J^n \subset J^k$.

The above is the consistency part. Now what about the trace map?

ω^t induces

$$X^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_\natural \otimes X(RB)$$

compatible with | supercomplex structure
graded T-module structure
I-filtration: $F^p X^t = F^p_{I_T(Q^t)} \longrightarrow L^t_\natural \otimes F^p_{IB}$

also induces

$$\Omega^t = \Omega_T(Q^t) \longrightarrow \Omega_{L^t}(L^t \otimes B) = L^t_\natural \otimes \Omega B$$

G Anyway what next?
Lets go over the argument ~~that~~ for filtration behavior of h_D and h_D.
We identify

<u>idea</u>: use degree operator to do the work.
Take D on Q $\quad D: Q \to Q \quad D(1) = 0$
$\exists!$ derivation D on RQ $\ni$ $D\rho = \rho D$

9/11-0524 Final stage: the grading part

~~Here~~ outline

$$Q \xrightarrow{t^D} T' \otimes Q$$

induces

$$X = X(RQ) \longrightarrow X_{T'}(R_{T'}(\cancel{\otimes} T' \otimes Q)) = T' \otimes X$$

what's the goal? ~~The goal is~~

two definitions for filtration to ~~~~ <u>reconcile</u>

background: I'm working on the T-theory section. First part begins with equiv.

filtrations on V $=$ graded T submodule of $T' \otimes V$

and induced $Q^t \subset T' \otimes Q$

$$X_T(R_T(Q^t)) \longrightarrow T' \otimes X \qquad \text{image } X^t$$

$$\Uparrow$$

$$\Omega_T(Q^t) \longrightarrow T' \otimes \Omega \qquad \text{image } \Omega^t$$

these agree via ~~&~~ s.c., injective yielding canonical isom.

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X^t$$

resp. super cx structure
graded T-mod structure
p-filtration $\quad F^p_{I_T(Q^t)} \xrightarrow{\sim} \cancel{\otimes} F^p X^t$

H

$$\Omega_T(Q^t) \xrightarrow{\sim} \Omega^t$$

resp. gr. T-mod

mixed complex

so far have used picture of X [redacted] coming from the s.s. (used $Q^t \subset T' \otimes Q$ homom.)

Next ρ picture.

use map $Q \xrightarrow{t^D} Q^t \subset T' \otimes Q$

$$T \otimes X \xrightarrow{\sim} X_T(R_T(Q^t)) \subset T' \otimes X$$

$$\Longrightarrow \quad T \otimes X \xrightarrow{\sim} X^t$$

means $X_{\geq k} = \bigoplus_{n \geq k} X_n$

$$T \otimes RQ \xrightarrow{\sim} RQ^t \subset T' \otimes RQ$$

~~Q~~

NTTH

N.7

N.9

Let's proceed.

$$Q \xrightarrow{t^D} Q^t \subset T' \otimes Q$$

induces

$$X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

whence

$$T \otimes X \twoheadrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

composition injective and the image is graded T-submodule corresp to filtration $\bigoplus_{n \geq k} X_n$.

Let's first get the relations straight for Q.

$Q = \bigoplus_n Q_n$, $1 \in Q_0$ graded v.s. with 1

$Q_{\geq k}$ is the assoc. filtration.

$$Q \subset T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

graded T-submodule

I ~~have~~ $\cancel{Q^t}$

the objects are

embeddings $Q \hookrightarrow Q^t \subset T' \otimes Q$

$$Q \subset T \otimes Q \longrightarrow Q^t \subset T' \otimes Q$$

$$X \subset T \otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes \cancel{X}$$

So what. $Q = \bigoplus_n Q_n \qquad Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

$$Q^t = \bigoplus t^k Q_{\geq k} \leftrightarrows T \otimes Q$$

9-12-0335 organize results.

$$Q \xrightarrow{t^D} Q^t \subset T' \otimes Q$$

$$X \xrightarrow{a} X_T(R_T(Q^t)) \xrightarrow{b} T' \otimes X$$

$$T \otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$$

grading homom.

logic. have Q with grading $Q = \bigoplus_n Q_n$

with filt $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

have $Q^t = \bigoplus_k t^k Q_{\geq k}$

have $Q \hookrightarrow Q^t \subset T' \otimes Q$

$x \longmapsto t^D x$

$$T \otimes Q \xrightarrow{\sim} Q^t$$

induced maps $X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T' \otimes X$

comp. t^{-D} first map surjective

J first stage is grading maps

~~[illegible]~~ grading $Q = \bigoplus Q_n$

$$t^D : Q \longrightarrow T' \otimes Q$$

linear resp. 1,
get induced maps

$$X = X(R\cancel{Q}) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$$

is the grading map t^{LD} ~~assoc. to~~ for the induced grading on X.

~~induced one~~

repeat. grading on Q determines injection

$$t^D : Q \longrightarrow T' \otimes Q$$

linear resp 1, so induces map

$$\cancel{RQ} \longrightarrow X_{T'}(R_{T'}(\overset{T'\otimes Q}{\cancel{Q}})) = T' \otimes X$$

$$\rho x_1 \cdots \rho x_r \longmapsto \rho(t^{|x_1|} x_1) \cdots \rho(t^{|x_r|} x_r)$$

$$= t^{|x_1| + \cdots + |x_r|} \rho x_1 \cdots \rho x_r$$

~~coincides agrees with~~
which $= t^{LD}$.

have $Q_{\geq k}$ and Q^t

t^D for Q factors

$$Q \longrightarrow Q^t \subset T' \otimes Q$$

$Q \longrightarrow Q^t$ induces $T \otimes Q \xrightarrow{\sim} Q^t$
T-module isom.

~~[illegible]~~

K objects

grading $Q = \oplus Q_n$, degree of D

filtration $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

$Q^t = \oplus t^k Q_{\geq k} \subset T' \otimes Q$

$Q \longrightarrow Q^t$, $Q^t \subset T' \otimes Q$, $Q \longrightarrow T' \otimes Q$

$T \otimes Q \longrightarrow Q^t$

X, grading $X = \oplus X_n$, filtr $X_{\geq k}$, X^t

$X \longrightarrow T' \otimes X$

---

objects:

Q, grading, D

filt, Q^t, $Q^t \subset T' \otimes Q$.

~~$Q \longrightarrow Q^t \subset T'$~~

$Q \longrightarrow Q^t$, $T \otimes Q \longrightarrow Q^t$

Q, grading, D, $t^D : Q \longrightarrow T' \otimes Q$,

induced T-mod. map $T \otimes Q \longrightarrow T' \otimes Q$

filtr. $Q_{\geq k}$, Q^t, $Q^t \subset T' \otimes Q$

Q, grading, D, $t^D : Q \longrightarrow T' \otimes Q$

induced T-module map $T \otimes Q \longrightarrow T' \otimes Q$

X, grading, L_D, $t^{L_D} : X \longrightarrow T' \otimes X$.

induced T-module map $T \otimes X \longrightarrow T' \otimes X$

assert. | $T \otimes Q \longrightarrow T' \otimes Q$ injective
| image is Q^t for filt. $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

L. assert | $T \otimes X \to T' \otimes X$ injective
image is graded T-submod corresp to filt. $\bigoplus_{n \geq k} X_n$

~~next what hap.~~ continue

---

repeat Q, grading, deg of D, $t^D : Q \to T' \otimes Q$

ind. T-mod. map $T \otimes Q \xrightarrow{j} T' \otimes Q$.

~~Claim~~: j injective and $\mathrm{Im}\, j = Q^t$ for filt $\bigoplus_{n \geq k} Q_n$

thus $T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$

X, grading, degree of L_D

$t^{L_D} : X \to T' \otimes X$

Claim: $t^{L_D} = (t^D)_* : X \to X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$.

$T \otimes X \to T' \otimes X$ induced T-module map.

claim injective & image = submod. corres to filt $\bigoplus_{n \geq k} X_n$

---

summarize

Q, grading, $t^D : Q \to T' \otimes Q$,

T-module ext. $T \otimes Q \to T' \otimes Q$

Q, grading, filt., Q^t

$t^D : Q \to T' \otimes Q$

Review: Q, grading, map $t^D : Q \to T' \otimes Q$

X, " , map $t^{L_D} : X \to T' \otimes X$

claim t^D lin. resp $\mathbb{1}$, so it induces

$X \to X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X$

this coincides with t^{L_D}, so conclude

M $\quad T\otimes X \longrightarrow T'\otimes X$ inj image

corresp. to filt assoc. to grading.

Consider $X_T(R_T(Q^t))$.

have $X \longrightarrow X_T(R_T(Q^t))$ induced by $t^0 : Q \to Q^t$

have maps

$$T\otimes X \longrightarrow X_T(R_T(Q^t)) \longrightarrow T'\otimes X$$

1108 T-theory proofs.

recall relative theory.

$T', T^{\cdot}$, filtration on $V \longleftrightarrow$ graded T-submod of $T'\otimes V$

notation V^t.

Consider Q with $Q_{\geq k}$, $Q^t \subset T'\otimes Q$ subalg.

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T'\otimes X$$

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T'\otimes Q) = T'\otimes\Omega$$

natural graded T module structures.

9/13 - 1055 ~~What is natu~~

do rest of T-theory proofs

grading + filtration.

so far I have introduced

filtration on V = graded T-submodule of $T'\otimes V$.

~~and~~ Q + $Q_{\geq k} \Longrightarrow Q^t \subset T'\otimes Q$

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X^t \subset T'\otimes X$$

$$\Omega_T(Q^t) \xrightarrow{\sim} \Omega^t \subset T'\otimes\Omega$$

N Now want to discuss grading

$$Q = \bigoplus_n Q_n \quad , \quad D ,$$

$$t^D : Q \longrightarrow T' \otimes Q$$

Let $T \otimes Q \longrightarrow T' \otimes Q$ be the T-module map extending t^D.

Claim: this map injective
image is graded T-submodule Q^t assoc. to $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$.

$\therefore \; T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$.

$X \longrightarrow T' \otimes X$ induced by t^D

Claim this map $t^{\wedge D}$ assoc. to grading on X.

$T \otimes X \longrightarrow T' \otimes X$ T-module extn. of $t^{\wedge D}$

~~Now~~ this inj + image is T submodule assoc. to $X_{\geq k} = \bigoplus_{n \geq k} X_n$.

next $\quad Q \xrightarrow{t^D} Q^t \subset T' \otimes Q$

$$\begin{tikzcd} T \otimes X \arrow[r, "\text{surj}"'] \arrow[rr, bend left, "\text{inj}"] & X_T(R_T(Q^t)) \arrow[r] & T' \otimes X \end{tikzcd}$$

~~Interesting point: Answer:~~

$$T \otimes X(R) \xrightarrow{\sim} X_T(R^t) \subset T' \otimes X(R)$$

$$X_T(T \otimes R) \xrightarrow{\sim} X_T(R^t) \subset X_{T'}(T' \otimes R)$$