Ⓛ 8/21 - 1507 start again I need the outline.

my ~~[illegible]~~ construction

$$A \xrightarrow{p+tq} S \otimes B$$

$$RA \xrightarrow{u} S \otimes RB \qquad IA \longrightarrow K \otimes RB + S \otimes IB$$

$$X(RA) \xrightarrow{u_*} X(S \otimes RB) \xrightarrow{\alpha} S_{\natural} \otimes X(RB) \xrightarrow{\mu_m} J_{\#}^{2m+1} \otimes X(RB)$$

$$F^p_{IA} \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \sum_{i \geq 0} \natural(K^i) \otimes F^{p-2i}_{IB}$$

$$\longrightarrow J_{\#}^{2m+1} \otimes F^{p-2m}_{IB}$$

$$\mathcal{X}_A \longrightarrow J_{\#}^{2m+1} \otimes \mathcal{X}_B [2m]$$

$$ch^{2m}(\theta, \theta') \in HC^{2m}(\mathcal{X}_A, J_{\#}^{2m+1} \otimes \mathcal{X}_B)$$

Nistor construction

$$Q = QA \qquad Q_{\geq k} = (qA)^k \qquad (\Omega Q)_{\geq k} \qquad \gamma$$

$$\iota_k \in HC^0((\Omega Q)_{\geq k+1}, (\Omega Q)_{\geq k}) \qquad \boxed{\gamma(\Omega Q)_{\geq k+1} = \gamma(\Omega Q)_{\geq k} \quad k \text{ even}}$$

$$\exists \quad \delta_k \in HC^0((\Omega Q)_{\geq k}, (\Omega Q)_{\geq k+1}) \qquad k \geq 1$$

$$\delta_k \iota_k = S \qquad \iota_k \delta_k = S \qquad \delta_k \text{ unique mod } \operatorname{Ker} S$$

$$\delta_k \longrightarrow \tfrac{1}{2}(\delta_k + \gamma \delta_k \gamma)$$

Ⓜ rest.

$$\iota'_k \in HC^0(\gamma_-(\Omega Q)_{\geq k+1}, \gamma_-(\Omega Q)_{\geq k})$$

$$s'_{2j-1} \in HC^2(\gamma_-(\Omega Q)_{\geq 2j-1}, \gamma_-(\Omega Q)_{\geq 2j+1})$$

same.

Put $ch^{2m}(\iota, \iota^\gamma) = s'_{2m-1} \cdot \cdots s'_3 \cdot s'_1 \cdot ch^0(\iota, \iota^\gamma)$

$$\in HC^{2m}(\Omega A, \gamma_-(\Omega Q)_{\geq 2m+1})$$

$ch^0(\iota, \iota^\gamma)$ class of $\Omega A \xrightarrow{\iota} \Omega Q \xrightarrow{\gamma_-} \gamma_- \Omega Q = \gamma_- \Omega Q_{\geq 1}$

finally θ, θ' induce homom. of filtered algs.

$Q \longrightarrow L \otimes B$, $\quad Q_{\geq k} \longrightarrow J^k \otimes B \quad \forall k$

whence $\quad \Omega Q \longrightarrow L \otimes \Omega B$, $\quad (\Omega Q)_{\geq k} \longrightarrow J^k \otimes \Omega B \quad \forall k$

Maps of mix cxs. : trace maps

$\boxed{l_k(\theta, \theta')}$: $(\Omega Q)_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$

define $\lambda \in HC^0((\Omega Q)_{\geq k}, J^k_\# \otimes \Omega B)$

$$ch^{2m}(\theta, \theta') = l_{2m+1}(\theta, \theta') \cdot ch^{2m}(\iota, \iota^\gamma)$$

$$\in HC^{2m}(\Omega A, J^{2m+1}_\# \otimes \Omega B).$$

X version of Nistor construction.

what are the essential points?

want to replace ΩQ by $X(RQ)$

Step 1 Define bifiltration $(F^p_{IQ} X(RQ))_{\geq k} = F^p X_{\geq k}$ of $X(RQ)$ by subcomplexes such that

$$X_{\geq k} \sim \theta((\Omega Q)_{\geq k})$$

Ⓝ ~~This,~~ We will use this, enables us to ~~realize the~~ construct bivariant classes used in the Nistor construction by suitable maps of

Step 2. bifiltration behavior of L_D, γ, h_D.

1) $L_D - k \;:\; F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

2) $\gamma - (-1)^k \;:\; F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

3) $h_D \;:\; F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$

By 1) have

$$1 - \frac{1}{k} L_D \;:\; \mathcal{X}_{\geq k} \longrightarrow \mathcal{X}_{\geq k+1}[2]$$

whence an elt

$$S_k \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$$

By 3) If $\iota_k \in HC^0(\mathcal{X}_{\geq k+1}, \mathcal{X}_{\geq k})$ class map induced by $X_{\geq k+1} \hookrightarrow X_{\geq k}$, then inv. to ι_k up to S, homotopy provided by h_D.

$$\gamma_- \mathcal{X}_{\geq k+1} = \gamma_- \mathcal{X}_{\geq k} \qquad \text{for } k \text{ even}$$

Concentrate:

Step 3: trace map

θ, θ' yield induce

$$Q \longrightarrow L \otimes B \qquad Q_{\geq k} \longrightarrow J^k \otimes B$$

Claim we get then an induced maps

$$X(RQ)_{\geq k} \longrightarrow J_{\#}^k \otimes X(RB)$$
$$F^p X_{\geq k} \longrightarrow J_{\#}^k \otimes F_{IB}^p$$

⓪ Go over Step 3

$$\begin{tikzcd} Q^t \arrow[r] & L^t \otimes B \\ T \arrow[u] \arrow[r] & L^t \arrow[u] \end{tikzcd}$$

homom. of graded algebras.

induced map

$$\begin{tikzcd} X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(R_{L^t}(L^t \otimes B)) \\ \cup & \cup \\ F^p_{I_T(Q^t)} X_T(R_T(Q^t)) & F^p_{I_{L^t}(L^t \otimes B)} X_{L^t}(R_{L^t}(L^t \otimes B)) \end{tikzcd}$$

might be clearer if you broke it.

$$Q^t \longrightarrow L^t \otimes B$$

induces

$$\begin{tikzcd} R_T(Q^t) \arrow[r] & R_{L^t}(L^t \otimes B) = L^t \otimes RB \\ \cup & \cup \\ I_T(Q^t) \arrow[r] & I_{L^t}(L^t \otimes B) = L^t \otimes IB \end{tikzcd}$$

induces

$$\begin{tikzcd} X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(L^t \otimes RB) = L^t_\eta \otimes X(RB) \\ \cup & \cup \\ F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r] & F^p_{L^t \otimes IB} X(L^t \otimes RB) = L^t_\eta \otimes F^p_{IB} \end{tikzcd}$$

Thus get

$$\begin{tikzcd} X(RQ)^t \arrow[r] & L^t_\eta \otimes X(RB) \\ \cup & \cup \\ (F^pX)^t & L^t_\eta \otimes F^p_{IB} \end{tikzcd}$$

yielding desired result.

Ⓟ There's some confusion in my mind. Suppose I start out with the end step. I know Nistor's class is given by the map of supercomplexes.

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "\alpha v_*"] \arrow[d, "P_m(\iota_D)\gamma_-"] & S_\natural \otimes X(RB) \arrow[d, "P_m(t\partial_t)\gamma^t_-"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r, "\alpha v_*"] \arrow[dr, "l_{2m+1}(\theta,\theta')"'] & S_{\natural,\geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
& & J^{2m+1}_{\#} \otimes X(RB)
\end{tikzcd}$$

What is v?

$$\begin{tikzcd}
A \arrow[r, "p+tq"] \arrow[d, "\iota"] & S \otimes B \\
Q \arrow[ur, "\xi"] \arrow[r, "t^D"'] & Q^{t,\geq 0} \arrow[u, "(\theta,\theta')^{t,\geq 0}"']
\end{tikzcd}
\qquad \xi = (\theta,\theta')^t_* t^D$$

Concretely $\quad Q \xrightarrow{\xi} L \otimes B$

$$a_0 da_1 \cdots da_n \longmapsto pa_0 qa_1 \cdots qa_n$$

MATHS INST

~~Magdalen College~~

Prof. D.G. Quillen,

What is going on is that we need a notation. I have

$$\begin{tikzcd}
A \arrow[rr, "p+tq"] \arrow[dr, "\iota"'] & & S \otimes B \\
& Q \arrow[ur, "\rho"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
RA \arrow[rr, "u"] \arrow[dr, "\iota_*"'] & & S \otimes RB \\
& RQ \arrow[ur, "v"'] &
\end{tikzcd}$$

Ⓠ So what do we have?

θ, θ' induce

$$\begin{tikzcd}
Q \arrow[r, "t^D"] \arrow[dr] & Q^t \arrow[r, "(\theta,\theta')"] \arrow[d, phantom, "\cup"] & L^t \otimes B \arrow[d, phantom, "\cup"] \\
 & Q^{t,\geq 0} \arrow[r] & S \otimes B
\end{tikzcd}$$

Anyway what do we have?
We have defined the trace map in a certain way namely

$$Q^t \longrightarrow L^t \otimes B$$

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, equal] \\
X(RQ)^t \arrow[r, "\circledast"] & L^t_\# \otimes X(RB)
\end{tikzcd}$$

so get $X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$ $\forall k$.

Next point however is that $*$ is T module map which means

$$Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

$$X(RQ) \xrightarrow{t^{LD}} X(RQ)^t$$

Sometimes college Non-University Teaching Officer (NUTO) Fellows are invited by the Faculty to participate in University lectures and classes, though neither the Faculty nor the Fellow has any formal obligation in this respect.

College Fellowship

College Fellowship carries an obligation to undertake various duties and offices. This will inevitably mean attending various Committees and, may in due course, entail the undertaking of such offices as Tutor, Librarian etc, for which an additional pensionable stipend is payable.

A Fellow will receive free rooms in College and dining rights.

Ⓡ Point:

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, equal] \\
X(RQ)^t \arrow[r, "*^t"] & L^t_{\hbar} \otimes X(RB)
\end{tikzcd}$$

$*$ is a T-module map and we have

$$T \otimes X(RQ) \xrightarrow{\sim} X(RQ)^t$$

this says something about l_k, namely

$$\cancel{X(RQ)_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n}$$

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r, "l_k"] & J^k_\# \otimes X(RB) \\
X(RQ)_{\geq k'} \arrow[u, phantom, "\cup"] \arrow[r, "l_{k'}"] & J^{k'}_\# \otimes X(RB) \arrow[u, "\iota_\# \otimes 1"']
\end{tikzcd}$$

whence $l_k \mid X(RQ)_n$ is

$$\begin{tikzcd}
X(RQ)_n \arrow[r, "l_n"] & J^n_\# \otimes X(RB) \arrow[d] \\
 & J^k_\# \otimes X(RB)
\end{tikzcd}$$

Ⓢ there seems to be no problem here but you must keep on going over things.

~~Consider~~

8/22 0404 Repeat the steps.

Consider $X(RQ)$ with L_D, h_D, γ

Define byfiltration $F^P X_{\geq k} = (F^P_{IQ} X(RQ))_{\geq k}$

~~Th~~ Prop 1. says $\mathcal{X}_{\geq k} \sim \theta((\Omega Q)_{\geq k})$

Second prop

$$\gamma_- \mathcal{X}_{\geq 2j+1} = \gamma_- \mathcal{X}_{\geq 2j}$$

$$1 - \frac{1}{2j-1} L_D : \gamma_- \mathcal{X}_{\geq 2j-1} \longrightarrow \gamma_- \mathcal{X}^{[2]}_{\geq 2j} = \gamma_- \mathcal{X}^{[2]}_{\geq 2j+1}$$

~~Hith~~

$$s'_{2j-1} \in HC^2(\gamma_- \mathcal{X}_{\geq 2j-1}, \gamma_- \mathcal{X}_{\geq 2j+1})$$

inverse ~~up to~~ up to S of the class ~~by~~ of inclusion

Ⓣ $X_A \xrightarrow{\iota} X_Q \xrightarrow{\gamma_-} \gamma_- X_Q = \cancel{2}\, \gamma_- X_{\geq 1}$

third result about ~~test~~ trace map

θ, θ' give rise to $Q \longrightarrow L \otimes B$

$Q_{\geq k} \longrightarrow J^k \otimes B$

$$Q^t \longrightarrow L^t \otimes B$$

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[r, phantom, "="] & X_{L^t}(L^t \otimes RB) \arrow[d, equal] \\
X(RQ)^t & \boxed{L^t_\natural \otimes X(RB)} \arrow[u] \arrow[r] & {}
\end{tikzcd}$$

in deg k:

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & F^p_{L^t \otimes IB} X_{L^t}(L^t \otimes RB) \arrow[d, equal] \\
(F^p X)^t & L^t_\natural \otimes F^p_{IB}
\end{tikzcd}$$

Ⓤ Repeat:

Θ, θ give rise to a homom of filt algs

$$Q \longrightarrow L\otimes B \qquad Q_{\geq k} \longrightarrow J^k\otimes B \quad \forall k$$

whence
$$Q^t \longrightarrow L^t\otimes B$$

$$R_T(Q^t) \longrightarrow R_{L^t}(L^t\otimes B) = L^t\otimes RB$$

$$\begin{array}{ccccc} X(RQ)^t = & X_T(R_T(Q^t)) & \longrightarrow & X_{L^t}(L^t\otimes RB) & = L^t_\natural \otimes X(RB) \\ & \cup & & \cup & \cup \\ (F^pX)^t = & F^p_{I_T(Q^t)} X_T(R_T(Q^t)) & \longrightarrow & F^p_{L^t\otimes IB} X(L^t\otimes RB) & = L^t_\natural \otimes F^p_{IB} X(RB) \end{array}$$

i.e.

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

$$F^pX_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB} X(RB)$$

$$\mathcal{X}_{\geq k} \longrightarrow J^k_\# \otimes \mathcal{X}_B \qquad \ell_k \in HC^0(\mathcal{X}_{\geq k}, J^k_\# \otimes \mathcal{X}_B)$$

Ⓥ So we have the classes

$$\gamma_- \cdot \iota_* \quad Ch^0(\iota, \iota^\gamma) \in HC^0(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 1})$$

$$s'_{2j-1} \in HC^2(\gamma_- \mathcal{X}_{\geq 2j-1}, \gamma_- \mathcal{X}_{\geq 2j+1})$$

$$Ch^{2m}(\iota, \iota^\gamma) = s'_{2m-1} \cdot s'_{2m-3} \cdots s'_1 \cdot \gamma_- \cdot \iota_*$$

$$\in HC^{2m}(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 2m+1})$$

$$l_{2m+1} \in HC^0(\mathcal{X}_{\geq 2m+1}, J_\#^{2m+1} \otimes \mathcal{X}_B)$$

$$Ch^{2m}(\theta, \theta^\gamma) = l_{2m+1} \cdot Ch^{2m}(\iota, \iota^\gamma)$$

$$\in HC^{2m}(\mathcal{X}_A, J_\#^{2m+1} \otimes \mathcal{X}_B)$$

and the ~~[illegible]~~ latter is the map of towers induced by

$$X(RA) \longrightarrow X(RQ) \xrightarrow{\rho_m(L_D)\gamma_-} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{l_{2m+1}} \bar{J}^{2m+1} \otimes X(RB)$$

Ⓦ So the remaining steps is to show this map agrees with mine. ~~So~~

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] \arrow[rr, bend left, "\alpha u_*"] & X(RQ) \arrow[r, "\alpha v_*"] \arrow[d, "P_m(\iota_D)\gamma_1"] & S_{b} \otimes X(RB) \arrow[d, "P_m(t\partial_t)\gamma_-^t"] \arrow[dd, bend left=60, "\mu_m"] \\
 & \gamma_- X(RQ)_{\geq 2m+1} \arrow[r, "\alpha v_*"] & S_{b,\geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
 & & J_{\#}^{2m+1} \otimes X(RB)
\end{tikzcd}$$

here

$$\begin{tikzcd}
A \arrow[r, "\iota"] \arrow[rr, bend left, "p+tq"] & Q \arrow[r] & S \otimes B \\
RA \arrow[r] \arrow[rr, bend left, "u"] & RQ \arrow[r, "v"] & S \otimes RB
\end{tikzcd}$$

lin resp 1, $D \to t\partial_t$

So what needs to be checked is that

(X)

$$X(RQ)_{\geq k} \xrightarrow{\alpha V_*} S_{\natural, \geq k} \otimes X(RB) \xrightarrow{ev_1} J^h_\# \otimes X(RB)$$

agrees with l_k

Something nagging me still about S versus L^t.

~~Take~~ Go back to $Q \longrightarrow S \otimes B$

$$\begin{tikzcd}
Q \arrow[r] \arrow[d, equal] & S \otimes B \arrow[r, phantom, "\subset"] & L^t \otimes B \arrow[d, equal] \\
Q \arrow[r] & Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d, equal] & S_\natural \otimes X(RB) \arrow[r, hook] & L^t_\natural \otimes X(RB) \\
x(RQ) \arrow[r] & X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes Q)) \arrow[d, equal] \\
 & x(RQ)^t & L^t_\natural \otimes X(RB).
\end{tikzcd}$$

Ⓨ It might be a good idea to draw

$$\begin{tikzcd}
Q \arrow[r] \arrow[d] & S \otimes B \arrow[d, phantom, "\cap"] \\
Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ) \arrow[r, "\alpha v_*"] \arrow[d, "t^{LD}"'] & S_\flat \otimes X(RB) \arrow[d, phantom, "\cap"] \\
X(RQ)^t \arrow[r, "\ell^t"] & L^t_\flat \otimes X(RB)
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r] \arrow[d, "t^{LD}"'] & S_{\flat, \geq k} \otimes X(RB) \arrow[d, equal] \\
\bigoplus_{m \geq k} t^m X(RQ)_{\geq m} \arrow[r] & L_{\flat, \geq k} \otimes X(RB)
\end{tikzcd}$$

$k \geq 0$

So here I am puzzled again. Recall that ~~there is no~~ an important ~~step~~ ~~one~~ point is that $L^t_\flat$ is a T-module

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Standing Committee on ad hominem and titular professorships and readerships

Following the meeting of Congregation yesterday it seems best to postpone the meeting of the Committee due to take place on May 24th.

I.M.J.
19.5.93

② Start again, again

Go over from the beginning.

θ, θ' induce hom. of filt. algs.

$$\begin{tikzcd} Q \arrow[r] & L \otimes B \\ Q_{\geq k} & J^k \otimes B \end{tikzcd}$$

whence

$$Q^t \longrightarrow L^t \otimes B$$

$$R_T(Q^t) \longrightarrow R_{L^t}(L^t \otimes B) = L^t_{\flat} \otimes RB$$

$$I_T(Q^t) \longrightarrow L^t \otimes IB$$

$$\begin{tikzcd} X(RQ)^t = X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(L^t \otimes RB) = L^t_{\flat} \otimes X(RB) \\ (F^pX)^t = F^p_{I_T(Q^t)} \arrow[u, phantom, "\cup"] \arrow[r] & F^p_{L^t \otimes IB} \arrow[r] & L^t_{\flat} \otimes F^p_{IB} \arrow[ul, phantom, "\cup"] \end{tikzcd}$$

whence ~~$\otimes$~~ in degree k you get

$$\begin{tikzcd} l_k : X(RQ)_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RB) \\ F^pX_{\geq k} \arrow[u, phantom, "\cup"] \arrow[r] & J^k_{\#} \otimes F^p_{IB} \arrow[u, phantom, "\cup"] \end{tikzcd}$$

whence $l_k \in HC^0(\mathcal{X}_{\geq k}, J^k_{\#} \otimes \mathcal{X}_B)$

On the other hand we have

$$\begin{tikzcd} A \arrow[r, "\iota"] \arrow[rr, bend left, "p+tq"] & Q \arrow[r, "\xi"] & S \otimes B \end{tikzcd}$$

$$\begin{tikzcd} RA \arrow[r, "\iota_*"] \arrow[rr, bend left, "u"] & RQ \arrow[r, "v"] & S \otimes B \end{tikzcd}$$

$\boxed{A}$

$$\begin{tikzcd}
X(RQ) \arrow[rr, "\alpha v_*"] & & S_\natural \otimes X(RB) \\
\cup & & \cup \\
X(RQ)_{\geq k} \arrow[rr, "(\alpha v_*)_{\geq k}"] \arrow[dr, "l_k"'] & & S_{\natural, \geq k} \otimes X(RB) \arrow[dd, "ev"] \\
 & \textcircled{?} & \\
 & & J^k_\# \otimes X(RB)
\end{tikzcd}$$

V comes from $Q \xrightarrow{\xi} S \otimes B$

l_k ——— $Q^t \longrightarrow L^t \otimes B$

square

$$\begin{tikzcd}
Q \arrow[r, "\xi"] \arrow[d, "t^D"'] & S \otimes B \\
 & \cap \\
Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}$$

$$\begin{tikzcd}
Q \arrow[r, "t^D"] & Q^{t, \geq 0} \arrow[r] & S \otimes B \\
 & \cap & \cap \\
 & Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}$$

commutes

$$\begin{tikzcd}
X(RQ) \arrow[r] & X(R(Q^{t \geq 0})) \arrow[r] \arrow[d] \arrow[dd] & S_\natural \otimes X(RB) \\
 & X(RQ)^{t, \geq 0} \arrow[ur] & \cap \\
 & X(RQ)^t \arrow[r, "l^t"] \arrow[u, phantom, "\cap"] & L^t_\natural \otimes X(RB)
\end{tikzcd}$$

get

$$\begin{tikzcd}
X(RQ) \arrow[r, "t^D"] \arrow[rr, bend left, "\alpha v_*"] & X(RQ)^{t, \geq 0} \arrow[r, "l^{t, \geq 0}"] & S_\natural \otimes X(RB) \\
 & \cap & \cap \\
 & X(RQ)^t \arrow[r, "l^t"] & L^t_\natural \otimes X(RB)
\end{tikzcd}$$

$\boxed{B}$ Still very confused

Try again. I need to prove

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r, "(\alpha V_*)_{\geq k}"] \arrow[dr, "l_k"'] & S_{q,\geq k} \otimes X(RB) \arrow[d, "\varpi_1"] \\
 & J^k_\# \otimes X(RB)
\end{tikzcd}$$

commutes. Now

l_k defined by
$$\begin{tikzcd}
 & Q^t \arrow[r] & L^t \otimes B \\
V \arrow[r, dash] & Q \arrow[r, "\xi"] & S \otimes B
\end{tikzcd}$$

These fit into

$$\begin{tikzcd}
Q \arrow[r, "t^D", "\lim."'] & Q^{t,\geq 0} \arrow[r] \arrow[d, "\cap"] & S \otimes B \arrow[d, phantom, "\cap"] \\
 & Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[rr, bend left, "t^{L_D}"] \arrow[rrr, bend left=40, "\alpha V_*"] & X(R(Q^{t,\geq 0})) \arrow[r] & X(RQ)^{t,\geq 0} \arrow[r, Rightarrow] \arrow[d, phantom, "\cap"] & S_q \otimes X(RB) \arrow[d, phantom, "\cap"] \\
 & & X(RQ)^t \arrow[r, "l^t"] & L^t_q \otimes X(RB)
\end{tikzcd}$$

So what do we find - only that αV_* is $l^t t^{L_D}$. But l^t is a T-module map and $t^{L_D} : X(RQ) \to X(RQ)^t$ induces an isom $J \otimes X(RQ) \to X(RQ)^t$. The conclusion is that l^t is the T-module extension of the map $X(RQ) \xrightarrow{\alpha V_*} S_q \otimes X(RB) \subset L^t_q \otimes X(RB)$.

$\boxed{C}$ I should check certain things carefully. First I should check carefully that $S_\natural \longrightarrow L^t_\natural$ is injective.

$$S = L^{t,\geq 0} = \bigoplus_{k\geq 0} t^k J^k$$

$$L^t = \bigoplus_{k\in\mathbb{Z}} t^k J^k$$

S is generated by L and tJ

so $[S,S]$ ~~is generated by~~ $= [L,S] + [tJ,S]$

so $[S,S]_n = \cancel{t^n\left([L,J^n] + [J,J^{n-1}]\right)}$

$$= \begin{cases} t^n\left([L,J^n] + [J,J^{n-1}]\right) & n\geq 1 \\ [L,L] & n=0. \end{cases}$$

so $S_\natural = \bigoplus_{n\geq 0} t^n J^n_\#$

~~But~~ Now L^t is generated by L, t^{-1}, tJ

so $[L^t,L^t] = [L,L^t] + \underbrace{[t^{-1},L^t]}_{0} + [tJ,L^t]$

$\therefore$ $[L^t,L^t]_n = [L,L^t_n] + [tJ,L^t_{n-1}]$

$$= \begin{cases} t^n\left([L,J^n] + [J,J^{n-1}]\right) & n\geq 1. \\ [L,L] \cancel{+[J,L]} & n=0. \end{cases}$$

$\therefore$ $S_\natural = \left(L^t_\natural\right)^{\geq 0}$

$\boxed{D}$ Claim that

$$\begin{tikzcd} X(RQ) \arrow[r, "\alpha V_*"] \arrow[d, "t^{L_D}"'] & S_\natural \otimes X(RB) \arrow[d, phantom, "\cap"] \\ X(RQ)^t \arrow[r, "\ell^t"] & L^t_\natural \otimes X(RB) \end{tikzcd}$$

ⓧ

commutes. Check this carefully. We have commutative

$$\begin{tikzcd} Q \arrow[r, "\xi"] \arrow[d, "D"'] & S \otimes B \arrow[d, phantom, "\cap"] \\ Q^t \arrow[r, "\mathrm{hom}"'] & L^t \otimes B \end{tikzcd}$$

D, ξ linear resp. $\mathbb{C}$.

$$\begin{tikzcd} \mathbb{C} \arrow[r, hook] \arrow[d] & S \arrow[d, hook] \\ T \arrow[r, hook] & L^t \end{tikzcd}$$

This gives

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d] & X_S(R_S(S \otimes B)) \arrow[d] \\ X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(R_{L^t}(L^t \otimes B)) \end{tikzcd}$$

yielding ⓧ. Since $T \otimes X(RQ) \xrightarrow{\sim} X(RQ)^t$ we find ℓ^t is T-mod extension of αV_* essentially.

8/22 - 1226 Now I have to go back again

$$\begin{tikzcd} \mathbb{C} \arrow[r, phantom, "\subset"] \arrow[d, phantom, "\cap"] & S \arrow[d, phantom, "\cap"] \\ T \arrow[r, phantom, "\subset"] & L^t \end{tikzcd}$$

$S/K \simeq L \times_{L/J} L$

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d, "t^{L_D}"'] & S_\natural \otimes X(RB) \arrow[d, phantom, "\cap"] \\ X(RQ)^t \arrow[r] & L^t_\natural \otimes X(RB) \end{tikzcd}$$

$\boxed{E}$ Start again – it seems that only S occurs at the end. The other steps should be doable with the $\mathbb{Z}$-grading.

See how this works. Consider again Q and its structure

grading as v.s. with 1

assoc. filt- | compat. with alg str.
assoc. $\mathbb{Z}/2$ grading |

Let's explore this picture again.

grading : $Q \xrightarrow{t^D} T' \otimes Q$

described by the subspace $t^D Q \subset T' \otimes Q$.

filtration described by the T-submodule

$$\begin{array}{c} T \cdot t^D Q \subset T' \otimes Q \\ \shortparallel \\ Q^t \end{array}$$

which is a subalgebra of $T' \otimes Q$.

Image (t^D) : $RQ \longrightarrow T' \otimes RQ$

describes the grading of RQ.

Image (t^{LD}) : $X(RQ) \longrightarrow T' \otimes X(RQ)$

describe grading of $X(RQ)$.

Summarize facts:

(grading on Q induces a grading on RQ
degree op. is a derivation extending the degree operator on Q, necessarily unique

Pf. $t^D : Q \longrightarrow T' \otimes Q$ lin. resp 1.

induces $RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$

F

image homogeneous as RQ gen by $\rho(Q)$ isom. to RQ under spec.

(filt. on Q induces a filt. on RQ

$Q^t \subset T' \otimes Q$ T-linear resp / comp. grading.

induces $R_T(Q^t) \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$

image is a graded T'-subalgebra, whence get filtration on RQ. ~~[illegible]~~

same works for $X \cdot R$.

$$RQ \longrightarrow T' \otimes RQ$$

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

image gives grading on $X(RQ)$.

$$R_T(Q^t) \longrightarrow T' \otimes RQ$$ graded T-alg map

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

graded T-module map

∴ image gives filtration

~~[illegible]~~

special features if filt assoc. to grading

$T \otimes Q \xrightarrow{\sim} Q^t$ T-module isom.

$\Rightarrow$ $T \otimes RQ \simeq R_T(T \otimes Q) \xrightarrow{\sim} R_T(Q^t) \longrightarrow T' \otimes RQ$

$\Rightarrow$ filt on RQ assoc. to grading

but more, you get $\underline{R_T(Q^t) \simeq (RQ)^t}$.

This is general if filt. ~~given~~ compat. with alg. structure because that isom to even part of $S_T(Q^t) \xrightarrow{\sim} (RQ)^t$.

$\boxed{G}$ Assertions in the case of $Q = QA$.
(more generally, when [illegible] filt. on Q assoc. to grading.

RQ inherits grading/filtration

these are ~~associated deg~~ described by

$$\mathrm{Im}\left(t^D : RQ \longrightarrow T' \otimes RQ\right)$$

$\mathrm{Im}\,(t'$

I need to organize this pretty carefully.

Take $Q = QA$.

grading $t^D Q \subseteq T' \otimes Q$

Take $Q = QA$

has grading as v.s.

assoc. filt. compat. alg structure

grading described by $t^D : Q \longrightarrow T' \otimes Q$

induces grading on RQ, grading on $X(RQ)$

described by induced maps

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

filtration described by $Q^t \subset T' \otimes Q$

induced filt. on RQ, $X(RQ)$ described by images of induced maps

$$R_T(Q^t) \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ).$$

$\boxed{\text{H}}$ But we have

$$T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$\Uparrow$ induced by t^D on Q.

$$\begin{tikzcd} T \otimes RQ \arrow[r, phantom, "="] \arrow[rrr, bend right=20, "\text{induced by } t^D \text{ on } RQ"'] & R_T(T \otimes Q) \arrow[r, "\sim"] & R_T(Q^t) \arrow[r] & T' \otimes RQ \end{tikzcd}$$

this injective, so get

$$T \otimes RQ \overset{\sim}{\text{——}} R_T(Q^t) \overset{\sim}{\text{——}} (RQ)^t$$

Similarly for X.

Summary:

$$\begin{tikzcd} T \otimes X(RQ) \arrow[r, "\sim"] \arrow[r, bend right=30, "\text{induced by } t^D : Q \to Q^t"'] \arrow[rrr, bend right=50, "\text{induced by } t^{h_D}"'] & X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[rr, bend right=30, "\text{induced by } Q^t \subset T' \otimes Q"'] & X(RQ)^t \arrow[r, phantom, "\subset"] & T' \otimes X(RQ) \end{tikzcd}$$

Now use the fact that $Q^t \subset T' \otimes Q$ is a subalgebra, get

$$\begin{aligned} I_T(Q^t) &= \operatorname{Ker}\big(\underset{\|}{R_T(Q^t)} \longrightarrow \underset{\|}{Q^t}\big) \\ (IQ)^t &= \operatorname{Ker}\big((RQ)^t \longrightarrow Q^t\big) \end{aligned}$$

Now look: We have this D around,

extend D to $Q^t \subset T' \otimes Q$

Q^t stable under $t\partial_t = t\partial_t \otimes 1$ and $D = 1 \otimes D$

~~Consider~~ Agree on $t^D Q$

D on Q^t is T linear $1 \to 0$

get D on $R_T(Q^t)$ consistent with $1 \otimes D$ on $T' \otimes Q$

L_D on $X_T(R_T(Q^t))$ consistent with $1 \otimes L_D$

h_D $\qquad\qquad\qquad\qquad$ $1 \otimes h_D$

$\boxed{I}$ need

$$\begin{tikzcd} \phi : R_T(Q^t) \arrow[r] \arrow[d] & \Omega^2_T(R_T(Q^t)) = \Omega^2_T((RQ)^t) \arrow[d] \\ T' \otimes RQ \arrow[r] & \Omega^2_{T'}(T' \otimes RQ) \end{tikzcd}$$

~~[illegible]~~ Note that once $R_T(Q^t) = (RQ)^t$ then by the ~~fact that we~~ result

$*$ $\Omega_T(Q^t) \subset T' \otimes \Omega Q$

applied to RQ we have

$**$ $\Omega_T((RQ)^t) \subset T' \otimes \Omega(RQ)$

$*$ should be

$$\Omega_T(Q^t) \xrightarrow{\sim} (\Omega Q)^t \subset T' \otimes \Omega Q$$

$**$

$$\Omega_T((RQ)^t) \xrightarrow{\sim} \Omega(RQ)^t \subset T' \otimes \Omega(RQ)$$

In particular

$$\begin{tikzcd} R_T(Q^t) \arrow[r, "\phi"] \arrow[d, equal] & \Omega^2_T(R_T(Q^t)) \arrow[d, equal] \\ (RQ)^t \arrow[r] & (\Omega^2(RQ))^t \end{tikzcd}$$

so the ϕ for $R_T(Q^t)$ is equivalent to the fact that ϕ for RQ preserves filtration, something I have checked.

ANNEXE

Colleges invited to bid for the Professorship of American Literature

All Souls College
Balliol College
Brasenose College
Christ Church
Corpus Christi College
Exeter College
Jesus College
Lincoln College
Magdalen College
Merton College
New College
Oriel College
Pembroke College
The Queen's College
St John's College
Trinity College
University College
Wadham College
Worcester College

$\boxed{J}$ So where am I? The point is that D has be lifted from Q to Q^t as T-module, hence there is an extension to deriv D on $R_T(Q^t)$, and L_D on $X_T(R_T(Q^t))$. Moreover there is a ϕ for $R_T(Q^t)$. ~~[illegible]~~!

~~I agree to argue that one~~

I propose to use

$$X_T(R_T(Q^t)) = X(RQ)^t$$

$$F^p_{I_T(Q^t)} = (F^p_I X_Q)^t$$

to prove things about the bifiltration $F^pX_{\geq k}$.

D given on Q extend to $Q^t = T\otimes Q$ ~~go~~ as $1\otimes D$. Then

$$Q^t \subset T'\otimes Q$$

compatible with D on both sides. Now do relative theory for $\left|\begin{array}{l} Q^t \text{ rel } T \\ T'\otimes Q \text{ rel } T' \end{array}\right.$

and you get D on $R_T(Q^t)$ compat w. D on $T'\otimes RQ$

L_D on $X_T(R_T(Q^t))$ —— $1\otimes L_D$ on $T'\otimes$

h_D

So when this is done we have

$$\cancel{}\ h_D, L_D : F^p_{I_T(Q^t)} \longrightarrow F^{p-2}_{I_T(Q^t)}$$

Other point is to ~~consider~~ $t\partial_t$ on Q^t

Big question: If all this stuff is true ~~[illegible]~~ about $X_T(R_T(Q^t))$, then ~~is it possible to write~~ what's the meaning of the ~~[illegible]~~

$\boxed{K}$ Nistor construction?

Repeat: Big Question: We seem to have formulated everything in terms of the relative complex $X_T(R_T(Q^t))$. If this is the case, what is the meaning of Nistor construction in these terms?

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{P_m(L_D)\gamma_-} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{\ell_{2m+1}} J_{\#}^{2m+1} \otimes X(RB)$$

Of course the relative complex $X_T(R_T(Q^t))$ together with $F^P_{I_T(Q^t)} \cdots$ is mainly a tool for handling the filtration. But it's possible that there's something deeper happening. So what might it be?

Instead of $P_m(L_D)\gamma_-$ [redacted] we should replace this by ~~something more happening~~ something like $t^{L_D} : X(RQ) \longrightarrow X(RQ)^t$ followed by some operator like $P_m(t\partial_t)\gamma_-^t$

8/23 - 0532 Go over something from yesterday

grading in Q described by image $t^0 : Q \xrightarrow{1)} T' \otimes Q$

filt. in Q " " " $T \otimes Q \xrightarrow{2)} T' \otimes Q$
unique T-module extn. of t^0

1) induces $RQ \xrightarrow{3)} R_{T'}(T' \otimes Q) = T' \otimes R$
whose image describes the induced grading on RQ

2) induces

$$\begin{tikzcd} R_T(T \otimes Q) \arrow[r] \arrow[d, equal] & R(T' \otimes Q) \arrow[d, equal] \\ T \otimes RQ \arrow[r, "4)"] & T' \otimes RQ \end{tikzcd}$$

whose image describes the induced filt. on RQ

$\boxed{L}$ Similarly get

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

in describing grading

and $$T \otimes X(RQ) = X_T(T \otimes RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

in describes filt.

Summary

$$\begin{tikzcd} T \otimes X(RQ) \arrow[r, "\sim"] \arrow[r, bend right, "\text{induced by } Q \to Q^t"'] \arrow[rrr, bend left, "\text{induced by } t^{L_D}"] & X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[rr, bend right, "\text{induced by } Q^t \subset T' \otimes Q"'] & X(RQ)^t \arrow[r, phantom, "\subset"] & T' \otimes X(RQ) \end{tikzcd}$$

---

big problem is this. You use $X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t$ and $F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \xrightarrow{\sim} (F^pX)^t$ to establish properties of L_D, h_D, γ wrt $(F^pX_{\geq k})$.

You have the map

$$X(RA) \xrightarrow{L_*} X(RQ) \xrightarrow{P_m(L_D)\gamma_-} \gamma_- X_{\geq 2m+1} \xrightarrow{e_{2m+1}} J_{\#}^{2m+1} \otimes X(RB)$$

Together with the bifiltration properties you get the X-version of Nistor const. The question is really whether this map can be understood in a better way using $X_T(R_T(Q^t))$.

$$\boxed{P_m(L_D)\gamma_- = P_m(L_D)\frac{1}{2}\left(1 - (-1)^{L_D}\right)}$$

$$\begin{tikzcd} X(RA) \arrow[r] & X(RQ) \arrow[r, "t^{L_D}"] \arrow[d, "P_m(L_D)\gamma_+"] & X(RQ)^t \arrow[d, "\wr\, P_m(t\partial_t)\gamma^t_-"] & \\ & {} & X(RQ)^t \arrow[r] & L^t_y \otimes X(RB) \end{tikzcd}$$

$\boxed{M}$ Ignore filtration for the moment and try to understand the map

$$X(RQ) \xrightarrow{t^{L_D}} X(RQ)^t \xrightarrow{e^t} L^t \otimes X(RB)$$

is induced by

$$\begin{tikzcd}
Q \arrow[r, "t^D"] \arrow[drr, bend right, "\xi"] & Q^t \arrow[r] & L^t \otimes B \\
& & S \otimes B \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

$$\begin{tikzcd}
C \arrow[r, dash] \arrow[d, dash] & S \arrow[d, dash] \\
T \arrow[r, dash] & L^t
\end{tikzcd}
\qquad
\begin{tikzcd}
Q \arrow[r, "\xi"] \arrow[d, "t^D"'] & S \otimes B \arrow[d, phantom, "\cap"] \\
Q^t \arrow[r] & L^t \otimes B
\end{tikzcd}
\qquad
\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d, "t^{L_D}"'] & S_q \otimes X(RB) \arrow[d, phantom, "\cap"] \\
X(RQ)^t \arrow[r] & L^t_q \otimes X(RB)
\end{tikzcd}$$

~~It seems that $X(RQ$~~

$$\begin{tikzcd}
X(R'A) \arrow[r] & X(RQ) \arrow[r] \arrow[d, "P_m(L_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\gamma^t_-"] \arrow[r, phantom, "\subset"] & L^t_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\gamma^t_-"] \\
& & S_{q, \geq 2m+1} \otimes X(RB) \arrow[r, phantom, "\subset"] & L^t_q \otimes X(RB)
\end{tikzcd}$$

problem seems to be how to translate the result $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

want to look at $L_D - t\partial_t$

on $X_T(R_T(Q^t)) = X(RQ)^t$

Let's see if I can do the following. You have a certain map $X(RQ) \longrightarrow J^{2m+1}_{\#} \otimes X(RB)$ with certain filtration properties, namely $F^p_{IQ} \longrightarrow J^{2m+1}_{\#} \otimes F^{p-2m}_{IB}$ and the question is to understand this. You use as a tool $X_T(R_T(Q^t))$ and it's canonical filtration, e.g.

$X_T(R_T(Q^t)) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_q \otimes X(RB)$

$\boxed{N}$ induced by $Q^t \longrightarrow L^t \otimes B$.

You might ~~use~~ try to use

$$Q \longrightarrow Q^{t,\geq 0} \longrightarrow L^{t,\geq 0}\otimes B$$

$$X(RQ) \longrightarrow X(R(Q^{t,\geq 0})) \longrightarrow S_\natural \otimes X(RB)$$

~~In the end~~ the composition is the effect of $\underset{\text{graded}}{Q \xrightarrow{\xi} S \otimes B}$ lin. resp 1.

$$\begin{tikzcd}
X(RQ) \arrow[r] & X_S(R_S(S\otimes B)) \arrow[r, phantom, "="] & S_\natural \otimes X(RB) \\
 & X_S(S\otimes RB) \arrow[u] \arrow[ur, "\alpha"'] &
\end{tikzcd}$$

So go back to

$$Q \xrightarrow{\xi} S\otimes B \qquad \text{lin. resp 1.}$$

$$RQ \xrightarrow{v} S \otimes RB \qquad IQ \longrightarrow K\otimes RB + S\otimes IB$$

$$X(RQ) \xrightarrow{v_*} X(S\otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB)$$

I don't understand the function of Q. It plays a role like $\mathcal{D}(RA)$

$$\begin{tikzcd}
RA \arrow[r] \arrow[d] & \underbrace{S\otimes RB}_{\text{graded}} \\
\mathcal{D}(RA) \arrow[ur, dotted] &
\end{tikzcd}$$

~~B~~ $RQ \longrightarrow$

$\boxed{D}$ 0858 try to get to the bottom

When you really ask what's going on its this roughly.

$$\begin{tikzcd} X(RA) \arrow[r] & X(RQ) \arrow[r] & X(S \otimes RB) \arrow[r, Rightarrow, "\alpha"] & S_q \otimes X(RB) \arrow[r, "\mu_m"] & J_{\#}^{2m+1} \otimes X(RB) \end{tikzcd}$$

The map and its filtation properties are ~~easiest~~ seen more easily my way via α

Check carefully that

$$\begin{tikzcd} IQ \arrow[r] & K \otimes RB + S \otimes IB \\ RQ \arrow[u, phantom, "\cap"] \arrow[d] & S \otimes RB \arrow[u, phantom, "\cap"] \arrow[d] \\ Q & S/K \otimes B \end{tikzcd}$$

So is

$$\begin{tikzcd} Q \arrow[r] \arrow[dr, "{(\theta,\theta')}"'] & (S/K) \otimes B \\ & (L \times L) \otimes B \arrow[u, phantom, "\cap"] \end{tikzcd}$$

a homom. Yes

So we can forget about A.

So we end up with worrying about

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d] & S_q \otimes X(RB) \arrow[d] \\ {} & {} \end{tikzcd}$$

What can you say about $X(RQ)$ in general? Can you really see everything you want on the ~~left~~ level of $X(RQ)$ with L_D.

~~on the~~ You have L_D, $t\partial_t$ on $X_T(R_T(Q^t)) = X(RQ)^t$ and you know $L_D - t\partial_t$ carries $F^p_{I_T(Q^t)}$ into $t^{-1}F^{p-2}_{I_T(Q^t)}$

$\boxed{P}$ So how does one proceed?

Äp

You know that $L_D - t\partial_t$ carries $(F^pX)^t$ into $t^{-1}(F^{p-2}X)^t$. ~~So you should get from this youse result~~ Actually you know $L_D - t\partial_t : X(RQ)^t \longrightarrow t^{-1}X(RQ)^t$. How does this help? Maybe it doesn't. ~~Can you get P_m~~ You get something for each m. Strange:

$$\begin{tikzcd}
X(RQ)_{\geq 1} \arrow[d] & \\
X(RQ)_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RQ).
\end{tikzcd}$$

Only thing to try would be to put these together. ~~You have~~

1153. Start review

X-version of Nistor's construction

grading of Q

assoc. filt.	comp. with alg. structure
assoc $\mathbb{Z}/k$ grading	

induced gradings $((RQ)_n)$, $(X(RQ)_n)$

induced filtrations $((RQ)_{\geq k})$, $(X(RQ)_{\geq k})$

~~and~~ $(F^p_{IQ}\, X(RQ))_{\geq k}$

properties $X(RQ) \sim \Omega Q$ induces

1. $(F^p_{IQ}\, X(RQ))_{\geq k} \sim F^p((\Omega Q)_{\geq k})$
2. behavior of L_D, h_D, δ wrt $(F^pX_{\geq k})$.
3. $Q \xrightarrow{(\theta,\theta')} L \otimes B$ induces
$F^pX_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB}$

$\boxed{Q}$ $\ni$

$$\begin{tikzcd}
F^p X_{\geq k} \arrow[r] & J^k_\# \otimes F^p_{IB} \\
F^p(\Omega Q)_{\geq k} \arrow[r] \arrow[u, phantom, "\int"] & J^k_\# \otimes F^p \Omega B \arrow[u, phantom, "\int"]
\end{tikzcd}$$

$$\mathcal{X}_{\geq k} = \left(X_{\geq k} / F^p X_{\geq k} \right)_p \sim \Theta\left(\Omega Q_{\geq k} \right)$$

$$L_0 - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$$

says

$$1 - \frac{1}{k} L_0 : \mathcal{X}_{\geq k} \longrightarrow \mathcal{X}_{\geq k+1} \qquad k \geq 1.$$

Go over 3 rd part.

θ, θ' induce $Q \longrightarrow L \otimes B$ hom. f. algs $\mathbb{Z}/_1$

$\qquad\qquad\qquad\quad Q_{\geq k} \longrightarrow J^k \otimes B$

whence $Q^t \longrightarrow L^t \otimes B$ hom of f. algs

where

$$R_T(Q^t) \longrightarrow R_{L^t}(L^t \otimes B) = L^t \otimes RB$$

$$I_T(Q^t) \longrightarrow L^t \otimes IB$$

$$X(RQ)^t = X_T(R_T(Q^t)) \longrightarrow X_{L^t}(L^t \otimes RB) = L^t_\natural \otimes X(RB)$$

$$(F^p X)^t = F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \longrightarrow L^t_\natural \otimes F^p_{IB} X(RB)$$

Moreover

$$\begin{tikzcd}
X(RQ)^t = X_T(R_T(Q^t)) \arrow[r] & L^t_\natural \otimes X(RB) \\
(\Omega Q)^t = \Omega_T(Q^t) \arrow[r, Rightarrow] \arrow[u, phantom, "\int"] & L^t_\natural \otimes \Omega B \arrow[u, phantom, "\int"]
\end{tikzcd}$$

Go over consistency

$$\begin{tikzcd}
Q \arrow[r] \arrow[d, mapsto] & S \arrow[d] & Q \arrow[r, "\xi"] \arrow[d, mapsto, "\ell^p"'] & S \otimes B \\
T \arrow[r] & L^t & Q^t \arrow[r, "\ell^t"] & L^t \otimes B \arrow[u, phantom, "\cap"]
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ) \arrow[r, "\alpha v_*"] \arrow[d, mapsto] & S_\natural \otimes X(RB) \\
X(RQ)^t \arrow[r, "\ell^t"] & L^t_\natural \otimes X(RB) \arrow[u, phantom, "\cap"]
\end{tikzcd}$$

$\boxed{R}$ So what are the difficult points? What do you need for a first draft?

h_D : D

So now I have to straighten this out.

Outline

my const. $\theta, \theta' : A \longrightarrow L \otimes B$ cong mod $J \otimes B$

$$p = \tfrac{1}{2}(\theta + \theta') : A \longrightarrow L \otimes B$$

$$q = \tfrac{1}{2}(\theta - \theta') : \bar{A} \longrightarrow J \otimes B$$

$$p + tq : A \longrightarrow S \otimes B \qquad S = \bigoplus_{n \geq 0} t^n J^n$$

K ideal $(1-t^2) J^2 S \subset S$

curvature $(1-t^2) q^2 : \bar{A}^{\otimes 2} \longrightarrow K \otimes B$

$p + tq$ lin resp 1 induces

$$u : RA \longrightarrow R_S(S \otimes B) = S \otimes RB$$

$$IA \qquad\qquad K \otimes RB + S \otimes IB$$

$$S_\natural = \bigoplus_{n \geq 0} t^n J^n_\# \qquad J^n_\# = \begin{cases} J^n / [J^{n-1}, J] & n \geq 1 \\ L / [L, L] & n = 0. \end{cases}$$

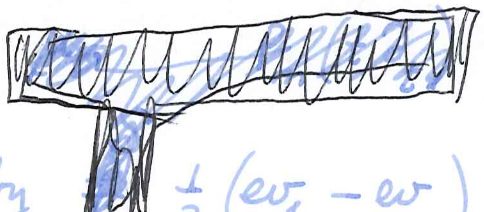
define $\mu_m : S_\natural \longrightarrow J^{2m+1}_\#$

$$\mu_m(t^n x) = \underbrace{P_m(n)}_{\prod_{i=1}^{m} \left(1 - \frac{1}{2i-1} n\right)} \frac{(1-(-1)^n)}{2} \#_{2m+1}(x) \qquad x \in J^n_\#$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{vanishes for } n = 0, \dots, 2m}$$

so this is well-defined. Also it's a trace because it factors through $S_\natural$.

$\boxed{S}$ Next note μ_m ~~is the composition~~ can be written as $P_m(t\partial_t)$ followed



by $\frac{1}{2}(ev_1 - ev_{-1})$ followed by $\#_{2m+1}$

so it vanishes on K^{m+1}.

Then we have

$$X(RA) \xrightarrow{u_*} X(S \otimes RB) \xrightarrow{\alpha} S_q \otimes X(RB) \xrightarrow{\mu_m} J_\#^{2m+1} \otimes X(RB)$$

$$F^p_{IA} \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \sum h(K^i) \otimes F^{p-2i}_{IB} \to J_\#^{2m+1} \otimes F^{p-2m}_{IB}$$

So we get

$$\mathcal{X}_A \longrightarrow J_\#^{-2m+1} \otimes \mathcal{X}_B[2m]$$

i.e. $ch^{2m}(\theta, \theta') \in HC^{2m}(\mathcal{X}_A, J_\#^{-2m+1} \otimes \mathcal{X}_B)$

Nistor construction

$Q = QA$

~~graded as vector space $Q = \oplus Q_n$ where $Q_n = \Omega^n A$, the filtration assoc. and the $\mathbb{Z}/2$ grading assoc. are com~~

$Q = QA$

graded as v.s. $Q = \oplus Q_n$ $\quad Q_n = \Omega^n A$

assoc. filt. | comp. with alg. structure

assoc. $\mathbb{Z}_2$ gr. |

induced filtration $(\Omega Q_{\geq k})$

also. induced autom γ of order 2.

mixed subcomplexes.

$\boxed{\begin{array}{l}\gamma\_\Omega Q_{\geq 2j+1} \\ = \gamma\_\Omega Q_{\geq 2j}\end{array}}$

$\iota_k \in HC^0(\Omega Q_{\geq k+1}, \Omega Q_{\geq k})$

$\exists\ s_k \in HC^2(\Omega Q_{\geq k}, \Omega Q_{\geq k+1})$

$\ni s_k \iota_k = S, \quad \iota_k s_k = S.$

$\boxed{T}$ Replace s_k by $\frac{1}{2}(s_k + \gamma s_k \gamma)$ can assume s_k commutes with γ. Get rest.

$$s'_{2j-1} \in HC^2(\gamma_- \Omega Q_{\geq 2j-1}, \gamma_- \Omega Q_{\geq 2j+1})$$

inverse up to S for the class

$$t'_{2j-1} \in HC^2(\gamma_- \Omega Q_{\geq 2j+1}, \gamma_- \Omega Q_{\geq 2j-1})$$

Define

$$ch^{2m}(\iota, \iota^\gamma) = s'_{2m-1} \cdot s'_{2m-3} \cdots s'_1 \cdot ch^0(\iota, \iota^\gamma)$$

$$\in HC^{2m}(\Omega A, \gamma_- \Omega Q_{\geq 2m+1})$$

$ch^0(\iota, \iota^\gamma)$ = class of

$$\Omega A \xrightarrow{\iota} \Omega Q \xrightarrow{\gamma_-} \gamma \Omega Q = \gamma_- \Omega Q_{\geq 1}$$

finally use $QA = A * A$ to get

$$\left.\begin{matrix} Q \longrightarrow L \otimes B \\ Q_{\geq k} \longrightarrow J^k \otimes B \end{matrix}\right) \text{hom. of filt. algs}$$

induces

$$\Omega Q \longrightarrow \Omega_L(L \otimes B) = L \otimes \Omega B$$

$$\Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

compose with $\#_k : J^k \to J^k_\#$ to get

$$\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$

this turns out to be a map of mixed cx, whence

$$l_k(\theta, \theta') \in HC^0(\Omega Q_{\geq k}, J^k_\# \otimes \Omega B)$$

Define

$$ch^{2m}(\theta, \theta') = l_k(\theta, \theta') \cdot ch^{2m}(\iota, \iota^\gamma)$$

$$\in HC^{2m}(\Omega A, J^k_\# \otimes \Omega B)$$

$\boxed{4}$ Want to compare my construction with Nistor's, so need to develop an <u>X analogue of Nistor construction.</u>

Let's find what it is we have to do. Start with $\Omega Q_{\geq k}$

8/24 - 0528.

X analogue of Nistor construction.

$F^p\Omega Q_{\geq k} = F^p(\Omega Q_{\geq k})$ gives dec. bifilt of ΩQ

Recall that $X(RQ) = \Omega Q$ such $F^p_{IQ}X(RQ) = F^p\Omega Q$

Define $F^p_{IQ}X(RQ)_{\geq k}$, $F^pX_{\geq k}$ for short, to be subspace of $X(RQ)$ corresp to $F^p\Omega Q_{\geq k}$.

properties. 1) bifilt. of $X(RQ)$ by subcomplexes. (because $F^p\Omega Q_{\geq k}$ stable under b, d, κ, etc.)

2) canonical htpy equiv $X(RQ) \sim \Omega Q$ induces $F^pX_{\geq k} \sim F^p\Omega Q_{\geq k}$ $\forall p, k$

$\mathcal{X}_{\geq k} \overset{def}{=} (X_{\geq k}/F^pX_{\geq k}) \sim$ Hodge tower of $\Omega Q_{\geq k}$

3) $p=-1$ get filt. of $X(RQ)$ by subcxs $X_{\geq k} = X(RQ)_{\geq k}$ even subspace RQ, get filtration $RQ_{\geq k} = \Omega^{ev}Q_{\geq k}$ compat with Fedosov product.

4) comment $F^pX_{\geq k} \subset F^p_{IQ} \cap X_{\geq k}$, but $\neq$

goes up with $F^p\Omega Q_{\geq k}$

More sophisticated approach

$T' = \mathbb{C}[t, t^{-1}]$ graded with degree $t = 1$.

$T = \mathbb{C}[t^{-1}]$

Can identify a (dec) filtration $(F^kV)_{k\in\mathbb{Z}}$ of V with the graded T-submodule

$$\bigoplus_{k\in\mathbb{Z}} t^kF^kV \subset T'\otimes V.$$

$\boxed{V}$ ~~[illegible]~~

V vector space

grading of V equivalent to a graded (i.e. homogeneous) subspace $W \subset T'\otimes V$ such that $W \subset T'\otimes V \xrightarrow{t\mapsto 1} V$ is bijection.

(dec) filt. of V equiv. to a graded T-submodule ~~[illegible]~~ $M \subset T'\otimes V$.

I don't need the ~~above~~ graded until 2nd part I think.

Start again: Given V w dec filt $V_{\geq k}$ put

$$V^t = \bigoplus t^k V_{\geq k} \subset T'\otimes V$$

Then V^t is a graded T-submod of $T'\otimes V$. ~~which is equivalent to the filtration~~.
This construction gives an equiv. between filtrations on V and graded T-submodules of $T'\otimes V$.

At this point I find myself recalling many things. Mental state is listing in preparation for something. Actually you should concentrate on assertions.

Have filt. $Q_{\geq k}$ so get ~~[illegible]~~ graded T-subalg ~~[illegible]~~ Q^t of the T'-algebra $T'\otimes Q$.

rel. forms

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T'\otimes Q) = T'\otimes \Omega Q$$

Lemma: Injective (this map)

1-forms

$$\begin{tikzcd}
\Omega^1_T(Q^t) \arrow[r] \arrow[d, shift right] & T'\otimes \Omega^1 Q \arrow[d, shift right] \\
Q^t\otimes_T Q^t \arrow[r] \arrow[u, shift right] & T'\otimes Q\otimes Q \arrow[u, shift right]
\end{tikzcd}$$

$$Q^t\otimes_T Q^t \underset{Q^t \text{ flat}}{\hookrightarrow} Q^t\otimes_T (T'\otimes Q) \underset{T'\otimes Q \text{ flat}}{\hookrightarrow} (T'\otimes Q)\otimes_T (T'\otimes Q)$$

$\boxed{W}$ Conclude $\Omega_T(Q^t) \xrightarrow{\sim} \{\Omega Q\}^t \subset T' \otimes \Omega Q$

~~Next recall [illegible] relative~~

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{T'}(R_{T'}(T' \otimes Q)) \arrow[d, equal] \\
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T' \otimes Q)
\end{tikzcd}$$

$\therefore$ get

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, equal] & X(RQ)^t \arrow[d, equal] \\
\Omega_T(Q^t) \arrow[r, "\sim"] & \text{[illegible]}\,\Omega Q^t
\end{tikzcd}$$

next

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, equal] & (F^pX)^t \arrow[d, equal] \\
F^p(\Omega_T(Q^t)) \arrow[r, "\sim"] & \boxed{(F^p\Omega Q)^t} \quad F^p(\Omega Q^t)
\end{tikzcd}$$

defn of $F^pX_{\geq k}$

rel form of $F^p_{IA}X(RA) = F^p\Omega A$

---

trace map.

$$Q^t \subset T' \otimes Q$$

induces

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega Q \\
F^p(\Omega_T(Q^t)) \arrow[u, phantom, "\cup"] \arrow[r] & {}
\end{tikzcd}$$

No not trace map

You must start with

$$Q^t \longrightarrow L^t \otimes B$$

and then you get a cube

$\boxed{X}$

$$\begin{tikzcd}
& X_T(R_T(Q^t)) \arrow[rr] \arrow[d, equal] & & L^t_\phi \otimes X(RB) \arrow[d, equal] \\
& \Omega_T(Q^t) \arrow[rr] & & L^t_\phi \otimes \Omega B \\
F^p_{I_T}(Q^t) \arrow[rr] \arrow[d, equal] \arrow[uur, hook] & & L^t_\phi \otimes F^p_{IB} \arrow[d, equal] \arrow[uur, hook] & \\
F^p(\Omega_T(Q^t)) \arrow[rr] \arrow[uur, hook] & & L^t_\phi \otimes F^p\Omega B \arrow[uur, hook] &
\end{tikzcd}$$

vertical arrows are canonical idents.

& in the case of $\left|\begin{array}{l} Q^t \text{ rel } T \\ L^t \otimes B \text{ rel } L^t \end{array}\right.$

can write this

$$\begin{tikzcd}
& X(RQ)^t \arrow[rr] \arrow[d, equal] & & L^t_\phi \otimes X(RB) \arrow[d, equal] \\
& \Omega Q^t \arrow[rr] & & L^t_\phi \otimes \Omega B \\
(F^pX)^t \arrow[rr] \arrow[d, equal] \arrow[uur, hook] & & L^t_\phi \otimes F^p_{IB} \arrow[d, equal] \arrow[uur, hook] & \\
F^p(\Omega Q^t) \arrow[rr] \arrow[uur, hook] & & L^t_\phi \otimes F^p\Omega B \arrow[uur, hook] &
\end{tikzcd}$$

$\boxed{Y}$ 8/25-0511

X version of Nistor construction.

$\Omega Q_{\geq k}$, γ already defined as well as trace map

$$\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$

maps of mixed complexes.

~~introduce~~ consider superc.x + subcomplexes $X(RQ)$ $F^p_{IQ} X(RQ)$ at this point

~~claim the filtration $(Q_{\geq k})$ induces a filtrations on these compatible with~~

claim these inherit filtrations from the filtration $(Q_{\geq k})$ compat with structure

define $F^p \Omega Q_{\geq k} = F^p(\Omega Q_{\geq k})$

bifiltation of ΩQ

Define

$$\left(F^p_{IQ} X(RQ)\right)_{\geq k} = F^p \Omega Q_{\geq k}$$

I would like to say RQ

Too confused still, again I am trying to mix motivation, defintion, and ~~results~~ proofs, and my mind can't handle it.
first get logical structure straight with all the steps.

In the Nistor construction have introduced filtration + γ on Q and the induced filtration & γ on ΩQ as well as trace maps

$$l_k : \Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B \qquad \forall k$$

Ignore γ first.

$\boxed{2}$ objects Q, $Q_{\geq k}$, ΩQ, $\Omega Q_{\geq k}$, l_k

relation: filt. $(Q_{\geq k})$ comp. w. alg st. on Q

filt $(\Omega Q_{\geq k})$ " | DG ——— ΩQ
| operators b, d, κ.

l_k comp. as k varies.

l_k map of mixed complexes.

---

object

RQ, IQ, $X(RQ)$, $F^p_{IQ}X(RQ)$

$RQ_{\geq k}$, $X_{\geq k}$, $F^pX_{\geq k}$, $\mathcal{X}_{\geq k}$

relation: canon. ident. $X(RQ) = \Omega Q$

identifies $F^pX_{\geq k} = F^p(\Omega Q_{\geq k})$.

also for can ~.

$\mathcal{X}_{\geq k} \sim$ Hodge tower of $\Omega Q_{\geq k}$.

$$l_k : F^pX_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB}X(RB)$$

$$\mathcal{X}_{\geq k} \longrightarrow J^k_{\#} \otimes \mathcal{X}_B$$

0832 X-version

first point: $(Q_{\geq k})$ induces filtration on ΩQ

RQ, $X(RQ)$, $F^p_{IQ}X(RQ)$ consistent with structure

Also $F^pX(\;)_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB}X(RB)$

All you can do is make this point and list examples. Diagram you want is

$$\begin{tikzcd}
F^pX_{\geq k} \arrow[r] \arrow[d, "\sim"'] & J^k_{\#} \otimes F^p_{IB} \arrow[d, "\sim"] \\
F^p\Omega Q_{\geq k} \arrow[r] & J^k_{\#} \otimes F^p\Omega B
\end{tikzcd}$$