a 7/25 -0635

~~Set up~~ Set up Nistor character for the universal quasi-hom.

Preliminaries.

Q alg $\Rightarrow$ we have $\Omega = \Omega Q$, $F^p\Omega$, $R = RQ$, $I = IQ$

$X = X(RQ)$, $F^pX = F^p_{IQ}X(RQ)$.

explicit heq $X \sim \Omega$ $\quad F^p\Omega \sim F^pX$.

Q filtered alg., dec. filt by subspaces $Q_{\geq k}$

$1 \in Q_{\geq 0}$, $\quad Q_{\geq i} \cdot Q_{\geq j} \subset Q_{\geq i+j}$

Claim: $\Omega, F^p\Omega, R, I, X, F^pX$ inherit filtrations $\Omega_{\geq k}$, $F^p\Omega_{\geq k}$ compatible with structure.

In particular $\mathcal{X}_{\geq k} = \left(X_{\geq k}/F^pX_{\geq k}\right) \sim \theta\left(\Omega_{\geq k}\right)$.

Assume v.s grading $Q = \oplus Q_n$, $1 \in Q_0$ $\ni$ $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$.

Claim: R, X inherit gradings.

D on Q, R, L_D, h_D on X

Suppose $Q = Q_{\geq 0} \supset Q_{\geq 1} \supset \cdots \supset Q_{\geq N} = Q_{\geq N+1} = \cdots$

Define $D = N$ on $Q_{\geq N}$.

Observe $h_D, L_D : F^pX \longrightarrow F^{p-2}X$

$h_D, L_D : X_{\geq k} \longrightarrow X_{\geq k}$

Thus $h_D, L_D : F^pX \cap X_{\geq k} \longrightarrow F^{p-2}X \cap X_{\geq k}$

$$\begin{array}{ccc} F^pX \cap X_{\geq k} & \longrightarrow & F^{p-2}X \cap X_{\geq k} \\ \cup & & \cup \\ F^pX_{\geq k} & \overset{?}{\dashrightarrow} & F^{p-2}X_{\geq k} \end{array}$$

But

$$F^pX \cap X_{\geq k} \xrightarrow{L_D - k} F^{p-2}X \cap X_{\geq k+1}$$

same problem holds. What next?

Review:

Q alg $\Longrightarrow$ we have $\Omega = \Omega Q$, $F^p\Omega$, R, I, X, F^pX

Assume Q filtered $\Rightarrow$ we have $\Omega_{\geq k}$, $F^p\Omega_{\geq k}$, etc.

$$\mathcal{X}_{\geq k} = \left(X_{\geq k}/F^pX_{\geq k}\right) \sim \Theta(\Omega_{\geq k})$$

Next suppose splitting of filtration given

$Q = \bigoplus Q_n$, $1 \in Q_0$, $\ni$ $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$.

D on Q, R, L_D, h_D on X:

$$L_D, h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$$

Take D = degree op $\quad D = n$ on Q_n

$$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$$

$k \neq 0$. $\quad S_k = 1 - k^{-1}L_D$:

$$\begin{tikzcd}
\mathcal{X}_{\geq k+1} \arrow[r, "S"] \arrow[d, "i_k"'] & \mathcal{X}_{\geq k+1}[2] \arrow[d, "i_k"] \\
\mathcal{X}_{\geq k} \arrow[r, "S"] \arrow[ur, "S_k"] & \mathcal{X}_{\geq k}[2]
\end{tikzcd}$$

c There's a problem with what to say.
Consider

Monday 7/26 — 0906

Review. $Q = QA$ has structure

filtration dec.: $Q_{\geq k}$

grading $Q = \bigoplus Q_n$

superalg: γ auto, $\gamma^2 = 1$

relations. filtr compatible with superalg structure:

$$Q_{\geq i} Q_{\geq j} \subset Q_{\geq i+j}, \quad 1 \in Q_{\geq 0}, \quad \gamma Q_{\geq k} = Q_{\geq k}$$

filt cmp with grading

$$Q_{\geq k} = \bigoplus_{n \geq k} Q_n, \quad 1 \in Q_0.$$

grading + γ compatible: $\gamma = (-1)^n$ on Q_n

$X = X(RQ)$ $\qquad F^pX = F^p_{IQ} X(RQ)$

D degree op on Q, extends to deriv. D on RQ

$L_D = L(1, D)$, $h_D = h^\phi(1, D)$ on X, ϕ canonical $RQ \longrightarrow \Omega^2(RQ)$.

Assert:

$$h_D : F^pX_{\geq k} \longrightarrow F^{p-2}_{\geq k} \qquad \text{same for } L_D = [\partial, h_D]$$

$$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}_{\geq k+1}$$

$$\gamma - (-1)^k : F^pX_{\geq k} \longrightarrow F^pX_{\geq k+1}.$$

d

$$\begin{tikzcd}
F^pX_{\geq k+1} \arrow[r, "\delta"] \arrow[d, "\iota_k"'] & F^{p-2}X_{\geq k+1} \arrow[d, "\iota_k"] \\
F^pX_{\geq k} \arrow[r] \arrow[ur, "s_k"] & F^{p-2}X_{\geq k}
\end{tikzcd}$$

$s_k = 1 - \frac{1}{k} L_D$. Observe

$$\delta - s_k \circ \iota_k = \frac{1}{k} L_D = [\partial, \tfrac{1}{k} h_D] : F^pX_{\geq k+1} \longrightarrow F^{p-2}X_{\geq k+2}$$

There's a question of how to say this efficiently.

7/26 - 1032

I have to find the steps then organize efficiently. So let's start with essential assertions.

$$1 - \frac{1}{k} L_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1} \quad \forall p.$$

$$L_D = [\partial, h_D] \quad \text{where} \quad h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k} \quad \forall p, k$$

op. $s_k \equiv 1 - \frac{1}{k} L_D$ yields

$$[s_k] \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$$

& $[\iota_k] \in HC^0(\mathcal{X}_{\geq k+1}, \mathcal{X}_{\geq k})$

e Try again.

$$1-\frac{1}{k}L_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1} \qquad \forall p$$

so $s_k = 1-\frac{1}{k}L_D$ yields a class

$$[s_k] \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$$

The inclusion $\iota_k : X_{\geq k+1} \longrightarrow X_{\geq k}$ yields

$$[\iota_k] \in HC^0(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$$

One has

$$[s_k][\iota_k] = S \in HC^2(\mathcal{X}_{\geq k+1}, \mathcal{X}_{\geq k+1})$$

$$[\iota_k][s_k] = S \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k})$$

since $L_D = [\partial, h_D]$ where $h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$ $\forall p, k$.

Def of Nistor.

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma^-} X(RQ)$$

$$\gamma^- \iota_* = \frac{1}{2}(\iota_* - \iota_*^\gamma)$$

1115 $ch^0 \in HC^0(\mathcal{X}_A, \gamma^- \mathcal{X}_{\geq 1})$

$$ch^{2m} \in HC^0(\mathcal{X}_A, \gamma^- \mathcal{X}_{QA, \geq 2m+1})$$

$$ch^{2m} = [S^-_{2m-1}][S^-_{2m-3}] \cdots [S^-_1]\, ch^0$$

f What's happening is that I have a mental problem summarizing things, listing only the important points, deciding between different notations for the same things. Different notations. I can ~~speak of~~ write

$$S_k \, , \quad 1-\tfrac{1}{k}L_D \quad \left(\begin{array}{l} \text{operator on } X \\ \text{carrying } F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1} \\ \qquad\qquad\quad \forall p \end{array} \right.$$

$$[S_k] : \mathcal{X}_{\geq k} \longrightarrow \mathcal{X}_{\geq k+1}[2]$$

$$[S_k] \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+2})$$

I have to decide between maps and bivariant classes.

Let's begin again. To construct Nistor's bivariant Chern character for the univ. quasi-hom.

$$Ch^{2m} \in HC^{2m}(A, \Omega(QA)_{\geq 2m+1})$$

Then to construct for $A \rightrightarrows L \otimes B$ cong mod $J \otimes B$ ~~class~~ biv. classes

$$ch^{2m}(\theta, \theta') \in HC^{2m}(\Omega A, J_{\#}^{2m+1} \otimes \Omega B)$$

9 To construct Nistor's

$$\begin{array}{c} ch^{2m}(\theta,\theta') \in HC^{2m}(A^b, J_{\#}^{2m+1}\otimes B^b) \\ \downarrow S \\ HC^{2m+2}(A^b, J_{\#}^{2m+1}\otimes B^b) \\ \uparrow \iota_{\#} \\ ch^{2m+2}(\theta,\theta') \in HC^{2m+2}(A^b, J_{\#}^{2m+3}\otimes B^b) \end{array}$$

$$\begin{tikzcd} \mathcal{X}_A \arrow[r, "ch^{2m}"] \arrow[d, "ch^{2m+2}"'] & J_{\#}^{2m+1}\otimes \mathcal{X}_B[2m] \arrow[d, "S"] \\ J_{\#}^{2m+3}\otimes \mathcal{X}_B[2m+2] \arrow[r, "\iota_{\#}"] & J_{\#}^{2m+1}\otimes \mathcal{X}_B[2m+2] \end{tikzcd}$$

Proceed as follows

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X_{\geq 0}(RQ) = \gamma_- X_{\geq 1}$$

$$\xrightarrow{s_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$$\xrightarrow{s_{2m-1}} \gamma_- X_{2m} = \gamma_- X_{\geq 2m+1}.$$

In down to earth terms

$$P_m(L_D)\gamma_- \iota_*$$

$$X(RA) \longrightarrow X(RQ) \xrightarrow{\gamma_-} \gamma_- X_{\geq 1} \xrightarrow{s_1} \gamma_- X_{\geq 3} \longrightarrow \cdots \longrightarrow \gamma_- X_{\geq 2m+1}$$

h Composition

$Ch^{2m} \mathcal{I} = [S_{2m-1}][S_{2m-3}] \cdots [S_1][\gamma_- \iota_*]$

$\in HC^{2m}(A^b, Q^b_{\geq 2m+1})$

~~What~~ 7/26 - 1629

Start going over the proofs.

Assert. Q alg with filtration $Q_{\geq k}$, $k \in \mathbb{Z}$
+ grading $Q = \oplus Q_n$. Assume

1) filt comp. with alg structure

2) grading ~~...~~ splits the filtration

D derivation of RQ $\ni$ $D = n$ on Q_n | h_D on X assoc. to D

$X = X(RQ)$, $F^p X_{\geq k} = \left(F^p_{IQ} X(RQ)\right)_{\geq k}$ induced filtration

Then a) $h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$, hence also for $L_D = [\partial, h_D]$

b) $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

<u>Proof:</u> $Q^t = \bigoplus_{k \in \mathbb{Z}} t^k Q_{\geq k} \subset \mathbb{C}[t, t^{-1}] \otimes Q$

$Q^t \otimes_T \overset{n}{\cdots} \otimes_T Q^t \subset \mathbb{C}[t, t^{-1}] \otimes Q^{\otimes n}$

$\Omega^n_T Q^t \subset \mathbb{C}[t, t^{-1}] \otimes \Omega^n Q$

$\boxed{R_T Q^t \subset \mathbb{C}[t, t^{-1}] \otimes RQ}$

$X(R_T Q^t) \subset \mathbb{C}[t, t^{-1}] \otimes X(RQ)$.

i ~~Define~~ Consider

$$F^{p}_{I_T Q^t} X_T(R_T Q^t) =$$

$p = 2n+1$ $$(I_T Q^t)^{n+1} \leftrightarrows \natural\left((I_T Q^t)^n d(I_T Q)\right)$$

$p = 2n$ $$(I_T Q)^{n+1} + [(I_T Q)^n, R_T] \leftrightarrows \natural\left((I_T Q^t)^n d R_T Q)\right)$$

In degree k

$$(I_T Q^t)^n_{(k)} = \sum_{\sum k_i = k} (I_T Q^t)_{(k_1)} \cdots (I_T Q^t)_{(k_n)}$$

or $$(IQ^n)_{\geq k} = \sum_{\sum k_i = k} (IQ)_{\geq k_1} \cdots (IQ)_{\geq k_n}$$

A basic fact is that

$IQ_{\geq k} / IQ_{\geq k+1}$ spanned by

$$\rho(x_0)\,\omega(x_1, x_2) \cdots \omega(x_{2n-1}, x_{2n}) \qquad n \geq 0$$

x_0 homogeneous $\quad \sum |x_i| = k$

And you know

$$L_D\, \omega(x_1, x_2) = \rho\left(D(x_1 x_2) - Dx_1\, x_2 - x_1 D x_2\right) + \omega(Dx_1, x_2) + \omega(x_1, Dx_2)$$

$$(L_D - k)\,\omega(x_1, x_2) = \rho\Big(\cancel{\quad x_1, x_2}$$

higher components of $D(x_1 x_2)$

j This it should be clear

The good proof is ~~that~~ to look at

$I_T Q^t = (IQ)^t$ and $F^P_{I_T Q^t} X_T (R_T Q^t) = (F^P X)^t$

Know that $h_D F^P \subset F^{P-2}$ where $F^P = F^P_{I_T Q^t}$ etc.

$\therefore h_D(F^P X_{\geq k}) \subset F^{P-2} X_{\geq k}$.

Also we know that $L_D - D_t$

$$D - t\partial_t : (RQ)^t \longrightarrow t^{-1} RQ^t$$

$RQ^t = R_T Q^t$ gen. by $\rho(Q^t) = \sum t^n \rho(Q_n)$.

~~and~~ Also ~~...~~ t^{-1}. Then

$$(D - t\partial_t)(t^n \rho(x_n)) = t^n n \rho(x_n) - n t^n \rho(x_n) = 0.$$

But $(D - t\partial_t)(t^{-1}) = -t(-t^{-2}) = t^{-1}$.

From this we conclude that

~~$L_D - t\partial_t$~~ $L_D - t\partial_t : (F^P X)^t \longrightarrow t^{-1}(F^{P-2} X)^t$

i.e. $L_D - k : F^P X_{\geq k} \longrightarrow F^{P-2} X_{\geq k+1}$

Thus all is clear.

Next consider ~~...~~ $(-1)^{L_D}$ $(-1)^{t\partial_t}$

These are autos of Q^t that agree on $\rho(Q^t)$
$= \sum t^n \rho(Q_n)$. $(-1)^{L_D}(t^{-1}) = t^{-1}$

$(-1)^{t\partial_t} t^{-1} = -t^{-1}$

k Look carefully: we have two autos γ and $(-1)^{t\partial_t}$ of X^t preserving $F^p X^t$. Now about these autos we know that

$$\begin{matrix} Q_0 & Q_1 & Q_2 \\ & tQ_1 & tQ_2 \\ & & t^2 Q_2 \end{matrix}$$

~~Look~~ These autos commute. The product minus the identity is divisible by t^{-1}, but we need to check this for the submodule $F^p X^t$

$\gamma' - 1$

7/27 0557

Joachim's Construction

$Q = QA \qquad Q_n = \Omega^n A \qquad Q_{\geq k} = \Omega^{\geq k} A$

$(\Omega^n Q)_{\geq k}$ spann. $x_0\, dx_1 \ldots dx_n \qquad \sum \operatorname{ord}(x_i) \geq k.$

The confusion in my mind. 0622

Let's try to list the main points.

induced filtration

map $X(RA) \longrightarrow X(RQ)_{\geq 2m+1}$

$\cup \qquad\qquad \cup$

$\ni \quad F^p_{IA} X(RA) \longrightarrow F^{p-2m}_{IQ} X(RQ)_{\geq 2m+1} \qquad \forall p$

gives a class in $HC^{2m}(A^b, Q^b_{\geq 2m+1})$

l main points

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X(RQ) = \gamma_- X(RQ)_{\geq 1}$$

$$\xrightarrow{1-L_D} \gamma_- X(RQ)_{\geq 2} = \gamma_- X(RQ)_{\geq 3}$$

$$\cdots\cdots$$

$$\xrightarrow{1-\frac{1}{2m-1}L_D} \gamma_- X(RD)_{\geq 2m} = \gamma_- X(RQ)_{\geq 2m+1}.$$

~~What~~ objects and relations.

I have to study the end map in more detail

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "\diamond"] \arrow[d, "\gamma_-"] & S_q \otimes X(RB) \arrow[d] \\
& \gamma_- X(RQ)_{\geq 1} \arrow[d, "P_m(L_D)"] & \pi_- S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r, "\diamond"] & \pi_- S_{q,\geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
& & J_{\#}^{2m+1} \otimes X(RB)
\end{tikzcd}$$

$$\diamond : X(RQ) \xrightarrow{t^{L_D}} \bigoplus_{n\geq 0} t^n X(RQ)_{\geq n} \xrightarrow{\text{end map}} S_q \otimes X(RB).$$

I need

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\diamond} S_q \otimes X(RB)$$

same as $X(RA) \xrightarrow{\iota_*} X(S \otimes RB) \xrightarrow{\alpha} S_q \otimes X(RB)$

$$X(RQ)_{\geq k} \xrightarrow{\diamond} S_{q,\geq k} \otimes X(RB)$$

m

$$\begin{tikzcd}
X(RQ) \arrow[r, "t^{L_D}"] \arrow[d, "P_m(L_D)\gamma_-"] & \bigoplus_{k\geq 0} t^k X(RQ)_{\geq k} \arrow[r, "E"] \arrow[d, "P_m(t\partial_t)\pi_-"] & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
\gamma_-^{\theta} X(RQ)_{\geq 2m+1} \arrow[r, "t^D"] & \bigoplus_{\substack{k \text{ odd} \\ k\geq 2m+1}} t^k X(RQ)_{\geq k} \arrow[r, "E"] \arrow[d, "ev_1"] & \pi_- S_{q, \geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
 & X(RQ)_{\geq 2m+1} \arrow[r, "E"] & J_\#^{2m+1} \otimes X(RB)
\end{tikzcd}$$

Note that this diagram does not mention the IQ adic filtration. E the end map is a family of maps $X_{\geq k} \xrightarrow{E_k} J_\#^k \otimes X(RB)$ which are compatible in the sense that

$$\begin{tikzcd}
X_{\geq k+1} \arrow[r, "E_{k+1}"] \arrow[d, phantom, "\cap"] & J_\#^{k+1} \otimes X(RB) \arrow[d, "\iota_\# \otimes 1"] \\
X_{\geq k} \arrow[r, "E_k"] & J_\#^k \otimes X(RB)
\end{tikzcd}$$

commutes. Commutativity of diagram at the top of this page is then clear.

n

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r, "t^{L_D}"] \arrow[d, "P_m(L_D)\gamma_*"] & \bigoplus\limits_{k\geq 0} t^k X_{\geq k} \arrow[r, "E"] \arrow[d, "P_m(t\partial_t)\pi_-"] \arrow[dr, phantom, "\text{obv. comm.}"] & S_\natural \otimes X(RB) \arrow[d, "P_m(\partial_t)\pi_-"] \\
& \gamma_- X_{\geq 2m+1} \arrow[r] \arrow[dr, bend right, "\text{obv. comm.}"'] & \bigoplus\limits_{\substack{k\ \text{odd} \\ \geq 2m+1}} t^k X_{\geq k} \arrow[r, "E_{\text{odd},\geq 2m+1}"] \arrow[d, "\delta_1"] \arrow[dr, phantom, "\text{requires comp. of } E_k."] & S_{\natural,\ \text{odd}\ \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1 \otimes 1"] \\
& & X_{\geq 2m+1} \arrow[r, "E_{2m+1}"] & J_\natural^{2m+1} \otimes X(RB)
\end{tikzcd}$$

So my problem is now to define E and check all the properties.

~~We have~~ There's a good way to define E, using $X^t \underset{\text{defn}}{=} \bigoplus_{k\in\mathbb{Z}} t^k X_{\geq k}$, $L^t = \bigoplus_{k\in\mathbb{Z}} t^k J^k$. and fact that $X^t = X_T(R_T Q^t)$. & Namely we have

$$A \xrightarrow[\text{hom}]{i} Q \xrightarrow[\text{lin.}]{t^D} Q^t \xrightarrow[\text{hom}]{} L^t \otimes B.$$

$$X(RA) \xrightarrow{\iota_*} X \xrightarrow{t^{L_D}} X^t \longrightarrow L^t_\natural \otimes B.$$

hom. $\begin{array}{ccc} Q & \longrightarrow & L\otimes B \\ \cup & & \cup \\ Q_{\geq k} & \longrightarrow & J^k \otimes B \end{array}$ of filtered alg.

extends $\begin{array}{ccc} RQ & \longrightarrow & L \otimes RB \\ (RQ)_{\geq k} & \longrightarrow & J^k \otimes RB \\ X_{\geq k} & \longrightarrow & J^k_\natural \end{array}$

0 Idea is this: Introduce $Q = QA$, $R = RQ$ and $X = X(RQ)$. grading on Q yields gradings on R, X.

I have $Q \xrightarrow[\text{hom.}]{} L \otimes B$

whence $X \longrightarrow L_\natural \otimes X(RB)$

I want a filtered version. I have

~~I have~~
Recall

$$Q \xrightarrow[\text{hom}]{} L \otimes B$$

$$RQ \xrightarrow[\text{hom}]{} L \otimes RB$$

$$X(RQ) \xrightarrow[\text{map of super-cxs.}]{} L_\natural \otimes X(RB)$$

I want a filtered version, namely

$Q \longrightarrow L \otimes B$ $\qquad Q_{\geq k} \longrightarrow J^k \otimes B$

$RQ \longrightarrow L \otimes RB$ $\qquad RQ_{\geq k} \longrightarrow J^k \otimes RB$

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB).$$

ultimately want $IQ_{\geq k} \longrightarrow J^k \otimes IB$

$$F^p_{IQ} X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB} X(RB).$$

How much do I actually need.

P Basically we have a ~~map~~ hom. of filtered algebras

$$R_{\geq k} \longrightarrow J^k \otimes RB$$

so we can conclude that there are induced maps

$$X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

compatible. Why form $R^t = \oplus t^k R_{\geq k}$, then you have $R^t \longrightarrow L^t \otimes RB$

whence $X_T(R^t) \longrightarrow X_{L^t}(L^t \otimes RB) = L^t_y \otimes X(RB)$.

You want to be able to conclude $X_T(R^t) \xrightarrow{\sim} X(R)^t$.

Not obvious in general.

$$\begin{array}{ccc} \Omega_T(R^t) & \longrightarrow & L^t \otimes \Omega(RB) \\ \wr\!\| & & \\ (\Omega R)^t & & \end{array}$$

$$X_T(R^t) = \underbrace{\Omega_T(R^t) / F^1 \Omega_T(R^t)}_{\text{might have torsion}} \longrightarrow L^t_y \otimes \Omega(RB)/F^1\Omega(RB)$$

$\Omega^1_T(R^t) / [\Omega^1_T R^t, R^t]$ quotient, so one must be careful.

8 7/27-1391. I'm working on the end map.

$$\begin{array}{ccc} Q & \longrightarrow & L\otimes B \\ \cup & & \cup \\ Q_{\geq k} & \longrightarrow & J^k \otimes B \end{array} \qquad \forall k.$$

to ~~[illegible]~~ construct $\forall k$ maps

$$\begin{array}{ccc} X_{\geq k} & \longrightarrow & J^k_{\#} \otimes X(RB) \\ \cup & & \cup \\ F^p X_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p_{IB} X(RB) \end{array} \qquad \text{comp. with filtr.}$$

Consistent.

$$\begin{array}{ccc} X_{\geq k+1} & \longrightarrow & J^{k+1}_{\#} \otimes X(RB) \\ \cap & & \downarrow \\ X_{\geq k} & \longrightarrow & J^k_{\#} \otimes X(RB) \end{array}$$

commutes.

2 methods: simplest way is to go from

$$Q \longrightarrow L \otimes B \qquad \text{hom. of filtered algs.}$$

to

$$\begin{array}{ccc} \Omega Q & \longrightarrow & L \otimes \Omega B \\ \cup & & \cup \\ \Omega Q_{\geq k} & \longrightarrow & J^k \otimes \Omega B \end{array} \qquad \text{hom of filt. DG alg.}$$

to

$$\Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B \longrightarrow J^k_{\#} \otimes \Omega B$$

compatible with d, b, κ etc.

then to

$$\begin{array}{ccc} X_{\geq k} & \longrightarrow & J^k_{\#} \otimes X(RB) \\ \cup & & \cup \\ F^p X_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p_{IB} X(RB) \end{array}$$

r alternative method

from $Q \to L \otimes B$ $\qquad Q_{\geq k} \longrightarrow J^k \otimes B$

to $Q^t \longrightarrow L^t_\# \otimes B$ $\qquad$ comp with $\mathbb{C}[t^{-1}] \longrightarrow L_t$

to

$$\begin{tikzcd}
X_T(R_T Q^t) \arrow[r] \arrow[dd, "\cong"'] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, equal] \\
 & X_{L^t}(L^t \otimes RB) \arrow[d, equal] \\
X(RQ)^t \arrow[r, dotted] & L^t_\# \otimes X(RB)
\end{tikzcd}$$

In any case the end map is not as obvious as Joachim thought.

1444 So it seems that we have a problem about writing up Joachim's construction. The thing you might be looking for is a direct definition of $X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$, which would be based on a description of $X = X(RQ)$, say the graded description. If I could define the map directly, then I could shove all the filtration details to the end. It looks possible because of the isom.

$$R = T(V)$$

$$\Omega^1 R_\natural = T(V) \otimes V$$

here $V = \rho(Q')$ $\qquad Q' = Q_0' \oplus Q_1 \oplus \cdots$

So you have $\rho(V)^{\otimes n} \longrightarrow (L \otimes \rho B)^{\otimes n}$

S

Maybe go back to

$$A \xrightarrow{\iota} Q \xrightarrow{a_0 da_1 \ldots da_n \mapsto t^n \rho a\, \rho a_1 \cdots \rho a_n} S \otimes B$$

$$RA \longrightarrow RQ \xrightarrow{u'} S \otimes RB$$

$$X(RA) \longrightarrow X(RQ) \longrightarrow S_q \otimes X(RB).$$

$$X(RA) \longrightarrow X(RQ) \xrightarrow{u'_*} X(S \otimes RB) \longrightarrow S_q \otimes X(RB)$$

Observe u' is a graded ~~[illegible]~~ alg. hom.
So we do get the map we want

$$X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$$

Anyway this might help to define the basic diagram of supercomplexes. To define things carefully.

Start again 1539. Diagram

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r, "\alpha u'_*"] \arrow[d, "P_m(t_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r] \arrow[dr, bend right] & \pi_- S_{q,\geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J^{2m+1}_\# \otimes X(RB)
\end{tikzcd}$$

The point to concentrate upon is that so far one is only considering the graded aspect of the setup t ~~[illegible]~~ 1623

Where do we stand?

1712 where are we at present?

I have my map αu_*:

$$\begin{tikzcd} X(RA) \arrow[r, "u_*"] \arrow[d, "\iota_*"] & X(S\otimes RB) \arrow[r, "\alpha"] & S_\natural \otimes X(RB) \\ X(RQ) \arrow[r, "u'_*"] & X(S\otimes RB) \arrow[r, "\alpha"] & S_\natural \otimes X(RB) \end{tikzcd}$$

Recall that $u: RA \longrightarrow S\otimes RB$ is induced by $p+tq : A \longrightarrow S\otimes B$. Go back to $\theta\theta' : A \longrightarrow L\otimes B$ cong mod $J\otimes B$. Get

$$Q \xrightarrow{\Theta} L\otimes B \qquad a_0 da_1 \cdots da_n \mapsto pa_0\, qa_1 \cdots qa_n$$

Now use the grading of Q to get

$$Q \xrightarrow{\Theta t^D} S\otimes B \qquad \text{(compatible with grading)}$$

$$a_0 da_1 \cdots da_n \longmapsto t^n pa_0\, qa_1 \cdots qa_n .$$

Alternative $Q \xrightarrow{t^D} \bigoplus t^n Q_n \xrightarrow{\Theta} \bigoplus t^n J^n \otimes B$

$$A \xrightarrow{\iota} Q \xrightarrow{\Theta t^D} S\otimes B$$

$$a \longmapsto a + da \longmapsto pa + tqa$$

Anyway now extend Θt^D to

$$X(RQ) \xrightarrow{u'_*} X(S\otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB)$$

n [redacted] Conclude

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] \arrow[rr, bend left, "\text{my map}"] & X(RQ) \arrow[r, "\alpha u'_*"] \arrow[d, "P_m(D)\gamma_-"] & S_\hbar \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
& X(RQ)_{\geq 2m+1} \arrow[r, "\alpha u'_*"] \arrow[dr, "\delta_1 \alpha u'_*"'] & S_{\hbar, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_\#^{2m+1} \otimes X(RB)
\end{tikzcd}$$

yes this writes very well.

$\alpha u'_*$ in degree n: [redacted]

$$X(RQ)_n \longrightarrow J_\#^n \otimes X(RB)$$

$\delta_1 \alpha u'_*$ is the [redacted] sum over $n \geq 2m+1$ of the maps

$$X(RQ)_n \longrightarrow J_\#^n \otimes X(RB) \xrightarrow[\pi]{} J_\#^{2m+1} \otimes X(RB)$$

induced by $J^n \subset J^{2m+1}$

This is a good start. Remains to [redacted] link

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{P_m(D)\gamma_-} X(RQ)_{\geq 2m+1}$$

with Nistor's biv. Chern character for the universal quasi-hom. and $\delta_1 \alpha u'_*$ with rest.

~~Point is that Nistor has~~

End map in Nistor's case is a map of mixed complexes

$$\Omega Q_{\geq k} \longrightarrow J_\#^k \otimes \Omega B$$

$$F_{-k}(Q^b, g) \longrightarrow J_\#^k \otimes B^b$$

Start. Define $(\Omega^{\bullet} Q)_{\geq k}$ to be spanned by $x_0 dx_1 \cdots dx_n$ with $\sum \operatorname{ord}(x_i) \geq k$.

Check $\Omega Q_{\geq k} = \bigoplus_n \Omega^n Q \cap (\Omega^{\bullet} Q)_{\geq k}$

$$\Omega Q_{\geq i} \cdot \Omega Q_{\geq j} \subset \Omega Q_{\geq i+j}$$

$$d(\Omega Q_{\geq k}) \subset \Omega Q_{\geq k}$$

$$b(\Omega Q_{\geq k}) \subset \Omega Q_{\geq k}$$

etc. So what?

$RQ = \bigoplus_n \Omega^{2n}$ under $\circ$

$RQ_{\geq k} = \bigoplus_n \Omega^{2n}_{\geq k}$ defines a filtration compatible with product in RQ.

$IQ^m = \bigoplus_{n \geq m} \Omega^{2n}$ under $\circ$

$$(IQ)^{m}_{\geq k} \overset{\text{defn}}{=} IQ^m \cap RQ_{\geq k} = \bigoplus_{n \geq m} \Omega^{2n}_{\geq k}$$

Claim this is $\sum_{\sum k_i = k} (IQ)_{\geq k_1} \cdots (IQ)_{\geq k_m}$ contained inside

$$\underbrace{x_0 dx_1 \cdots dx_{2m}}_{\in RQ_{\geq \sum_0^{2m} \operatorname{ord} x_i}} \underbrace{(dx_{2m+1} dx_{2m+2})}_{\in IQ_{\geq \operatorname{ord}(x_{2m+1}) + \operatorname{ord}(x_{2m+2})}} \cdots (dx_{2n-1} dx_{2n})$$

W Take $x_0\, dx_1 \dots dx_{2n} \in IQ^m \cap RQ_{\geq k}$

so $n \geq m$ $\sum \mathrm{ord}(x_i) \geq k$. Then

$$\underbrace{(x_0 dx_1 dx_2)}_{\in IQ_{\geq \mathrm{ord} x_1 + \mathrm{ord} x_2}} \underbrace{(dx_3 dx_4)}_{\in IQ_{\geq \mathrm{ord}(x_3) + \mathrm{ord}(x_4)}} \cdots \underbrace{(dx_{2m-1} dx_{2m}\, dx_{2m+1} \dots dx_{2n})}_{\in IQ_{\geq \sum_{i=2m-1}^{2n} \mathrm{ord}(x_i)}}$$

It seems I have to define $FPX_{\geq k}$ brutally.

Try to make precise what you need to proceed. I want to identify $X(RQ)_{\geq k}$ with $\bigoplus_{n \geq k} X(RQ)_n$ where $X(RQ)_n$ spanned by products $\rho(x_1) \cdots \rho(x_r)$ or $\rho(x_1) \cdots \rho(x_r)\, d\rho(x_{r+1})$ such that $\sum |x_i| = n$.

How do I define $X(RQ) \longrightarrow S_\natural \otimes X(RB)$ — As the ext of

$$Q \longrightarrow S \otimes B$$
$$Q_n \longmapsto t^n J^n \otimes B.$$

You want to check that

X 7/28 - 0553

Yesterday I found I can define ~~the~~

1) $Q \longrightarrow S \otimes B$ graded linear map

$a_0 da_1 \cdots da_n \longmapsto t^n \rho a_0 \, g a_1 \cdots g a_n$

2) $RQ \xrightarrow{u'} S \otimes RB$ graded alg homom. ext. 1)

3) $X(RQ) \xrightarrow{\alpha u'_*} S_\natural \otimes X(RB)$ Compat. with grading i.e. $L_D \leftrightarrow t\partial_t$

~~[illegible]~~ Now 3) is ~~[illegible]~~ the direct sum of maps

4) $(\alpha u'_*)_n : X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$

Have $J^n_\# \longrightarrow J^k_\#$ for ~~[illegible]~~ $n \geq k$, whence maps

5) $X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$

So far have discussed vector space grading on Q. ~~[illegible]~~ Now study filtration $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ $\qquad Q_n = \Omega^n A$

Consider more gen a filtered alg ~~[illegible]~~ Q $\quad Q_{\geq k}$

Put $X = X(RQ)$, $F^p X = F^p_{IQ} X(RQ)$.

$\boxed{\Omega = \Omega Q, \quad F^p\Omega = F^p(\Omega Q)}$

~~[illegible]~~

$\exists$ induced filtrations $\Omega_{\geq k}$, $F^p\Omega_{\geq k}$, $X_{\geq k}$, $F^p X_{\geq k}$

compatible with corresp $X \simeq \Omega$, $F^p X \simeq F^p \Omega$

6) $\mathcal{X}_{\geq k} = \varprojlim (X_{\geq k}/F^p X_{\geq k}) \sim \varprojlim (\Omega_{\geq k}/F^p\Omega_{\geq k}) =$ Hodge tower of $\Omega_{\geq k}$.

A problem is to figure out how much detail to give about this filtration.

Ex. $(I^m)_{\geq k} = I^m \cap R_{\geq k}$

$$\natural(I^m dR)_{\geq k} = \natural(I^m dR) \cap (\Omega^1 R_\natural)_{\geq k}$$

So what do we want to say?

You need various facts.

$$L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$$

$p = 2m$, $\quad F^{2m}X : \quad I^{m+1} + [I^m, R] \leftrightarrows \natural(I^m dR)$

$$F^{2m}X_{\geq k} : \quad I^{m+1}_{\geq k} + \sum_{i+j=k} [I^m_{\geq i}, R_{\geq j}] \leftrightarrows \sum_{i+j=k} \natural(I^m_{\geq i}\, dR_{\geq j})$$

First you check with I

$$I_{\geq k} = I \cap R_{\geq k}$$

Consider $L_D - k : I^m \longrightarrow I^{m-1}$

$$R_{\geq k} \longrightarrow R_{\geq k+1}$$

$$L_D - k : I^m_{\geq k} = I^m \cap R_{\geq k} \longrightarrow I^{m-1} \cap R_{\geq k+1} = I^{m-1}_{\geq k+1}.$$

$$\natural(I^m dI)_{\geq k} = \sum_{i+j=k} \natural(I^m_{\geq i}\, dI_{\geq j})$$

$$\xrightarrow{L_D - k} \sum_{i+j=k} \natural\left((L_D - i) I^m_{\geq i} \cdot dI_{\geq j} + I^m_{\geq i} \cdot (L_D - j)\, dI_{\geq j}\right)$$

$$\subset \sum_{i+j=k} \natural\left(I^{m-1}_{\geq i+1}\, dI_{\geq j} + I^m_{\geq i}\, dR_{\geq j+1}\right).$$

I still have to figure out the details.

You are dealing with $\Omega = \Omega Q$

You have filtr $Q_{\geq k}$ of Q compatible with alg. st.

$\Longrightarrow$ $\Omega Q_{\geq k}$ on ΩQ | compat. with grading, alg structure, d, b, k, etc.

~~Next point~~ $\Omega Q_{\geq k}$ mixed sub complexes

bifiltration $F^p \Omega Q_{\geq k} = F^p(\Omega Q_{\geq k})$

In degree p $\quad F^p \Omega_{\geq k}$ is $\quad b\,\Omega^{p+1}_{\geq k} = \sum_{i+j=k} [\Omega^p_{\geq i}, Q_{\geq j}]$

Use correspondence $X \sim \Omega$ to transport filtration to X: $X_{\geq k}$, $F^p X_{\geq k}$. At this point the objects $R_{\geq k}$, $I^m_{\geq k}$, $X_{\geq k}$, $F^p X_{\geq k}$ have been defined. Next need properties. What properties.

$\mathcal{X}_{\geq k} \underset{def}{=} (X_{\geq k}/F^p X_{\geq k}) \sim (\Omega_{\geq k}/F^p \Omega_{\geq k}) = \underline{\theta(\Omega_{\geq k})}$.

next concern behavior relative to L_D.

$$X_{\geq k} = \bigoplus_{n \geq k} X_n \quad \text{where } L_D = n \text{ on } X_n$$

(a) $h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$ same for L_D

(b) $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

(c) $\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

a) Conseq of ⓐ ⓑ

$$\begin{tikzcd} X_{\geq k+1} \arrow[r, "S"] \arrow[d, "\iota_k"'] & X_{\geq k+1}[2] \arrow[d, "\iota_k"] \\ X_{\geq k} \arrow[r, "S"'] \arrow[ur, dashed, "1-\frac{1}{k}L_D"] & X_{\geq k}[2] \end{tikzcd}$$

$$[S_k] \in HC^2(\cancel{Q^b_{\geq k}, Q^b_{\geq k+1}}\ X_{\geq k}, X_{\geq k+1})$$
$$= HC^2(Q^b_{\geq k}, Q^b_{\geq k+1})$$

$\cancel{(1-k^{-1}L_D)}\iota_k$

$S_k = 1 - k^{-1}L_D : X_{\geq k} \to X_{\geq k+1}$. $\qquad k \geq 1$.

$S_k\iota_k = 1 - k^{-1}L_D : X_{\geq k+1} \to X_{\geq k+1}$
$= 1 - [\partial, k^{-1}h_D]$

$\iota_k : X_{\geq k+1} \to X_{\geq k}$ $\qquad \iota_k : X_{\geq k} \to X_{\geq k+1}$

$S_k = 1 - k^{-1}L_D : X_{\geq k} \to X_{\geq k+1}$ $\qquad S_k : X_{\geq k+1} \to X_{\geq k}[2]$

$S_k\iota_k \sim S : X_{\geq k+1} \to X_{\geq k+1}[2]$

$\iota_k S_k \sim S : X_{\geq k} \to X_{\geq k}[2]$

Now bring is $\gamma_- = \frac{1}{2}(1-\gamma)$ on X preserves grading bifiltration all ~~structure~~. ⓒ

$$\gamma_- F^p X_{\geq 2n} = \gamma_- F^p X_{\geq 2n+1}$$

b)

$$X(RA) \xrightarrow{\iota_*} X(RQ) \simeq X_{\geq 0} \xrightarrow{\gamma_-} \gamma_- X_{\geq 0} = \gamma_- X_{\geq 1}$$

$$\xrightarrow{s_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$$\xrightarrow{s_m} \gamma_- X_{\geq 2m} = \gamma_- X_{\geq 2m+1}.$$

same as Nistor's bivariant Chern character for the universal quasi-hom. with differences

factor of 2: $\gamma_- \iota_* = \frac{1}{2}(\iota_* - \iota_*^{\gamma})$

N

$$\mathcal{X}_A \xrightarrow{\gamma_- \iota_*} \gamma_- \mathcal{X}_{\geq 1} \xrightarrow{s_1} \gamma_- \mathcal{X}_{\geq 3} \to \cdots \to \gamma_- \mathcal{X}_{\geq 2m+1}$$

$$\in HC^{2m}(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 2m+1}) = HC^{2m}(\Omega A, \gamma_- \Omega Q_{\geq 2m+1})$$

Nistor instead of using γ_- uses

$$\mathcal{X}_A \xrightarrow{\iota_* - \iota_*^{\gamma}} \mathcal{X}_{\geq 1} \xrightarrow{s_1} \mathcal{X}_{\geq 2} \xrightarrow{s_2} \qquad \xrightarrow{s_n} \mathcal{X}_{\geq n+1}$$

$$HC^{2n}(A, \Omega Q_{\geq n+1}).$$

All this seems to be OK.

What remains is the end map.

1137 $$\Omega Q_{\geq 2m+1} \longrightarrow J_{\#}^{2m+1} \otimes \Omega B$$

Let's go over methods. Recall that this depends on $Q \longrightarrow L \otimes B$ a hom. of filtered algebras.

$Q_{\geq k} \qquad J^k \otimes B$

c) Induces a hom. of filtered DG algs

$$\Omega Q \longrightarrow L \otimes \Omega B$$

$$\Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

and then $\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$

compatible with d, b, etc.

Relative version: have homom.

$$Q^t \longrightarrow L^t \otimes B \qquad \text{comp. w } T \to L^t$$

$$\begin{tikzcd}
\Omega_T Q^t \arrow[r] \arrow[d, "\wr"] & \Omega_{L^t}(L^t \otimes B) \arrow[d, "\wr"] \\
\Omega Q^t \arrow[r] \arrow[d, equal] & L^t \otimes \Omega B \arrow[d] \\
\Omega Q^t \arrow[r] & L^t_{\natural} \otimes \Omega B
\end{tikzcd}
\qquad
\begin{tikzcd}
X_T(R_T Q^t) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, equal] \\
X(RQ)^t \arrow[d, equal] & X_{L^t}(L^t \otimes RB) \arrow[d, equal] \\
X(RQ)^t \arrow[r] & L^t_{\natural} \otimes X(RB)
\end{tikzcd}$$

Recall where we are. I have in mind a map

$$\begin{array}{ccccccc}
X(RA) & \xrightarrow{\iota_*} & X(RQ) & \xrightarrow{P_m(L_D)\gamma_-} & \gamma_- X(RQ)_{\geq 2m+1} & \longrightarrow & J^{2m+1}_{\#} \otimes X(RB) \\
\cup & & \cup & & \cup & & \\
F^p & \longrightarrow & F^p & \longrightarrow & \gamma_- F^{p-2m} & & \text{provid}
\end{array}$$

$$\underbrace{\chi_A \longrightarrow \chi_Q \longrightarrow \gamma^- \chi_{\geq 2m+1}[2m]}_{\text{this is directly related to the Nistor character for universal quasi-hom.}}$$

d) So all that remains is to relate my end map to Nistor's. My end map:

$$Q \xrightarrow{\Theta\, t^D} S \otimes B$$

is defined using the grading of Q as vector space and functoriality of $X(RQ)$ in Q as a v.s. + 1.

1254 The problem is now to ~~handle which~~. link the follow.

Trace map

$$\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B.$$

starting from $Q \longrightarrow L \otimes B$ hom. of filtered algebras

How do I proceed

$$\Omega Q_{\geq k} \longrightarrow \underline{\Omega(L \otimes B)_{\geq k}} \longrightarrow J^k \otimes \Omega B$$

spanned by

$(x_0 \otimes b_0)\, d(x_1 \otimes b_1) \ldots d(x_n \otimes b_n)$

$\sum$ order $x_i \geq k$.

$$RQ_{\geq k} \longrightarrow J^k \otimes RB$$

~~$X(R$~~

filtered alg. hom $Q \longrightarrow L \otimes B$

$Q_{\geq k}$ $\quad J^k \otimes B$

get $X(RQ)_{\geq k} \longrightarrow X(\cancel{L \otimes B})_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$

$R(L \otimes B)$

e) $\Omega Q_{\geq k} \longrightarrow \Omega(L \otimes B)_{\geq k} \longrightarrow J^k \otimes \Omega B \rightarrow J^k_{\#} \otimes \Omega B$

comp. with d, b, K, etc.

get map of ~~supercomplexes~~ supercomplexes

OK

$\therefore \quad X(RQ)_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$.

Start again: $Q \longrightarrow L \otimes B$, $\quad Q_{\geq k} \longrightarrow J^k \otimes B$

Given a filtered alg homom.

you get a filtered DG alg hom.

$$\Omega Q \longrightarrow L \otimes \Omega B, \quad \Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B.$$

check that
$$\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$$
comp. with d, b, κ etc.

$$RQ_{\geq k} \longrightarrow J^k \otimes RB$$

$$RQ \longrightarrow L \otimes RB$$

Review: 1530 Anyway end map for Nistor construction should give a map of special towers

$$\mathcal{X}_{\geq 2m+1} \longrightarrow J^{2m+1}_{\#} \otimes \mathcal{X}_B$$

enough to give

$$\begin{array}{ccc} X(RQ)_{\geq k} & \longrightarrow & J^k_{\#} \otimes X(RB) \\ \cup & & \cup \\ F^p X(RQ)_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p_{IB} X(RB) \end{array}$$

f) So what is needed? My t method starts from $Q \longrightarrow L \otimes B$, $Q_{\geq k} \longrightarrow J^k \otimes B$

$$Q^t \longrightarrow L^t \otimes B$$

$$\cancel{X_T(R_T Q^t)}$$

$$R_T Q^t \longrightarrow R_{L^t}(L^t \otimes B) = L^t \otimes RB$$

$$I_T Q^t \longrightarrow I_{L^t}(L^t \otimes B) = L^t \otimes IB.$$

$$\cancel{X_T(R_T Q^t) \longrightarrow X}$$

$$X_T(R_T Q^t) \longrightarrow X_{L^t}(L^t \otimes RB) = L^t_q \otimes X(RB)$$

$$F^p_{I_T Q^t} X_T(R_T Q^t) \longrightarrow L^t_q \otimes F^p_{IB} X(RB)$$

$$\shortparallel$$

$$F^p_{L^t \otimes IB} X_{L^t}(L^t \otimes RB)$$

So I can start with

$$Q \longrightarrow L \otimes B, \quad Q_{\geq k} \longrightarrow J^k \otimes B$$

$$\Omega Q \longrightarrow L \otimes \Omega B, \quad \Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

check $\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$
comp. with d, b, K etc. whence you get

$$RQ \longrightarrow L \otimes RB, \quad RQ_{\geq k} \longrightarrow J^k \otimes RB$$

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

g) Do again. Start with filtered alg hom.

$$Q \longrightarrow L \otimes B\,, \qquad Q_{\geq k} \longrightarrow J^k \otimes B$$

get filtered DG alg hom

$$\Omega Q \longrightarrow L \otimes \Omega B\,, \qquad \Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

whence rest. to even forms + Fed. a filtered alg hom

$$RQ \longrightarrow L \otimes RB\,, \qquad RQ_{\geq k} \longrightarrow J^k \otimes RB$$

Check $\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$ comp with b

$\therefore$ with b, d, κ, etc.

get filtered supercomplex map

$$\begin{array}{ccc} X(RQ)_{\geq k} & \longrightarrow & J^k_{\#} \otimes X(RB) \\ \cup & & \cup \\ F^p X_{\geq k} & \longrightarrow & J^k_{\#} \otimes F^p_{IB} X(RB) \end{array}$$

whence map of special towers

$$\mathcal{X}_{\geq k} \longrightarrow J^k_{\#} \otimes \mathcal{X}_B$$

as desired.

The main question now is why is $X(RQ)_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$ defined in this way via the trace map $\Omega Q_{\geq k} \longrightarrow J^k_{\#} \otimes \Omega B$ consistent with the map defined via the grading of $X(RQ)$. Recall that ~~this~~ we have

$$\begin{array}{ccc} Q & \longrightarrow & S \otimes B \\ RQ & \longrightarrow & S \otimes RB \\ X(RQ) & \longrightarrow & S_{\natural} \otimes X(RB) \\ \therefore\ X(RQ)_n & \longrightarrow & J^n_{\#} \otimes X(RB). \end{array}$$

D on left $\Longleftrightarrow$ $t\partial_t$ on right.

h) We have two maps

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

which we have to identify. First method proceeds via the identification

~~2nd method~~

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r] \arrow[d, "\wr"'] & X(R(L\otimes B))_{\geq k} \arrow[dd, "\wr"] & \\
(\Omega Q)_{\geq k} \arrow[rr] \arrow[dr] & & J^k_\# \otimes \Omega B \\
& \Omega(L\otimes B)_{\geq k} \arrow[ur] &
\end{tikzcd}$$

First map proceeds via identification

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r] \arrow[d, "\wr"'] & X(R(L\otimes B))_{\geq k} \arrow[r] \arrow[d, "\wr"'] & J^k_\# \otimes X(RB) \arrow[d, "\wr"'] \\
(\Omega Q)_{\geq k} \arrow[r] & \cancel{\Omega(L\otimes B)_{\geq k}} \arrow[r] & J^k_\# \otimes \Omega B
\end{tikzcd}$$

Can take $Q = L\otimes B$ to define it
$Q_{\geq k} = J^k \otimes B$

~~Defined~~ Given by formula

$$\natural\big(\rho(x_0\otimes b_0)\,\omega(x_1b_1, x_2b_2)\cdots\omega(x_{2n-1}b_{2n-1}, x_{2n}b_{2n})\,d\rho(x_{2n+1}b_{2n+1})\big)$$

$$\longrightarrow \#_k(x_0\cdots x_{2n}) \otimes \natural\big(\rho(b_0)\,\omega(b_1,b_2)\cdots\omega(b_{2n-1},b_{2n})\,d\rho(b_{2n+1})\big)$$

where $\sum \operatorname{ord} x_i \geq k$.

i) 2nd ~~[illegible]~~ method.

$$\begin{tikzcd} Q \arrow[rr] \arrow[dr, "\wr"'] & & S \otimes B \\ & \bigoplus_n t^n Q_n \arrow[ur, hook] & \end{tikzcd}$$

$x \in Q_n \longmapsto \rho_n x \in J^n \otimes B$

$$RQ \longrightarrow S \otimes RB$$
$$X(RQ) \longrightarrow S_{\natural} \otimes X(RB)$$

Somehow to focus on the missing step.

Here $X(RQ)_n$ spanned by elts.

$$\natural\left(\rho(y_0) \cdots \rho(y_s)\, d\rho(y_{s+1})\right)$$

where y_i homog in Q and $\sum |y_i| = n$.

y_i in $Q_{|y_i|}$ and resolve the image in $J^{|y_i|} \otimes B$

~~This might be~~:

Look ~~[illegible]~~ more abstractly. Suppose we ~~[illegible]~~ have understand $X(RQ)^t \longrightarrow L^t_{\natural} \otimes X(RB)$ in some fashion as induced by the filtered alg hom. $Q \to L \otimes B$, $Q_{\geq k} \to J^k \otimes B$. Then we understand it as

$$\begin{array}{ccccc} X(RQ)^t & \longrightarrow & X(R(L\otimes B))^t & \longrightarrow & L^t_{\natural} \otimes X(RB) \\ \| & & \| & & \| \\ X_T(R_T(Q^t)) & \longrightarrow & X_T(R_T(L^t \otimes B)) & \to & X_{L^t}(R_{L^t}(L^t \otimes B)) \end{array}$$

So ~~[illegible]~~ next compose with

$$X(RQ) \xrightarrow{t^{L_D}} X(RQ)^t$$

where does this come from?

j) ~~So next compose~~

$$A \xrightarrow{\iota} Q \xrightarrow{t^{D}} \bigoplus t^n Q_n \subset \bigoplus t^k Q_{\geq k} \longrightarrow L^t \otimes B$$

$$X(RA) \longrightarrow X(RQ) \xrightarrow{t^{L_D}} \bigoplus t^n X_n \subset \bigoplus t^k X_{\geq k} \longrightarrow L^t_{\natural} \otimes X(RB)$$

So the funny business occurs when we use the functorality of $X(RQ)$ in the vector space with 1 Q.

Consider $\quad Q \xrightarrow[\text{linear resp } 1]{t^{D}} Q^t \xrightarrow[\text{hom.}]{} L^t \otimes B$

$$\begin{tikzcd}
X(RQ) \arrow[r, "t^{L_D}"] \arrow[dr] & X_T(R_T Q^t) \arrow[r] \arrow[d, phantom, "\parallel"] & X_{L^t_\natural}(R_{L^t}(L^t \otimes B)) \arrow[d, phantom, "\parallel"] \\
& X(RQ)^t \arrow[r] & L^t_\natural \otimes X(RB)
\end{tikzcd}$$

So what did we learn before. The point is that we know how to define the map

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d] & S_\natural \otimes X(RB) \\
X(S \otimes RB) \arrow[ur, "\alpha"'] &
\end{tikzcd}$$

namely

Thus you have $\quad Q \xrightarrow[\text{lin. resp.}]{t^{D}} \bigoplus_{n \geq 0} t^n Q_n \longrightarrow S \otimes B$

$$RQ \xrightarrow{\quad u' \quad} S \otimes RB$$

$$X(RQ) \xrightarrow{u'_*} X(S \otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB).$$

Compatible with grading hence get

$$X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$$

k)

$$\begin{tikzcd} Q \arrow[r, "t^D"] & Q^{t,\geq 0} \arrow[r] & S \otimes B \\ RQ \arrow[r, "(t^D)_*"] \arrow[dr, bend right, "t^D"'] & R(Q^{t,\geq 0}) \arrow[r] \arrow[d] & S \otimes RB \\ & RQ^{t,\geq 0} & \end{tikzcd}$$

$$\boxed{\begin{tikzcd} X(RQ) \arrow[r, "t^{h_D}"] & X(RQ^{t,\geq 0}) \arrow[r] \arrow[d] & S_\hbar \otimes X(RB) \\ & X(RQ)^t & ? \end{tikzcd}}$$

So far I have

$$Q \xrightarrow[\text{based lin}]{} \bigoplus t^n Q_n \longrightarrow S \otimes B$$

$$X(RQ) \longrightarrow X(S \otimes RB) = S_\hbar \otimes X(RB).$$

$$X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$$

Then you can piece together to get

$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB).$$

obvious playing the ~~[illegible]~~ structure of $S_\hbar$ as $\mathbb{C}[t^{-1}]$ module.

So far what do you do?? Answer!

~~Point $Q \longrightarrow \bigoplus t^n Q$~~

Go over the situation. You have

$$Q = \bigoplus Q_n \xrightarrow{t^D} \bigoplus t^n Q_n \longrightarrow S \otimes B$$

based linear map compatible with D on right and $t\partial_t$ on the left.

l) Point for tomorrow: Ultimately you are after $X(RQ)_{\geq k} \longrightarrow J_{\#}^{k} \otimes X(RB)$ compatible with $F^p X_{\geq k} \longrightarrow J_{\#}^{k} \otimes F_{IB}^{p}$

This comes from $X(RQ) = \Omega Q$ ~~identification~~ description

But on the other hand you have a direct construction based on the grading

$$X(RQ) \longrightarrow X(S \otimes RB) \longrightarrow S_{\natural} \otimes X(RB)$$

L_D $\qquad t\partial_t$

You have $X(RQ)_n \longrightarrow J_{\#}^{n} \otimes X(RB)$

Need some how to link the filtered map construction with this graded one.

7/29 - 0536 Try again.

First define filtered case.

$Q^t \longrightarrow L^t \otimes B$ hom. yields

$$R_T Q^t \longrightarrow R_{L^t}(L^t \otimes B) = L^t \otimes RB$$

$$X_T(R_T Q^t) \longrightarrow X_{L^t}(L^t \otimes B) = L_{\natural}^{t} \otimes RB$$

$\downarrow s$

$X(RQ)^t \xrightarrow{\text{map of } T\text{-modules}} L_{\natural}^{t} \otimes RB$

whence a family of maps

$$X_{\geq k} \longrightarrow J_{\#}^{k} \otimes X(RB)$$

compatible as k varies.

Also have $\Omega Q^t \longrightarrow L_{\natural}^{t} \otimes \Omega B$

$\qquad\wr \qquad\qquad \wr$

$X(RQ)^t \longrightarrow L_{\natural}^{t} \otimes X(RB)$

m) so there is compatibility with trace Mistor uses.

Other point is that ~~[illegible]~~

$$T \otimes Q \xrightarrow{\sim} Q^t \qquad \text{extends} \quad Q \overset{t^D}{\hookrightarrow} Q^t$$

so

$$T \otimes RQ \xrightarrow{\sim} R_T Q^t \simeq RQ^t$$

$$T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t) \xrightarrow{\sim} X(RQ)^t$$

Repeat:

$$A \longrightarrow S \otimes B$$

$$RA \longrightarrow R_S(S \otimes B) = S \otimes RB$$

$$X(RA) \longrightarrow X_S(S \otimes RB) = S_q \otimes X(RB)$$

But can factor

~~$A \to Q \to S \otimes B$~~

$$A \xrightarrow[\text{hom}]{} Q \xrightarrow[\text{b. lin}]{t^D} Q^t \xrightarrow[\text{hom}]{} L^t \otimes B$$

$$A \xrightarrow[\text{hom}]{} Q \underset{\text{hom}}{\subset} T \otimes Q \xrightarrow[\text{b. lin over } T]{\sim} Q^t \xrightarrow[\text{hom}]{} L^t \otimes B$$

$$\begin{array}{ccccccccc} RA & \to & RQ & \to & R_T(T \otimes Q) & \xrightarrow{\sim} & R_T Q^t & \longrightarrow & R_{L^t}(L^t \otimes B) \\ & & & & \| & & & & \| \\ & & & & T \otimes RQ & & & & L^t \otimes RB \end{array}$$

$$\begin{array}{ccccccccc} X(RA) & \to & X(RQ) & \to & X_T(T \otimes RQ) & \xrightarrow{\sim} & X_T(R_T Q^t) & \longrightarrow & X_{L^t}(L^t \otimes RB) \\ & & & & \| & & \| & & \| \\ & & & & T \otimes X(RQ) & & X(RQ)^t & & L^t_q \otimes X(RB) \end{array}$$

n) ~~so what actually happens?~~
Repeat the steps in general mode

$$A \xrightarrow{p+tq} S \otimes B \subset L^t \otimes B$$

$$A \longrightarrow Q \hookrightarrow T \otimes Q \underset{\text{of } T\text{-lin}}{\xrightarrow{\simeq}} Q^t \longrightarrow L^t \otimes B$$

$$RA \to RQ \longleftrightarrow T \otimes RQ \simeq R_T Q^t \longrightarrow L^t \otimes RB$$

$$X(RA) \to X(RQ) \longrightarrow \underbrace{T \otimes X(RQ) \simeq X_T(R_T Q^t)}\longrightarrow L^t_q \otimes X(RB)$$

isom respecting
graded T-module structure
supercomplex structure
But not ~~structure~~ the Hodge
related filtrations $T \otimes F^p X$ $\quad F^p_{I_T Q^t}$

Go back now to Nistor

$$\Omega A \underset{\text{mix}}{\longrightarrow} \Omega Q \underset{\text{mix}}{\xrightarrow{\gamma_-}} \gamma_- \Omega Q = \gamma_- \Omega Q_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- \Omega Q_{\geq 2} = \gamma_- \Omega Q_{\geq 3}$$

$$\xrightarrow{S_{2m-1}} \gamma_- \Omega Q_{\geq 2m} = \gamma_- \Omega Q_{\geq 2m+1}$$

$$\subset \Omega Q_{\geq 2m+1} \underset{\text{mix}}{\longrightarrow} J_\#^{-2m+1} \otimes \Omega B .$$

$$\mathcal{X}_A \longrightarrow \mathcal{X}_Q \xrightarrow{\gamma_-} \gamma_- \mathcal{X} = \gamma_- \mathcal{X}_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- \mathcal{X}_{\geq 2} = \gamma_- \mathcal{X}_{\geq 3}$$

$$\xrightarrow{S_{2m-1}} \gamma_- \mathcal{X}_{\geq 2m} = \gamma_- \mathcal{X}_{\geq 2m+1}$$

$$\subset \mathcal{X}_{\geq 2m+1} \longrightarrow J_\#^{-2m+1} \otimes \mathcal{X}_B .$$

Alternative notation $A^\flat$ for $\mathcal{X}_A$ $\quad Q^\flat_{\geq k}$

0) 0915 time to organize main statements.

my map

(*) $A \xrightarrow[\text{b. linear}]{p+tq} S \otimes B \subset L^t \otimes B$

induces $X(RA) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_q \otimes X(RB)$

factor (*) into

$$A \xrightarrow{\iota} Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

which induces

$$X(RA) \longrightarrow X(RQ) \longrightarrow X_T(R_T Q^t) \longrightarrow L^t_q \otimes X(RB)$$

Beforehand organize

$$T \otimes Q \xrightarrow[\text{based } T\text{-linear}]{\sim} Q^t \subset \overset{U}{\cancel{\mathbb{C}[t,t^{-1}]}} \otimes Q$$

induced

$$\begin{array}{ccccc} X_T(R_T(T \otimes Q)) & \xrightarrow{\sim} & X_T(R_T Q^t) & \longrightarrow & X_U(R_U(U \otimes Q)) \\ \| & & & & \| \\ T \otimes X(RQ) & \xrightarrow{\sim} & X_T(R_T Q^t) & \longrightarrow & U \otimes X(RQ) \end{array}$$

identifies $X_T(R_T Q^t) = \bigoplus t^k X(RQ)_{\geq k}$

1015 Q graded + filtered

$$Q \xrightarrow[\sim]{t^D} \bigoplus t^n Q_n \subset \bigoplus t^k Q_{\geq k} \subset \bigoplus t^k Q = \mathbb{C}[t,t^{-1}] \otimes Q$$

RA and X(RA) depend only on A as based v.s.

$$RQ \xrightarrow[\sim]{t^D} \bigoplus t^n (RQ)_n \subset \bigoplus$$

$$RQ \xrightarrow{t^D} \bigoplus t^n RQ_n \subset \underset{T \otimes Q}{RQ^t} \subset \mathbb{C}[t,t^{-1}] \otimes RQ.$$

φ) objects Q graded v.s. $1 \in Q_0$

$$Q^t = \bigoplus t^k Q_{\geq k} \subset \mathbb{C}[t,t^{-1}] \otimes Q$$
$$\| $$
$$\mathbb{C}[t^{-1}] \otimes \bigoplus_n t^n Q_n$$

$$Q^t \overset{\text{defn}}{=} \bigoplus_k t^k Q_{\geq k} \subset \bigoplus_k t^k Q = \mathbb{C}[t,t^{-1}] \otimes Q$$

Note $T \otimes Q \xrightarrow{\sim} Q^t$

$t\partial_t + D \qquad t\partial_t$

Have

$$X(RQ) \xrightarrow{(t^D)_*} X_T(R_T(T \otimes Q)) \xrightarrow{\sim} X_T(R_T Q^t) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q))$$
$$X(RQ) \xrightarrow{t^{L_D}} T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t) \longrightarrow T' \otimes X(RQ)$$

$$\boxed{\begin{array}{l} \text{Conclude} \quad X_T(R_T Q^t) \xrightarrow{\sim} \bigoplus t^k X(RQ)_{\geq k} \\ \text{specifically} \quad Q^t \hookrightarrow T' \otimes Q \quad \text{a hom.} \\ \text{induces an identifications} \quad R_T Q^t \longrightarrow RQ^t \subset T' \otimes RQ \\ \qquad X_T(R_T Q^t) \longrightarrow X(RQ)^t \subset T' \otimes X(RQ). \end{array}}$$

How do I organize all this? My stuff

$$A \xrightarrow{p+tq} S \otimes B$$

$$X(RA) \longrightarrow X_S(R_S(S \otimes B)) = S_\natural \otimes X(RB)$$
$$X(RA) \longrightarrow X(S \otimes RB) \longrightarrow S_\natural \otimes X(RB)$$
$$F^p \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \bigoplus_i \natural(K^i) \otimes F^{p-2i}$$

8) Next ~~stuff~~

$$\begin{tikzcd}
A \arrow[r, "p+tq"] \arrow[dr] & S\otimes B \arrow[d, equal] & \subset & L^t\otimes B \\
\text{[redacted]}\ & Q \arrow[r, "t^D"] & Q^t \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] \arrow[d] \arrow[dr, "t^{L_D}"] & X_T(R_T Q^t) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t\otimes B)) \arrow[d, equal] \\
& T\otimes X(RQ) \arrow[r, "\sim"] & \oplus t^k X_{\geq k} \arrow[r] & L^t_q \otimes X(RB)
\end{tikzcd}$$

1130 Let's concentrate on the objects.

Maybe I should tabulate the points.

Leave F^p filtrations to the end.

My map

$$A \xrightarrow{p+tq} S\otimes B \subset L^t \otimes B$$

$$X(RA) \longrightarrow S_q \otimes X(RB) \subset L^t_q \otimes X(RB).$$

Factor my map

$$A \xrightarrow{\iota} Q \xrightarrow{t^D} Q^t \xrightarrow{w} L^t\otimes B$$

induces

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "(t^D)_*"] \arrow[dr, "t^{L_0}"'] & X_T(R_T Q^t) \arrow[r, "w_*"] \arrow[d, phantom, "\wr"] & X_{L^t}(R_{L^t}(L^t\otimes B)) \arrow[d, phantom, "\wr"] \\
& & X(RQ)^t & L^t_q \otimes X(RB)
\end{tikzcd}$$

need:

[redacted] (In fact I can say that because $L^t_q \otimes X(RB)$ is a T-module, the map

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d] & L^t_q \otimes X(RB) \\
T\otimes X(RQ) = X(RQ)^t \arrow[ur] &
\end{tikzcd}$$

arising from $Q \to Q^t \to L^t\otimes B$

induces

2) Emphasizing fact that $X(RQ)$ functor on based vector spaces (based T-modules)

Now do everything ~~to~~ without F^p filtrations.

Have on $X = X(RQ)$, ~~to~~ the grading equiv L_D and the filtration assoc.

My map 1) $A \xrightarrow{p+tq} S \otimes B \subset L^t \otimes B$

induces

$$X(RA) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B)) = L^t_q \otimes X(RB)$$

My map factors

2) $$A \longrightarrow Q \longrightarrow Q^t \longrightarrow L^t \otimes B$$

which induces

$$\begin{tikzcd} X(RA) \arrow[r] & X(RQ) \arrow[r] \arrow[dr, "t^{L_D}"'] & X_T(R_T Q^t) \arrow[r] \arrow[d, "\wr"] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, "\wr"] \\ & & X(RQ)^t & L^t_q \otimes X(RB) \end{tikzcd}$$

we need to amplify this

The point in fact is that

$$T \otimes X(RQ) \xrightarrow{\sim} X(RQ)^t$$
$$1 \otimes \xi \longmapsto t^{L_D} \xi$$

which is a fancy way of saying that

$$X_{\geq k} = \bigoplus_{n \geq k} X_n$$

5) I want to keep on saying this in order to get it clear. Really in order to understand well enough to find the points to emphasize.

Joachim's version of Nistor's construction.

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X_{\geq 0} = \gamma_- X_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$S_k = 1 - \frac{1}{k} L_D : X_{\geq k} \to X_{\geq k+1}$.

$$\xrightarrow{S_{2m-1}} \gamma_- X_{\geq 2m} = \gamma_- X_{\geq 2m+1}.$$

Lemma: a) $h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$ also $L_D = [\partial, h_D]$

b) $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

c) $\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

($\gamma = (-1)^k$ on $F^p X_{\geq k} / F^p X_{\geq k+1}$.)

Observe clear if $F^p X_{\geq k} = F^p X \cap X_{\geq k}$

$$F^p_{IA} \xrightarrow{\iota_*} F^p_{IQ} \xrightarrow{\gamma_-} \gamma_- F^p X_{\geq 0} = \gamma_- F^p X_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- F^{p-2} X_{\geq 2} = \gamma_- F^{p-2} X_{\geq 3}$$

$$\xrightarrow{S_{2m-1}} \gamma_- F^{p-2m} X_{\geq 2m} = \gamma_- F^{p-2m} X_{\geq 2m+1}$$

Thus get

$$\mathcal{X}_A \xrightarrow{\iota_*} \mathcal{X}_Q \xrightarrow{\gamma_-} \gamma_- \mathcal{X}_{\geq 0} = \gamma_- \mathcal{X}_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- \mathcal{X}_{\geq 2}[2] = \gamma_- \mathcal{X}_{\geq 3}[2]$$

t)
$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[rr] \arrow[d, "P_m(D)\gamma_-"] & & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
& X_{\geq 2m+1} \arrow[rr] \arrow[drr] & & S_{q,\geq 2m+1} \otimes X(RB) \arrow[d] \\
& & & J_{\#}^{2m+1} \otimes X(RB)
\end{tikzcd}$$

Point I seem to want is that the map $Q \xrightarrow[\text{based lin}]{} L^t \otimes B$

induces $X(RQ) \longrightarrow L_q^t \otimes X(RB)$

compatible with $\left| \begin{array}{l} \text{grading} \\ \text{s.cx. structure} \end{array} \right.$

thus extends to

$$X(RQ)_{\geq k} \longrightarrow J_{\#}^k \otimes X(RB)$$

Different views of this extens.

1) $$\begin{tikzcd}
X(RQ)_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n \arrow[r] & \bigoplus_{n \geq k} J_{\#}^n \otimes X(RB) \arrow[d, "\iota_\#"] \\
& J_{\#}^k \otimes X(RB)
\end{tikzcd}$$

2) $$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d] & \underbrace{L_q^t \otimes X(RB)}_{T\text{-module}} \\
T \otimes X(RQ) \arrow[ur, dashed, "\exists"'] &
\end{tikzcd}$$

extn as T-module

$T \otimes X(RQ) = X(RQ)^t$

4) And the point is that this extn. is something depending on the filtration of Q.

~~Thus~~ So the idea maybe to

Start again. $Q = \oplus Q_n$ grading $1 \in Q_0$

$Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ comp. with alg st.

RQ, $X(RQ)$ depend on Q as based vector space

$$Q^t \overset{\text{def}}{=\!=} \bigoplus_{k \in \mathbb{Z}} t^k Q_{\geq k} \subset T' \otimes Q$$

$$T \otimes Q \xrightarrow{\sim} Q^t \qquad \text{isom. resp. graded } T \text{ module st.}$$

$$1 \otimes \xi \longmapsto t^D \xi$$

Relative ~~$R_T Q^t$~~

$$T \otimes RQ = R_T(T \otimes Q) \xrightarrow{\sim} R_T Q^t$$

Start again: I want all details in outline.

My map $A \xrightarrow{p+tq} S \otimes B \subset L^t \otimes B$

induces $X(RA) \longrightarrow S_\natural \otimes X(RB) \subset L^t_\natural \otimes X(RB)$.

Factor my map

$$A \xrightarrow[\text{hom.}]{i} Q \xrightarrow[\text{b. lin.}]{t^D} Q^t \xrightarrow[\text{hom}]{w} L^t \otimes B$$

induces

$$\begin{tikzcd} X(RA) \arrow[r, "i_*"] & X(RQ) \arrow[r, "(t^D)_*"] \arrow[dr, "t^{L_D}"'] & X_T(R_T Q^t) \arrow[r, "w_*"] \arrow[d, "\cong"] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, equal] \\ & & X(RQ)^t & L^t_\natural \otimes X(RB) \end{tikzcd}$$

v) part requiring amplification.

$$T\otimes Q \xrightarrow{\sim} Q^t$$

$Q \xrightarrow{t^D} Q^t$ extends to $T\otimes Q \xrightarrow{\sim} Q^t$ resp T-module structures, ~~this in turn yields~~ note composition

$$T\otimes Q \xrightarrow{\sim} Q^t \subset T'\otimes Q$$

is $i\otimes id_Q$, inverting t^{-1}. So

$$\begin{array}{ccccc} X_T(R_T(T\otimes Q)) & \xrightarrow{\sim} & X_T(R_T Q^t) & \longrightarrow & X_{T'}(R_{T'}(T'\otimes Q)) \\ \parallel & & & & \parallel \\ T\otimes X(RQ) & \xrightarrow{\sim} & \cancel{X(RQ)^t} & \longrightarrow & T'\otimes X(RQ) \end{array}$$

From my viewpoint. Have $Q \xrightarrow{t^D} Q^t$ b. linear extends to $T\otimes Q \xrightarrow{\sim} Q^t$ such that

$$T\otimes Q \xrightarrow{\sim} Q^t \subset T'\otimes Q$$

$$\begin{array}{ccccccc} RQ & \longrightarrow & R_T(T\otimes Q) & \xrightarrow{\sim} & R_T Q^t & \longrightarrow & R_{T'}(T'\otimes Q) \\ & & \parallel & & & & \parallel \\ & & T\otimes RQ & & & & T'\otimes RQ \end{array}$$

Point maybe is that $Q \longrightarrow Q^t$ induces $RQ \xrightarrow{(t^D)_*} R_T Q^t$

$$X(RQ) \longrightarrow$$

Q^t and $T'\otimes Q$ ~~are resp. the gra~~ together with

$t^D: Q \longrightarrow Q^t$ and $t^D: Q \longrightarrow T'\otimes Q$

are resp. the graded T and T' modules gen by

w) the graded v.s. Q. Thus we should also have that

$$t^D : RQ \longrightarrow R_T Q^t$$

$$t^D : RQ \longrightarrow R_{T'}(T'\otimes Q)$$

yield isos.

$$\begin{array}{ccc} T\otimes RQ & \xrightarrow{\sim} & R_T Q^t \\ \cap & & \cap \\ T'\otimes RQ & \xrightarrow{\sim} & T\otimes RQ \end{array} \qquad ?$$

The way to proceed is to assume that

grading on Q yields grading on RQ, X(RQ)
filtr on Q ——— filtr on RQ, X(RQ).

granting this we have then certain objects;

namely RQ_n, D, $RQ_{\geq k}$, $(RQ)^t$

$X(RQ)_n$, L_D, $X(RQ)_{\geq k}$, $X(RQ)^t$

relations: D degree op on RQ : $D = n$ on RQ_n

$$\begin{tikzcd} T\otimes RQ \arrow[r, "(t^D)_*", "\sim"'] \arrow[rr, bend right, "\iota\otimes 1"'] & RQ^t \subset & T'\otimes RQ \end{tikzcd}$$

R

You don't yet have the logic straight.

x) 7/30 - 0522 Start again.

$$A \underset{\theta'}{\overset{\theta}{\rightrightarrows}} L \otimes B \quad \text{cong mod } J \otimes B$$

$$p = \tfrac{1}{2}(\theta + \theta') : A \longrightarrow L \otimes B$$
$$q = \tfrac{1}{2}(\theta - \theta') : \bar{A} \longrightarrow J \otimes B$$

b. linear $\quad A \xrightarrow{p+tq} S \otimes B \qquad S = \overbrace{\bigoplus_{n \geq 0} t^n J^n}^{\subset \mathbb{C}[t] \otimes J}$

curv. $\quad (1-t^2)q^2 : \bar{A}^2 \longrightarrow (1-t^2) J^2 \otimes B \subset K \otimes B.$

$$u : \quad RA \xrightarrow{(p+tq)_*} R_S(S \otimes B) = S \otimes RB$$
$$IA \longrightarrow K \otimes RB + S \otimes IB$$

$$X(RA) \xrightarrow{u_*} X(S \otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB) \xrightarrow{\mu_m} J_\#^{2m+1} \otimes X(RB)$$
$$F^p_{IA} \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \sum_{i \geq 0} \natural(K^i) \otimes F^{p-2i}_{IB} \longrightarrow J_\#^{2m+1} \otimes F^{p-2m}_{IB}$$

get $\mathcal{X}_A \longrightarrow J_\#^{2m+1} \otimes \mathcal{X}_B[2m]$

whence a class in $HC^{2m}(A^b, J_\#^{2m+1} \otimes B^b)$

Nistor: $\quad Q = QA = A * A = (\Omega A, \circ)$

~~$p+tq : A \longrightarrow A^t_K$~~ $\qquad \theta, \theta'$ induce

$$Q \longrightarrow L \otimes B, \quad Q_{\geq k} \longrightarrow J^k \otimes B$$

factor ~~base~~ into $\quad p + tq : A \longrightarrow S \otimes B \subset L^t \otimes B$

$$A \xrightarrow{\iota} Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

Here I haven't found a good order in which to do things. Basically I want to introduce Q, then identify my map $X(RA) \longrightarrow J_\#^{2m+1} \otimes X(RB)$

y) with

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X = \gamma_- X_{\geq 1} \qquad\qquad X = X(RQ)$$

$$\xrightarrow{S_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$$\vdots$$

$$\xrightarrow{S_{2m-1}} \gamma_- X_{\geq 2m} = \gamma_- X_{\geq 2m+1} \longrightarrow J^{2m+1}_{\#} \otimes X(RB)$$

identification requires αu_*

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "\diamond"] \arrow[d, "P_m(L_D)\gamma_-"] & S_\natural \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \arrow[dd, bend left, "\mu_m"] \\
& \gamma_- X_{\geq 2m+1} \arrow[r] & S_{\natural,\geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J^{2m+1}_{\#} \otimes X(RB)
\end{tikzcd}$$

So how do I proceed?

$$A \longrightarrow Q \xrightarrow{t^D} Q^{t,\geq 0} \longrightarrow S \otimes B$$

$$A \longrightarrow Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

$$X(RA) \xrightarrow{\iota_*} \underbrace{X(RQ) \xrightarrow{t^{\iota D}} X(RQ)^t \longrightarrow L^t_\natural \otimes X(RB)}_{\text{where does this come from.}}$$

$$X(RQ) \xrightarrow{(t^D)_*} X_T(R_T Q^t) \longrightarrow X_{L^t}(R_{L^t}(L^t \otimes B))$$

OK, so I have hidden the understand in all this relative stuff. But how to really proceed?

2) Basically dealing with Q graded such that the assoc. filtr is compatible with the alg structure.

Grading on Q $\Longrightarrow$ grading on RQ as alg
$X(RQ)$ as supercx.

D $\Longrightarrow$ derivation D on RQ
L_D on $X(RQ)$
degree ops. for grading.

specific RQ_n sp by $\rho(x_1)\cdots\rho(x_s)$ $\sum|x_i|=n$.

$(\Omega^1(RQ)_\natural)_n$ —— $\natural(\rho(x_1)\cdots\rho(x_s)\,d\rho(x_{s+1}))$ ...

$RQ_{\geq k}$ sp by $\rho(x_1)\cdots\rho(x_s)$ $\sum \mathrm{ord}(x_i)\geq k$

$(\Omega^1(RQ)_\natural)_{\geq k}$ ——————

Now we have

$$Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

Keep as simple as possible. You have

$$Q_n \longrightarrow J^n \otimes B \qquad \forall n$$
$$a_0 da_1 \cdots da_n \longmapsto t^n p a_0\, q a_1 \cdots q a_n$$

hence

$$Q \longrightarrow S \otimes B \qquad \text{b. linear resp. grading}$$
$$D \longleftrightarrow t\partial_t.$$

Extends then to

$$X(RQ) \longrightarrow S_\natural \otimes X(RB)$$
$$L_D \qquad\qquad t\partial_t$$

(a) Summarize: Important are:

$$A \xrightarrow{p+t_\varepsilon} S \otimes B \implies X(RA) \longrightarrow S_\natural \otimes X(RB)$$

$$\underset{D}{Q} \longrightarrow \underset{t\partial_t}{S \otimes B} \quad D \longleftrightarrow t\partial_t \qquad (\text{made up of } Q_n \longrightarrow J^n \otimes B)$$

$$\implies X(RQ) \longrightarrow S_\natural \otimes X(RB)$$

$$L_D \longleftrightarrow t\partial_t$$

$$(\text{made up of} \quad X(RQ)_n \longrightarrow J^n_\# \otimes X(RB))$$

So far only the grading on Q, RQ, $X(RQ)$ have been considered. ~~Next, look at filtration.~~

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r] \arrow[d, "P_m(D)\gamma_-"] & S_\natural \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
 & \gamma_- X(RQ)_{\geq 2m+1} \arrow[r] \arrow[dl, equal] \arrow[dr] & \pi_- S_{\natural, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
\bigoplus\limits_{\substack{n \text{ odd} \\ \geq 2m+1}} X(RQ)_n & & J^{2m+1}_\# \otimes X(RB)
\end{tikzcd}$$

Conclude that my map $X(RA) \longrightarrow J^{2m+1}_\# \otimes X(RB)$ is ~~the~~ equal to the comp.

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{P_m(L_D)\gamma_-} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{\Diamond} J^{2m+1}_\# \otimes X(RB)$$

where $\Diamond$ ~~is the sum~~ on $X(RQ)_n$ is

$$\begin{tikzcd}
X(RQ)_n \arrow[r] \arrow[d] & J^n_\# \otimes X(RB) \arrow[r, "\iota_\#"] & J^{2m+1}_\# \otimes X(RB) \\
X(R(S \otimes B))_n \arrow[r] & X(S \otimes RB)_n \arrow[r] & S_{\natural, n} \otimes X(RB).
\end{tikzcd}$$

(b) Summarize again.

$$A \xrightarrow{p+t\theta} S \otimes B$$

factors into

$$A \xrightarrow{\iota} Q \xrightarrow{\sigma} S \otimes B$$

$$D \longleftrightarrow t\partial_t$$

based linear map σ made of $Q_n \longrightarrow J^n \otimes B$

σ induces

$$X(RQ) \longrightarrow X(R(S\otimes B)) \longrightarrow X(S \otimes RB) \longrightarrow S_\natural \otimes X(RB)$$

$$L_D \qquad\qquad t\partial_t$$

Thus we have $X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$ for all n. Also we have

$$X(RQ)_{\geq k} \longrightarrow S_{\natural, \geq k} \otimes X(RB) \xrightarrow{\delta_1} J^k_\# \otimes X(RB)$$

~~made~~ made of $X(RQ)_n \longrightarrow J^n_\# \otimes \cdots \xrightarrow{\iota_\#} J^k_\# \otimes \cdots$ for $n \geq k$.

So far everything is simple & depends only on Q as graded vector space with ι.

~~At this point I want to bring in the filtration. What's the filtration all about?~~

At the ~~...~~ moment I have my map equal to

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{P_m(L_D)\gamma_-} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{\ell_{2m+1}} J^{2m+1}_\# \otimes X(RB)$$

and the problem is to connect up with Nistor's construction. Definition of ℓ_{2m+1}, recall:

$$Q \xrightarrow{w} S \otimes B \qquad \text{based linear} \qquad D \longleftrightarrow t\partial_t$$

$$X(RQ) \longrightarrow X_S(R_S(S\otimes B)) = S_\natural \otimes X(RB)$$

$$\therefore \quad X(RQ)_n \longrightarrow J^n_\# \otimes X(RB).$$

(c)

So now I have map map.
bring in filtration.
point $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ is compatible with
alg. structure.

Points to establish: $F^p X_{\geq k} \sim F^p \Omega_{\geq k}$

$l_k : X_{\geq k} \longrightarrow J_{\#}^{k} \otimes X(RB)$
carries $F^p X_{\geq k}$ into $J_{\#}^{k} \otimes F_{IB}^{p} X(RB)$.

$h_D : X \circlearrowleft$ carries $F^p X_{\geq k}$ into $F^{p-2} X_{\geq k}$

$L_D - k$ carries $F^p X_{\geq k}$ into $F^{p-2} X_{\geq k+1}$

$\gamma = (-1)^k$ on $F^p X_{\geq k} / F^p X_{\geq k+1}$.

$\gamma^2 = 1$ so exact sequence

$$0 \longrightarrow \gamma_- F^p X_{\geq k+1} \longrightarrow \gamma_- F^p X_{\geq k} \longrightarrow \gamma_- (F^p X_{\geq k} / F^p X_{\geq k+1}) \longrightarrow 0$$

$\gamma_- (F^p X_{\geq k} / F^p X_{\geq k+1}) = 0$ if k even

Need to define $F^p X_{\geq k}$
Now I have to go over everything.

$X = X(RQ)$ Q graded $= \bigoplus_n Q_n$

$Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ comp with alg. st.

$1 \in Q_{\geq 0}$, $Q_{\geq i} \cdot Q_{\geq j} \subset Q_{\geq i+j}$

$Q^t = \bigoplus_k t^k Q_{\geq k} \subset \bigoplus_k t^k Q = T' \otimes Q$.

graded T-subalg. of $T' \otimes Q$.

(d) Focus on the essential.

Q^t graded alg over $T = \mathbb{C}[t^{-1}]$

So $X_T(R_T Q^t) \simeq \Omega_T Q^t$

are defined, and isomorphic as [redacted] graded T-modules. $I_T Q^t = \mathrm{Ker}\,(R_T Q^t \to Q^t)$

$\Omega_T^{ev,\geq 2} Q^t$

Moreover $F^p_{I_T Q^t} X_T(R_T Q^t) \simeq F^p \Omega_T Q^t$

$p = 2n$.

$$\underbrace{(I_T Q^t)^{n+1}} + [(I_T Q^t)^n, R_T Q^t] \simeq [\Omega_T^{2n} Q^t, Q^t] + \Omega_T^{ev,\geq 2n+2} Q^t$$

$$\sum t^k \Big(\sum_{\sum k_i = k} IQ_{\geq k_0} \cdots IQ_{\geq k_n} \Big] + \big[IQ_{\geq k_0} \cdots IQ_{\geq k_{n-1}}, RQ_{\geq k_n} \big] \Big)$$

What do I need to make things clear?

What do I need? So far have defined

$$Q \xrightarrow{\omega} S \otimes B$$ based linear graded

$$X(RQ) \xrightarrow{\omega_*} X_S(R_S(S \otimes B)) \to S_\# \otimes X(RB)$$

L_D $\qquad t\partial_t$

$$\omega_{*,n} : X(RQ)_n \longrightarrow J^n_\# \otimes X(RB)$$

so ~~far~~ consider on Q as a graded vector space. now consider filtration $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$. Compatible with alg. structure

(e) Make a lot of words, but what's the points? So what? The claim will be the existence of a bifiltration ~~$F^pX_{\geq k} = (F^p_{IQ}X(RQ))_{\geq k}$~~

$$F^pX_{\geq k} = F^p_{IQ}X(RQ)_{\geq k}$$

with various properties.

0) $F^pX_{\geq k} = \begin{cases} X_{\geq k} & p \leq -1 \\ F^p_{IQ}X(RQ) & k \leq 0 \end{cases}$

1) For k fixed $F^pX_{\geq k}$ is a decreasing filt. of $X_{\geq k} = X(RQ)_{\geq k}$ by subcomplexes such that

$$\mathcal{X}_{\geq k} = \left(X_{\geq k}/F^pX_{\geq k}\right) \sim \Theta(\Omega Q_{\geq k}).$$

2) $L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$

3) $h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$

4) $\gamma = (-1)^k$ on $F^pX_{\geq k}/F^pX_{\geq k+1}$.

5) $\iota_k : X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$ carries $F^pX_{\geq k}$ into $J^k_\# \otimes F^p_{IB}X(RB)$.

Define how $F^pX_{\geq k}$. First step

$$T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t) \hookrightarrow T' \otimes X(RQ)$$

$$1 \otimes \xi \longmapsto t^{L_D}\xi$$

Do this carefully

$$T \otimes Q \longrightarrow Q^t \subset T' \otimes Q$$

$$t^i \otimes \underset{\substack{\cap \\ Q_n}}{x} \longmapsto t^{i+n}x$$

$$T \otimes Q_n \longleftrightarrow T t^n Q_n = \sum_{k \leq n} t^k t^n Q_n \subset T' \otimes Q_n$$

mult. by t^n.

(f) Method I propose. I form $Q^t = \oplus t^{k} Q_{\geq k}$ and note this is a graded T-alg. Consider relative constructions $R_T Q$, $X_T(R_T Q)$, $\Omega_T Q$.

Now $T \otimes Q \xrightarrow{\sim} Q^t$ T-module isom.

$$\Longrightarrow \quad T \otimes RQ \xrightarrow{\sim} R_T Q^t$$

$$T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t)$$

Put another way. We have the based linear maps

$$t^D : Q \longrightarrow Q^t$$

and this induces

$$t^D : RQ \longrightarrow R_T Q^t$$

$$t^{LD} : X(RQ) \longrightarrow X_T(R_T Q^t)$$

But t^D extends to a T-bimodule isom.

$$T \otimes Q \xrightarrow{\sim} Q^t$$

and because of fmlas

$$R_T(T \otimes Q) = T \otimes RQ$$

$$X_T(T \otimes RQ) = T_4 \otimes X(RQ)$$

we conclude

$$T \otimes RQ \xrightarrow{\sim} R_T Q^t$$

$$T \otimes X(RQ) \xrightarrow{\sim} X_T(R_T Q^t)$$

In particular ~~we have~~ the canonical localization maps

$$R_T Q^t \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X_T(R_T Q^t) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$$

are injective.

(8) Technically the point to verify is that $X_T(R_T Q^t)$ is flat over T (equiv. mult. by t^{-1} is injective).

~~Thus~~ You do the following - define filtration on $\mathcal{L}Q$ then transport to $X(RQ)$, but the grading i.e. L_D and h_D are invisible it would seem.

Start with grading ~~on~~ $Q = \oplus Q_n$, $D =$ degree o get D on RQ, how?

$$Q \xrightarrow{t^D} T' \otimes Q \qquad \text{based linear}$$

$$RQ \xrightarrow{t^D} R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X(RQ) \xrightarrow{t^{h_D}} X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ).$$

filtration: $\quad Q \xrightarrow{t^D} Q^t \quad$ b. lin.

induces $\left| \begin{array}{l} RQ \xrightarrow{(t^D)_*} R_T Q^t \\ X(RQ) \xrightarrow{(t^D)_*} X_T(R_T Q^t) \end{array} \right.$

But $\quad T \otimes Q \xrightarrow{\sim} Q^t \quad$ from t^D

hence get $\quad T \otimes RQ \xrightarrow{\sim} R_T Q^t$

$\quad T \otimes X(RQ \xrightarrow{\sim} X_T(R_T Q^t).$

~~[illegible]~~ In particular $R_T Q^t$, $X_T(R_T Q^t)$ are T-flat

(h) See if your objection to ~~[illegible]~~ Joachim's argument is valid. Namely you claim that the inclusion

$$b(\Omega^{p+1}Q_{\geq k}) \subset (b\Omega^{p+1}Q) \cap (\Omega^p Q)_{\geq k}$$

can be strict. Consider a map

$$M \longrightarrow N$$

We have a filtered complex $\to \Omega^n_{\leq k} \xrightarrow{b} \Omega^{n-1}_{\leq k}$

We form $\Omega^{n,t} \longrightarrow \Omega^{n-1,t} \to .$

Better notation. $F_p M_n$ and $M_n^t = \bigoplus h^p F_p M_n$

filtered complex, get complex of graded $\mathbb{C}[h]$-modules

Look carefully

$$M^h_{n+1} \xrightarrow{d} M^h_n \xrightarrow{d} M^h_{n-1}$$

$$0 \to Z^h_n \longrightarrow M^h_n \longrightarrow B^h_{n-1} \longrightarrow 0 \qquad b(M^h_n)$$

$$0 \longrightarrow B^h_n \longrightarrow Z^h_n \longrightarrow H^h_n \longrightarrow 0$$

$$M^h_n = \bigoplus h^p F_p M_n$$

$$B^h_{n-1} = \mathrm{Im}\left\{M^h_n \xrightarrow{b} M^h_{n-1}\right\} = \bigoplus h^p\, b(F_p M_n)$$

$$Z^h_n = \mathrm{Ker}\left\{M^h_n \xrightarrow{b} M^h_{n-1}\right\} = \bigoplus h^p \underbrace{\mathrm{Ker}(F_p M_n \to F_p M_{n-1})}_{F_p M_n \cap Z_n}$$

$\therefore$ Z^h_n is assoc. to Z_n & induced filt.

(i)

$$B_n^h \subset Z_n^h \subset M_n^h$$

$$0 \to Z_n^h/B_n^h \longrightarrow M_n^h/B_n^h \longrightarrow \underbrace{M_n^h/Z_n^h}_{\text{torsion free}} \to 0$$

have same h torsion

I think M_n^h/B_n^h ~~has no torsion~~ flat

$\iff$ $b(F_pM_{n+1}) = bM_{n+1} \cap F_pM_n$

In general given $F_pB \subset F_pM$

then M^h/B^h torsion free $\iff$ $F_pB = B \cap F_pM$ all p.

$\Updownarrow$

$\forall p$ $F_pM/F_pB \hookrightarrow M/B$

$\Updownarrow$

$\forall p$

$$F_pB = F_pM \cap B.$$

$\boxed{\text{So that's very clear.}}$ Lets continue - Keep on going over it. Learned today this:

$$Q \xrightarrow{t^D} Q^t \quad \text{based lin so induces}$$

$$RQ \longrightarrow R_TQ^t, \quad X(RQ) \longrightarrow X_T(R_TQ^t)$$

which give $T \otimes RQ \xrightarrow{\sim} R_TQ^t$, $T \otimes X(RQ) \xrightarrow{\sim} X_T(R_TQ^t)$

In particular R_TQ^t, $X_T(R_TQ^t)$ T-flat.

Local. $\hat{X}_T R_TQ^t \longrightarrow \underset{\hat{X}_{T'}}{R_{T'}}(T' \otimes Q^t) = T' \otimes \underset{\hat{X}}{RQ}$

This identifies $X_T(R_T Q^t)$ with the image of

$$T\otimes X(RQ) \longrightarrow T'\otimes X(RQ)$$
$$f\otimes \xi \longmapsto f t^{-D}\xi$$

Thus we have $X_T(R_T Q^t) \xrightarrow{\sim} \oplus\, t^k X(RQ)_{\geq k} \subset T'\otimes X$

$$RQ \xrightarrow{t^D} Q^t \hookrightarrow T'\otimes Q$$

Maybe can say

$$T\otimes Q \xrightarrow{\sim} Q^t \hookrightarrow T'\otimes Q$$

may be treat separately.

Thus have $Q^t \subset T'\otimes Q$ which is the canonical map $Q^t \to T'\otimes_T Q^t$, so we have canonical local maps (base change rel to $T\to T'$)

$$R_T Q^t \longrightarrow R_{T'}(T'\otimes Q) = T'\otimes RQ$$
$$X_T(R_T Q^t) \longrightarrow X_{T'}(T'\otimes RQ) = T'\otimes X(RQ).$$

$R_T Q^t$, $X_T(R_T Q^t)$ identified with RQ^t $X(RQ)^t$

Still very hard to say.

Anyway lets go back to $\Omega_T Q^t = \oplus\, t^k(\Omega Q)_{\geq k}$

$$F^p\Omega_T Q^t = [\Omega_T^p Q^t, Q^t] \oplus \bigoplus_{n\geq p} \Omega_T^n Q$$

$$b(\Omega_T^{p+1}Q^t) = [\Omega_T^p Q^t, Q^t] = \oplus\, t^k\,[\Omega^p Q, Q]_{\geq k}$$

$[\Omega^p Q, Q]_{\geq k}$ spanned by $[x_0 dx_1 \ldots dx_p, x_{p+1}]$ with $\sum \operatorname{ord}(x_i) \geq k$.

In general $[\Omega^p Q, Q]_{\geq k}$ smaller than $[\Omega^p Q, Q] \cap \Omega Q_{\geq k}$.

(k)

$$I_T Q^t = \Omega_T^{ev, \geq 2} Q$$

ideal generated by $\omega(x,y)$ $\quad x, y \in Q^t$

What to say about $I_T Q^t = \bigoplus t^k (IQ)_{\geq k}$

Would really like to know that the ρ, ω picture of $X(RQ)$ is ~~consistenet~~ consistent with the Ω picture. Here's a typical problem.

Define $F^p X_{\geq k}$ for $p = 2n$ as

$$\underbrace{I^{n+1}_{\geq k}} + [I^n, R]_{\geq k} \qquad \natural \; (I^n dR)_{\geq k}$$

$$\sum_{\sum k_i = k} I_{\geq k_0} \cdots I_{\geq k_n} + [I_{\geq k_0} \cdots I_{\geq k_{n-1}}, R_{\geq k_n}] \qquad \natural \; \sum (I_{\geq k_0} \cdots I_{\geq k_{n-1}} dR_{\geq k_n})$$

~~Thm~~ Proof. $F^{2n}_{I_T Q^t} X_T(R_T Q^t)$

$$(I_T Q^t)^{n+1} + [I_T Q^t, R_T Q^t] \qquad \natural \; ((I_T Q^t)^n d(R_T Q^t))$$

$$I_T Q^t \sim \Omega_T^{ev, \geq 2} Q^t$$

(2) 7/31 - 0536

Summary: My construction

$$A \underset{\theta'}{\overset{\theta}{\rightrightarrows}} L \otimes B \quad \text{cong mod } J$$

$$A \xrightarrow{p + tg} S \otimes B \qquad \text{b. lin}$$

$$RA \xrightarrow{(p+tg)_*} R_S(S \otimes B) = S \otimes RB$$

$$X(RA) \longrightarrow X(S \otimes RB) \longrightarrow S_\natural \otimes X(RB) \longrightarrow J_\#^{2m+1} \otimes X(RB)$$

$$F^p_{IA} \longrightarrow F^p_{K \otimes RB + S \otimes IB} \longrightarrow \sum \natural(K^i) \otimes F^{p-2i}_{IB} \longrightarrow J_\#^{2m+1} \otimes F^{p-2m}_{IB}$$

$$Ch^{2m}(\theta, \theta') : \mathcal{X}_A \longrightarrow J_\#^{2m+1} \otimes \mathcal{X}_B[2m]$$

$$\tau \in \left(J_\#^{2m+1}\right)^* \qquad Ch^{2m}(\theta, \theta', \tau) \in HC^{2m}(A, B)$$

Introduce $Q = QA$, grading, D

b. lin. $\quad Q \xrightarrow{w} S \otimes B \qquad w(a_0 da_1 \dots da_n) = t^n p a_0 \, g a_1 \dots g a_n$

$D \longleftrightarrow t\partial_t$

$$RQ \xrightarrow{w_*} R_S(S \otimes B) = S \otimes RB$$

$$X(RQ) \xrightarrow{w_*} S_\natural \otimes X(RB) \qquad L_D \longleftrightarrow t\partial_t$$

$$X(RQ)_n \xrightarrow{w_{*,n}} J^n_\# \otimes X(RB).$$

(m)

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r] \arrow[d, "P_m(L_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r] & \pi_- S_{q, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_{\#}^{2m+1} \otimes X(RB)
\end{tikzcd}$$

Conclusion is my map coincides with

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X = \gamma_- X_{\geq 1}$$

$$\xrightarrow{s_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$$\cdots$$

$$\xrightarrow{s_{2m-1}} \gamma_- X_{\geq 2m} = \gamma_- X_{\geq 2m+1} \xrightarrow{e_{2m+1}} J_{\#}^{2m+1} \otimes X(RB)$$

How next to introduce bifiltrations. $F^p X_{\geq k}$

0824: ~~Next point~~ Next. Introduce bifiltration $F^p X_{\geq k}$

various ~~the~~ viewpoints

but the properties needed are clear.

0) $F^p X_{\geq k}$ decreasing in p, k

$F^p X_{\geq 0} = F^p_{IQ} X(RQ)$

$F^{-1} X_{\geq k} = X(RQ)_{\geq k}$.

1) $\mathcal{X}_{\geq k} = (X_{\geq k} / F^p X_{\geq k}) \sim \Theta(\Omega Q_{\geq k})$

2) $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

3) $h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$

4) $e_k : F^p X_{\geq k} \longrightarrow \mathcal{J}_{\#}^k \otimes F^p_{IB}$

(h) So let's find what's missing in my understanding.

Need to define $F^p X_{\geq k}$

First for $p=-1$: $X_{\geq k}$

Defn. 1: $X_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n$

Thus $X_{\geq k}$ spanned by

$\rho(x_1)\cdots\rho(x_s)$

$\natural(\rho(x_1)\cdots\rho(x_s)\, d\rho(x_{s+1}))$

$\sum \mathrm{ord}(x_i) \geq k$.

Defn. 1': Use identification

$$X_T(R_T Q^t) \to X(RQ)^t \subset T' \otimes X(RQ)$$

i.e. $X_{\geq k}$ is the degree k subspace of $\mathrm{Im}\left(X_T(R_T Q^t) \longrightarrow T' \otimes X(RQ)\right)$

$\therefore$ $X(RQ)^t$ spanned by $\rho(y_1)\cdots\rho(y_s)$

$\natural(\rho(y_1)\cdots\rho(y_s)\, d\rho(y_{s+1}))$

where $y_i \in Q^t$ homog: $y_i = t^{k_i} x_i$

$\mathrm{ord}(x_i) \geq k_i$ Then

$$\rho(y_1)\cdots\rho(y_s) = t^{\sum k_i} \rho(x_1)\cdots\rho(x_s)$$

$$\natural(\qquad \rho(y_{s+1})) = t^{\sum k_i} \natural(\rho(x_1)\cdots\rho(x_s)\, d\rho(x_{s+1}))$$

Σ

(0)

Defn. 2: Use $X \simeq \Omega Q$

define $X_{\geq k} \simeq \Omega Q_{\geq k}$

i.e. $X_{\geq k}$ spanned by

$$\rho(x_0)\,\omega(x_1,x_2)\cdots\omega(x_{2s-1},x_{2s})$$

$$b\big(\text{\_\_\_\_\_\_\_\_\_\_}\; d\rho(x_{2s+1})\big) \qquad \sum \operatorname{ord}(x_i) \geq k$$

Claim same as Defn. 1. Why?

$\Omega_T Q^t$ flat graded T-module

localizes to $T' \otimes_T \Omega_T Q^t = \Omega_{T'}(T' \otimes_T Q^t)$

$$= \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega Q$$

so therefore $\Omega_T Q^t \xrightarrow{\sim} \underbrace{\bigoplus t^k\, \Omega Q_{\geq k}}_{\Omega Q^t} \subset T' \otimes \Omega Q$

But then our first thm. says

$X_T(R_T Q^t) = \Omega_T Q^t$ identifying $X(RQ)^t = \Omega Q^t$

In practical terms says that

$\rho(y_1)\cdots\rho(y_s)$ can be written in terms of $\rho(y_0)\,\omega(y_1,y_2)\cdots\omega(y_{2r-1},y_{2r})$

and conversely.

(P) 1135 Repeat Q alg

$$Q = \bigoplus_n Q_n \quad \text{graded as v.s.} \quad 1 \in Q_0$$

$$Q_{\geq k} = \bigoplus_{n \geq k} Q_n \quad \text{assoc. filtration} \ni$$

$$Q_{\geq i} \cdot Q_{\geq j} \subset Q_{\geq i+j}, \quad 1 \in Q_{\geq 0}.$$

The grading on Q induces grading on RQ, ~~$X(RQ)$~~. comp with alg. structure; degree op is unique deriv. D

induced grading on $X(RQ)$; degree op is L_D.

~~What~~

$X_n = X(RQ)_n$ spanned by

$$\rho(x_1) \cdots \rho(x_n), \quad \natural\left(\rho(x_1) \cdots \rho(x_n)\, d\rho(x_{n+1})\right)$$

x_i homog + $\sum |x_i| = n$.

Next consider filtration $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ and similarly defined filtrations $RQ_{\geq k}$ $X(RQ)_{\geq k}$

$Q_{\geq k}$ comp w alg structure $\quad Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q$

graded T-subalgebra of $T' \otimes Q$ such that

$$T' \otimes_T Q^t \xrightarrow{\sim} T' \otimes Q$$

Also $\quad T \otimes Q \xrightarrow{\sim} Q^t \quad$ as T-modules.

Lost again

~~Poss~~ filtration Q^t

9

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Q graded as vector space, $1 \in Q_0$

induced gradings on RQ, $X(RQ)$

description of the degree operators

canonical ϕ, h_D, $L_D = [\partial, h_D]$, $[L_D, h_D] = 0$

filtrations $Q_{\geq k}$, $(RQ)_{\geq k}$, $X(RQ)_{\geq k}$

T, T', Q^t, $(RQ)^t$, $X(RQ)^t$

ident. $R_T(Q^t) \simeq (RQ)^t$, $X_T(R_T(Q^t)) \simeq X(RQ)^t$

I seem to be losing the thread again.

Start again with my map

$$A \xrightarrow{p+tq} S \otimes B \qquad \text{based linear}$$

$$X(RA) \xrightarrow{(p+tq)_*} X_S(R_S(S \otimes B)) = S_q \otimes X(RB)$$

$$A \xrightarrow{\iota} Q \xrightarrow{\omega} S \otimes B$$

$$\begin{tikzcd}
X(RA) \arrow[r, "\iota_*"] & X(RQ) \arrow[r, "\omega_*"] \arrow[d, "P_m(L_D)\gamma_-"] & S_q \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r, "\omega_*"] & \pi_- S_{q, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_{\#}^{2m+1} \otimes X(RB).
\end{tikzcd}$$

(2) much awkwardness with

$$w : Q \longrightarrow S \otimes B$$

$$a_0 da_1 \cdots da_n \longmapsto t^n p a_0 q a_1 \cdots q a_n.$$

should be understood as combination of

$$Q \xrightarrow{t^D} \bigoplus_{k\geq 0} t^k Q_{\geq k} \longrightarrow S\otimes$$

8/1 - 0458 my construction

$$A \overset{\theta}{\underset{\theta'}{\rightrightarrows}} L\otimes B \quad \text{cong mod } J\otimes B$$

$$A \xrightarrow{p+tq} S\otimes B \qquad \text{b. linear map}$$

homom. mod $K\otimes B$.

induces

$$\begin{array}{ccc} RA & \xrightarrow{u} & S\otimes RB \\ \cup & & \\ IA & \longrightarrow & K\otimes RB + S\otimes IB \end{array}$$

$$X(RA) \longrightarrow X(S\otimes RB) \longrightarrow S_{\natural}\otimes X(RB) \longrightarrow J_{\#}^{2m+1}\otimes X(RB)$$

$$F^p_{IA} \longrightarrow F^p_{K\otimes RB + S\otimes IB} \longrightarrow \sum_{i\geq 0} \natural(K^i)\otimes F^{p-2i}_{IB} \longrightarrow J_{\#}^{2m+1}\otimes F^{p-2m}_{IB}$$

yields $Ch^{2m}(\theta,\theta') : \mathcal{X}_A \longrightarrow J_{\#}^{2m+1}\otimes \mathcal{X}_B[2m]$

$\therefore$ given $\tau : J_{\#}^{2m+1} \to \mathbb{C}$ get class

$$Ch^{2m}(\theta,\theta',\tau) \in HC^{2m}(A,B)$$

Problem: to relate to Nistor's construction.

$Q = QA$

(5) instead of listing features of Q and $X(RQ)$, let's concentrate on what we need. [redacted]

The problem concerns the end. At present I write down

$$w : Q \longrightarrow S \otimes B$$

which is really

$$Q \longrightarrow Q^{t,\geq 0} \longrightarrow S \otimes B$$

So I need to figure out what to say. I have preliminaries. The ideal is that

$\theta, \theta' : A \longrightarrow L \otimes B$ lead to a homom. of filtered algebras

$$Q \longrightarrow L \otimes B$$

$$Q_{\geq k} \longmapsto J^k \otimes B$$

Let's try to get straight the filtration and grading.

Let's try to focus.

θ, θ' induce $\quad Q \longrightarrow L \otimes B \qquad a \mapsto pa, \quad$ ~~$a_0 da_1$~~ $\; da \mapsto ga$

$\therefore \quad Q_n \longrightarrow J^n \otimes B$

so analogous to $p + tg$ we ~~have~~ can form

$$Q \longrightarrow S \otimes B \qquad a_0 da_1 \cdots da_n \longmapsto t^n pa_0 \, ga_1 \cdots ga_n$$

induces $\quad RQ \longrightarrow S \otimes RB \qquad D \longleftrightarrow t\partial_t$

$$X(RQ) \longrightarrow S_{\natural} \otimes X(RB) \qquad h_D \longleftrightarrow t\partial_t$$

~~and~~ θ_s

(t) What comes ~~[illegible]~~ later? ~~[illegible]~~

Repeat: θ, θ' induce hom

$$Q \longrightarrow L \otimes B \qquad a_0 da_1 \cdots da_n \mapsto \rho a_0 \varrho a_1 \cdots \varrho a_n$$

$$Q_n \longrightarrow J^n \otimes B$$

based line. $$Q \longrightarrow S \otimes B \qquad a_0 da_1 \cdots da_n \mapsto t^n \rho a_0 \varrho a_1 \cdots \varrho a_n$$

$$RQ \longrightarrow S \otimes RB$$

$$X(RQ) \longrightarrow S_\natural \otimes X(RB)$$

$$L_D \longleftrightarrow t\partial_t$$

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] \arrow[d, "P_m(L_D)\gamma_-"] & S_\natural \otimes X(RB) \arrow[d, "P_m(t\partial_t)\pi_-"] \arrow[dd, bend left, "\mu_m"] \\
& \gamma_- X_{\geq 2m+1} \arrow[r] & \pi_- S_{\natural, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_\#^{2m+1} \otimes X(RB)
\end{tikzcd}$$

~~[illegible]~~ Point is my map equals

$$X(RQ) \xrightarrow{\gamma_-} \gamma_- X_{\geq 0} = \gamma_- X_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- X_{\geq 2} = \gamma_- X_{\geq 3}$$

$$\cdots$$

$$\xrightarrow{S_{2m-1}} \gamma_- X_{\geq 2m} = \gamma_- X_{\geq 2m+1} \longrightarrow J_\#^{2m+1} \otimes X(RB)$$

Stopped here for taxes