$\boxed{h}$ Can't use ink pen with this paper so instead maybe make a summary. What I need is to organize exactly what is needed.

My construction

$$A \longrightarrow L \otimes B \quad \text{cong mod } J \otimes B$$

$$S = \bigoplus_{n \geq 0} t^n J^n \qquad K = (1-t^2) J^2 S$$

$$A \xrightarrow{p+tq} S \otimes B \quad \text{hom. mod } K \otimes B$$

$$RA \xrightarrow{u} S \otimes RB \qquad IA \longrightarrow K \otimes RB + S \otimes IB$$

$$X(RA) \xrightarrow{u_*} X(S \otimes RB) \xrightarrow{\alpha} S_\natural \otimes X(RB) \xrightarrow{\mu_m} J_\#^{2m+1} \otimes X(RB)$$

$$F^p_{IA} \longrightarrow F^p_{K\otimes RB + S \otimes IB} \longrightarrow \sum_{i \geq 0} \natural(K^i) \otimes F^{p-2i}_{IB} \longrightarrow J_\#^{2m+1} \otimes F^{p-2m}_{IB}$$

μ_m composition

$$S_\natural \xrightarrow{\pi_-} \pi_- S_\natural \xrightarrow{P_m(t\partial_t)}$$

$$S_\natural \xrightarrow{\pi_-} \pi_- S_{\geq 1} \xrightarrow{1 - D_t} \pi_- S_{\geq 3} \xrightarrow{1 - \frac{1}{3} D_t} \cdots \xrightarrow{1 - \frac{1}{2m-1} D_t} \pi_- S_{\geq 2m+1} \xrightarrow{\delta_1} J_\#^{2m+1}$$

Joachim's version of Nistor Construction

First structure on $X(RQ)$. Describe what is needed. The output will be a map

$$X(RA) \longrightarrow J_\#^{2m+1} \otimes X(RB) \quad , \quad F^p_{IA} \longrightarrow J_\#^{2m+1} \otimes F^{p-2m}_{IB}$$

$\boxed{i}$ ~~It is obtained~~

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r, "\alpha V_*"] \arrow[d, "\gamma_-"] & S_4 \otimes X(RB) \arrow[d, "\pi_-"] \\
& \gamma_- X(RQ)_{\geq 1} \arrow[d, "1-L_D"] & \pi_- S_{4,\geq 1} \otimes X(RB) \arrow[d, "1-D_t"] \\
& \gamma_- X(RQ)_{\geq 3} \arrow[d, "1-\frac{1}{3}L_D"] & \vdots \\
& \vdots \arrow[d, "1-\frac{1}{2m-1}L_D"] & \vdots \arrow[d] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[dr] & \pi_- S_{4,\geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
& & J_\#^{2m+1} \otimes X(RB)
\end{tikzcd}$$

~~Notice that we need to know something about~~

$$\begin{tikzcd}
X(RQ) \arrow[r] & S \otimes X(RB) \\
X(RQ)_{\geq k} \arrow[r] \arrow[u, phantom, "\cup"] & S_{\geq k} \otimes X(RB) \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

~~This is something new. I've been working on~~

$$X(RQ)_{\geq k} \longrightarrow J_\#^k \otimes X(RB)$$

$\boxed{1}$ Logic: You take my map $X(RA) \to S_q \otimes X(RB)$ and interpolate $X(RQ)$

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] & S_q \otimes X(RB)
\end{tikzcd}$$

$$\begin{tikzcd}
RA \arrow[r] & RQ \arrow[r, "t^D"] & RQ^{t,\geq 0} \arrow[r] & S \otimes RB
\end{tikzcd}$$

So I have to work on understanding

$$X(RQ) \longrightarrow S_q \otimes X(RB)$$

and to understand why it is compatible with the grading, i.e. why L_D on the left is compatible with $D_t = t\partial_t$ on the right. Notice that when you've done this you get automatically

$$\begin{tikzcd}
X(RQ)_{q\geq k} \arrow[r] & S_{q,\geq k} \otimes X(RB) \arrow[d] \\
 & J^k_{\#} \otimes X(RB)
\end{tikzcd}$$

which I struggled over yesterday.

Yesterday the point was

$$X(RQ)^t = \bigoplus t^k X(RQ)_{\geq k}$$

$$X_T(R_T Q^t) \longrightarrow X(RQ)^t$$

$\boxed{k}$ Anyway yesterday I did this:

Q alg w filt $Q_{\geq k}$, hom of filt algs.

$$Q \longrightarrow L \otimes B \quad , \quad Q_{\geq k} \longrightarrow J^k \otimes B$$

$$Q^t \longrightarrow L^t \otimes B$$

$$\begin{tikzcd}
X_T(R_TQ^t) \arrow[r] \arrow[d, equal] & L^t_h \otimes X(RB) \\
X(RQ)^t &
\end{tikzcd}
\quad , \quad
\begin{tikzcd}
F^p_{I_TQ^t} \arrow[r] & L^t_h \otimes F^p_{IB} \\
\left(F^p_{IQ} X(RQ)\right)^t \arrow[r] & L^t_h \otimes F^p_{IB}
\end{tikzcd}$$

In addition $D : \bar{Q} \longrightarrow Q$ linear resp $Q_{\geq k}$

$$\begin{tikzcd}
h_D : \left(F^p_{I_TQ^t} X_T(R_TQ^t)\right) \arrow[r, phantom, "\subset"] \arrow[d, equal] & F^{p-2}_{I_TQ^t} X_T(R_TQ^t) \arrow[d, equal] \\
h_D : \left(F^p_{IQ} X(RQ)\right)^t \arrow[r] & \left(F^{p-2}_{IQ} X(RQ)\right)^t
\end{tikzcd}$$

~~[illegible]~~

~~How does this commute?~~

What does the above gives?

Given Q, $Q_{\geq k}$, $Q \longrightarrow L \otimes B$, $Q_{\geq k} \longrightarrow J^k \otimes B$, D, $D(Q_{\geq k}) \subset Q_{\geq k}$ we get

$$X(RQ)_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$$

$$\left(F^p_{IQ} X(RQ)\right)_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB} X(RB)$$

and $h_D\left(F^p_{IQ} \quad\right)_{\geq k} \subset \left(F^{p-2}_{IQ} \quad\right)_{\geq k}$

$\boxed{\ell}$ So the next thing is to link these two. Notice that so far D has not been related to the map $Q \to L \otimes B$

In fact suppose we ~~assume known the result on D. Choose~~ assume D arises from a splitting: $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$ of the filtration. Then

$$Q \xrightarrow{t^D} Q^t \longrightarrow L^t \otimes B$$

Start again. Yesterday ~~proved~~ understood two results:

$Q, Q_{\geq k}$ filtered alg.,

1) If $D: \bar{Q} \to Q$ linear resp. filtration, then

$$h_D: \left(F^p_{IQ} X(RQ)\right)_{\geq k} \longrightarrow \left(F^{p-2}_{IQ} X(RQ)\right)_{\geq k}$$

2) If $Q \longrightarrow L \otimes B$, $Q_{\geq k} \longrightarrow J^k \otimes B$ homom. of filtered algs, then one has induced maps

$$X(RQ)_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB), \quad \left(F^p_{IQ} X(RQ)\right)_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB} X(RB)$$

Now [redacted] suppose given $Q_n \subset Q \ni Q_{\geq k} = \bigoplus_{n \geq k} Q_n$, $1 \in Q_0$

take $D = n$ on Q_n

Need the rest of the picture. So return to the link. ~~All these~~ my map

$\boxed{m}$

$$\begin{tikzcd}
X(RA) \arrow[rr, "\alpha u_*"] \arrow[dr, "\iota_*"'] & & S_\natural \otimes X(RB) \\
& X(RQ) \arrow[ur, "\alpha v_*"'] &
\end{tikzcd}$$

and want to factor

need v: $RQ \longrightarrow S \otimes RB$

which like u arises from a linear map in this case

$$Q \longrightarrow S \otimes B$$

and really is the map

$$Q \xrightarrow{t^D} Q^{D,\geq 0} \longrightarrow S \otimes B$$

$$X(RQ) \xrightarrow{t^{L_D}} \bigoplus_{k \geq 0} t^k X(RQ)_{\geq k} \longrightarrow S_\natural \otimes X(RB)$$

Claim that L_D on the left ~~comm~~ is compat with D_t on the right.

Note that [redacted]

you have analyzed the 2nd map, you know that the [illegible] degree k component $X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$ carries $\left(F^P_{IQ} \cdots\right)_{\geq k}$ into $J^k_\# \otimes F^P_{IB}$.

Now you must understand the first map. It is perfectly transparent from the viewpoint

$\boxed{n}$ of the grading, namely $X(RQ)_n$ goes by mult. by t^n into $t^n X(RQ)_{\geq n}$.

Another point $Q \xrightarrow{t^D} S \otimes B$ is ~~a graded~~ compatible with grading ~~So where are we?~~ means D and D_t are compatible, so this extends immediately to X-R-a.s.

Consider

$$\begin{tikzcd}
Q \arrow[r, "t^D"] \arrow[d, equal] & Q^t \arrow[d, equal] \\
\bigoplus_k Q_k \arrow[r] & \bigoplus_k t^k Q_{\geq k} \\
x_k \arrow[r, mapsto] & t^k x_k
\end{tikzcd}$$

induces

$$\begin{tikzcd}
RQ \arrow[r] & R_{\mathbb{C}^t}(Q^t) = (RQ)^t \\
X(RQ) \arrow[r] \arrow[rr, bend right, "t^{L_D}"'] & X_{\mathbb{C}^t}(R_{\mathbb{C}^t} Q^t) & = X(RQ)^t
\end{tikzcd}$$

Now I don't know very much above t^{L_D} behavior wrt. $F^p_{IQ} X(RQ)$. Could I find out more

$\boxed{0}$ Back to earlier struggle. I'm trying to produce an account of Joachim's approach to Nistor. This is based on $X(RQ)$ Q filtered algebra $Q = Q_{\geq 0} \supset Q_{\geq 1} \supset \cdots$ etc. with linear splitting: $Q_n \subset Q_{\geq n} \twoheadrightarrow$ [illegible] $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$, $1 \in Q_0$. Get $D = n$ on Q_n extend to D on RQ & grading $RQ = \bigoplus_n (RQ)_n$ $D = n$ on $(RQ)_n$, and to $X(RQ) = \bigoplus_n X(RQ)_n$ $L_D = n$ on $X(RQ)_n$.

Question: You've defined $X(RQ)_{\geq k}$ via $X(RQ)^t = X_{\mathbb{C}^t}(R_{\mathbb{C}^t} Q^t)$. And defined $X(RQ)_n$ via D. How do you see that $X(RQ)_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n$?

There should be some mechanism for handling gradings. A grading on Q is a map $Q \xrightarrow{t^D} \mathbb{C}[t, t^{-1}] \otimes Q$ of some sort. We start with such a thing and ~~define~~ obtain Q^t

What I want is meaning of splitting a filtration. Filtration on Q is graded $\mathbb{C}^{t}$ ~~algebra~~ module

$\boxed{P}$ Lifting of Q into Q^t

Lifting of Q into Q^t.

$Q = \bigoplus_n Q_n$

Idea: A grading on Q yields a Can. map $Q \xrightarrow{t^D} \mathbb{C}[t,t^{-1}]\otimes Q$

Consider the graded $\mathbb{C}^t$ submodule generated by $t^DQ = \bigoplus t^n Q_n \subset \bigoplus t^n Q$. When we take the $\mathbb{C}^t$ submodule we have

$$\bigoplus_n \bigoplus_{k\leq n} t^k Q_n = \bigoplus_k t^k \bigoplus_{n\geq k} Q_n$$

Conversely given $Q_{\geq k}$ and Q^t, we have surjective $Q^t \longrightarrow Q$ sending $t \mapsto 1$ and a lifting should be the same as a grading: Not quite

$tQ_{\geq 0}$

graded complement for $t^{-1}Q^t \subset Q^t$

$Q_{\geq 0}$

$tQ_{\geq 1}$

$t^2Q_{\geq 2}$

when we've split Q we have an isom

$$\mathbb{C}[t^{-1}] \otimes Q \xrightarrow{\sim} Q^t$$

of graded $\mathbb{C}^t$-modules, then an isom.

$$\mathbb{C}[t^{-1}] \otimes RQ \xrightarrow{\sim} RQ^t$$

etc. ~~[illegible] consider gilch~~

Return now to Joachim's version of Mistor construction. Work with $X(RQ)$. On one hand Q has grading, D, and ~~these~~ induces similar structure on RQ, $X(RQ)$

$$Q \xrightarrow{t^D} \mathbb{C}[t,t^{-1}] \otimes Q \implies Q^t \subset \mathbb{C}[t,t^{-1}] \otimes Q$$

yields

$$RQ \longrightarrow \mathbb{C}[t,t^{-1}] \otimes RQ$$

yields

$$X(RQ) \longrightarrow \mathbb{C}[t,t^{-1}] \otimes X(RQ)$$

OKAY at this point we have the grading and filtration organized on $X(RQ)$. From the left side we get

$$h_D : X(RQ) \longrightarrow X(RQ)$$

$$\left(F^p_{IQ} X(RQ)\right)_{\geq k} \longrightarrow \left(F^{p-2} \ldots\right)$$

r 7/21 - 0530

Start with a review of Joachim's version of the Nistor construction.

Let $Q = QA$, $Q_{\geq k}$ ~~=~~ q^k filtration

$Q_n = \Omega^n$ grading

From the filtration get [redacted] filtration

[redacted] $X(RQ)_{\geq k}$ defined by

$$X(RQ)^t = \text{Image}$$

$$X_{\mathbb{C}^t}(R_{\mathbb{C}^t} Q^t) \longrightarrow \mathbb{C}[t, t^{-1}] \otimes X(RQ)$$

Also for $(F^P_{IQ} X(RQ))_{\geq k}$

From the grading we get a grading on $X(RQ)$ ~~i.e. [illegible] map~~ defined how?

The grading on Q gives an isom of $\mathbb{C}^t$ modules

$$\mathbb{C}[t^{-1}] \otimes Q \xrightarrow{\sim} Q^t$$

$$\begin{tikzcd}
X_{\mathbb{C}^t}(R_{\mathbb{C}^t}(\mathbb{C}^t \otimes Q)) \arrow[r, "\sim"] \arrow[d, equal] & X_{\mathbb{C}^t}(R_{\mathbb{C}^t} Q^t) \arrow[d, equal] \\
\mathbb{C}^t \otimes X(RQ) \arrow[r] & X(RQ)^t
\end{tikzcd}$$

because $X(RQ)$ depend on Q and $\mathbb{C} \to Q$

~~Define~~ ~~$\delta(\theta) = \theta^2$~~ [illegible] $= 0$

~~then~~

$\boxed{S}$ $Q = QA$, $Q_{\geq k} = q^k$, $Q_n = \Omega^n$

filtration yields filtrations on $X(RQ)$, $F^p_{IQ}X(RQ)$

$$\begin{array}{ccc} \left(F^p_{IQ}X(RQ)\right)^t & = & F^p_{I_{\mathbb{C}^t}Q^t}X_{\mathbb{C}^t}(R_{\mathbb{C}^t}Q^t) \\ \wr\| & & \wr\| \\ \left(F^p\Omega Q\right)^t & & F^p\,\Omega_{\mathbb{C}^t}Q^t \end{array}$$

— grading of Q as vector space with 1 ~~leads to~~ yields a grading of $X(RQ)$

$$Q \xrightarrow{t^D} \mathbb{C}[t,t^{-1}]\otimes Q$$

map of graded vector spaces with 1 induces

$$X(RQ) \longrightarrow \mathbb{C}[t,t^{-1}]\otimes X(RQ)$$

corresp to a grading on $X(RQ)$.

— D on Q gives rise to L_D, h_D on $X(RQ)$

focus, maybe list objects

Somehow you want to list the structure

filtration $X(RQ)_{\geq k}$, $\left(F^p_{IQ}X(RQ)\right)_{\geq k}$

grading $X(RQ)_n$

L_D, h_D

$\boxed{17}$ Order I would like I think is

filtration $Q_{\geq k}$ $\Longrightarrow$ filtration $X(RQ)_{\geq k}$, $\left(F^{P}_{IQ}\cdots\right)_{\geq k}$

$D: Q \to Q$ resp $Q_{\geq k}$ $\Longrightarrow$ L_D, h_D on $X(RQ)$

mapping $\mathcal{F}^{p}_{k} \longrightarrow \mathcal{F}^{p-2}_{k}$

Next ~~take~~ assume ~~trivial spl~~ given grading on Q splitting filtration, take $D =$ degree op.

grading Q_n $\Longrightarrow$ grading $X(RQ)_n$

$\{Q_n\} + \{Q_{\geq k}\}$ comp $\Longrightarrow$ $X(RQ)_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n$

$D = n$ on Q_n $\Longrightarrow$ $h_D = n$ on $X(RQ)_n$

~~$\delta(a_1 \otimes a_2) = (1 \otimes a_1 - a_1 \otimes 1) \otimes a_2$~~

~~$+ a_1 (1 \otimes 1 \otimes 1) a_2$~~

~~$- a_1 \otimes (1 \otimes a_2 - a_2 \otimes 1)$~~

~~$= 1 \otimes a_1 \otimes a_2 - a_1 \otimes 1 \otimes a_2 + a_1 \otimes a_2 \otimes 1$~~

~~$\delta(a_1 \otimes a_2 \otimes a_3) = (1 \otimes a_1 \otimes a_2 - a_1 \otimes 1 \otimes a_2 + a_1 \otimes a_2 \otimes 1) \otimes a_3$~~

~~$- a_1 \otimes a_2 (1 \otimes 1 \otimes 1) a_3$~~

~~$+ a_1 \otimes a_2 \otimes (1 \otimes a_3 - a_3 \otimes 1)$~~

~~$[\delta, \partial] = \delta\partial + \partial\delta$~~

~~$(\delta\partial + \partial\delta)(1) = \delta(1) + \partial(D^2) = 0$~~

~~$(\delta\partial + \partial\delta)(a) = \partial([0,a]) = [1,a] = 0.$~~

Repeat

Let Q, $(Q_{\geq k})$ be a filtered algebra

Then $X(RQ)$, $F^p_{IQ}X(RQ)$ inherit filtrations

Let $D : Q \to Q$ ~~resp~~ be linear resp. filt

Then L_D, h_D resp $X(RQ)_{\geq n}$, map $F^p_{\geq k} \to F^{p-2}_{\geq k}$

Let $Q_* = \bigoplus Q_n$, $1 \in Q_0$ be a linear grading of Q. Then $X(RQ)$ inherit grading

$$Q_{\geq k} = \bigoplus_{n \geq k} Q_n \Longrightarrow X(RQ)_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n$$

$$D = n \text{ on } Q_n \Longrightarrow L_D = n \text{ on } X(RQ)_n.$$

~~[illegible]~~. Now put this together

For each k have mixed complex $\Omega Q_{\geq k}$ ~~form a $\Omega Q_{\geq k}$~~ The cyclic homology type of $\Omega Q_{\geq k}$ is rep by the tower $X(RQ)_{\geq k} / (F^p \quad)_{\geq k}$

~~Note that~~

$$X(RQ)^t / \left(F^p_{IQ} X(RQ)\right)^t$$

tower of graded $\mathbb{C}^t$-modules, clearly special.

Notation needed,

Claim is that in derived cat of mixed complexes we have certain maps and properties.

$\boxed{V}$ 7/21 - 1342 run into a problem namely

$$S_n = 1 - \frac{1}{n} L_D : F^p X_{\geq n} \longrightarrow F^{p-2} X_{\geq n+1} \quad ?$$

I know $L_D : F^p X_{\geq n} \longrightarrow F^{p-2} X_{\geq n} / F^{p-2} X_{\geq n+1}$

I know that

$$\begin{array}{rcl} L_D : F^p X_{\geq n} & \longrightarrow & F^{p-2} X_{\geq n} \\ \cup & & \cup \\ L_D : F^p X_{\geq n+1} & \longrightarrow & F^{p-2} X_{\geq n+1} \end{array}$$

but I don't ~~necessarily~~ yet have $L_D = n$ on this quotient.

$$D : (IQ)_{\geq n} \longrightarrow (RQ)_{\geq n}$$

See if it works using the T-formalism. Consider

So what do I use at this point?

Go back to L_D on $X(RQ)^t$. We

$D = n$ on $Q_{\geq n} / Q_{\geq n+1}$

What you would like to do is to

~~Consider~~ Grading

$$\mathbb{C}[t^{-1}] \otimes Q \xrightarrow{\sim} Q^t$$

isomorphism of graded V.S.

$$t\partial_t \otimes 1 + 1 \otimes D \longleftrightarrow t\partial_t$$

$$L_D : F^p_{I_T Q^t} X_T(R_T Q^t) \longrightarrow F^{p-2}_{I_T Q^t} X_T(R_T Q^t)$$

$\boxed{W}$ So what to do.

$$\begin{tikzcd} t^D : Q \arrow[r] & \mathbb{C}[t,t^{-1}]\otimes Q \\ & Q^t \arrow[u, phantom, "\cup" description] \end{tikzcd}$$

So what happens. You have

$$\begin{tikzcd} \mathbb{C}[t^{-1}]\otimes Q \arrow[r, "\sim"] & Q^t = \bigoplus_k t^k Q_{\geq k} \\ Q \arrow[u, phantom, "\bigcup" description] \arrow[r, "t^D"] & \mathbb{C}[t,t^{-1}]\otimes Q \arrow[u, phantom, "\cap" description] \end{tikzcd}$$

So the fact is that

$$\begin{tikzcd} \overset{t\partial_t + L_D}{\mathbb{C}[t^{-1}] \otimes X(RQ)} \arrow[r, "\sim"] \arrow[d] & X(RQ)^t \quad t\partial_t \arrow[d] \\ \underset{L_D}{X(RQ)} \arrow[r] & \underset{t\partial_t}{X(RQ)^t / t^{-1} X(RQ)^t} \end{tikzcd}$$

But now . consider $\quad \boxed{F^p_{IQ^t} X(RQ)^t}$

Take L_D on $F^p_{IQ^t} X(RQ)^t$

~~L_D commu~~ L_D commutes with t^{-1}!

$$F^p_{IQ^t} X(RQ)^t \subset \underset{L_D}{X(RQ)^t}$$

$\boxed{x}$

$$\begin{tikzcd}
0 \arrow[r] & F^{p}_{IQ^t} X(RQ)^t \arrow[r, "t^{-1}"] \arrow[d, "L_D"] & F^{p}_{IQ^t} X(RQ)^t \arrow[r] \arrow[d, "L_D"] & gr\left(F^{p}_{IQ} X(RQ)\right) \arrow[r] \arrow[d, "(t\partial_t)\,?"] & 0 \\
0 \arrow[r] & F^{p-2}_{IQ^t} X(RQ)^t \arrow[r, "t^{-1}"] & F^{p-2}_{IQ^t} X(RQ)^t \arrow[r] & gr\left(F^{p-2}_{IQ} X(RQ)\right) \arrow[r] & 0
\end{tikzcd}$$

$t\partial_t$ × canonical map induced by the inclusion

[redacted]

Point is that

$$F^{p}_{IQ^t} X(RQ)^t \twoheadrightarrow gr\left(F^{p}_{IQ} X(RQ)\right)$$

should be a proof.

$$\left(F^{p}_{IQ} X(RQ)\right)_{\geq k}$$

What I want is to take $F^{p}_{IQ} X(RQ)_{\geq k}$ and to define

7/22-0522 Yesterday I encountered another problem. I need to know that $S_n = 1 - \frac{1}{n} L_D$ going from $X_{\geq n}$ to $X_{\geq n+1}$ actually carries $F^{p}_{*} X_{\geq n}$ into $F^{p-2} X_{\geq n+1}$. So far I only know that

$$S_n\left(F^p X_{\geq n}\right) \subset F^{p-2} X_{\geq n} \cap X_{\geq n+1}$$

~~The problem is this~~ First do it elementwise

Consider $\left(IQ^{\cancel{n+1}}\right)_{\geq k}$

$\boxed{y}$ Consider $(IQ^n)_{\geq k}$ spanned by

$$\xi = \rho(x_0)\,\omega(x_1, x_2) \cdots \omega(x_{2n-1}, x_{2n}) \qquad \sum \operatorname{ord}(x_i) \geq k$$

modulo $(IQ^n)_{\geq k+1}$ I can assume x_i homogeneous: $x_i \in Q_{k_i}$

$\sum k_i = k$.

Then $D\rho(x_0) = \rho(Dx_0) = k_0\,\rho(x_0)$

$$D\,\omega(x_1, x_2) = D\big(\rho(x_1 x_2) - \rho(x_1)\rho(x_2)\big)$$

$$= \rho\big(D(x_1x_2)\big) - \rho(Dx_1)\rho(x_2) - \rho(x_1)\rho(Dx_2)$$

$$= \underbrace{\rho\big(D(x_1x_2) - Dx_1\,x_2 - x_1Dx_2\big)}_{\in\, \rho(Q_{>k_1+k_2})} + \underbrace{\omega(Dx_1, x_2) + \omega(x_1, Dx_2)}_{(k_1+k_2)\,\omega(x_1, x_2).}$$

Thus ~~$D\,(IQ^n)_{\geq k} \subset (IQ^{n-1}$~~

$$D\xi - k\xi \in (IQ^{n-1})_{\geq k+1}.$$

This should work. Let's find an abstract argument. ~~So what?~~ What is needed is to amplify

$h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$ to include

$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$.

Thus you want $L_D - t\partial_t$ to map $(F^pX)^t$ into $t^{-1}(F^{p-2}X)^t$

What is the point?

$X = X(RQ) \qquad F^P = F^P_{IQ}$

$$X^t = X_T(R_T Q^t) \qquad (F^P X)^t = F^P X_T(R_T Q^t)$$

This time you want: $T/t^{-1}T \xrightarrow{\sim} \mathbb{C}$.

$$X^t/t^{-1}X^t = X(R(\mathrm{gr}\,Q))$$

How are ~~$(F^P X)^t/t^{-1}(F^P X)^t$~~

$$F^P(X^t)/t^{-1}F^P(X^t) \quad \text{and} \quad F^P X(R(\mathrm{gr}\,Q))$$

related. I want a

First look at the algebras. I have R^t with ideal I^t over $T = \mathbb{C}^t$.

I am ~~R^t~~ interested in R^t/mR^t $\qquad m = t^{-1}$

actually in $(I^t)^n/m(I^t)^n \underset{\substack{\uparrow \\ \text{NO}}}{=} (I^t/mI^t)^n$?

$$I^t = \oplus\, t^k I_{\geq k}$$

$$(I^t)^n = \oplus\, t^k \Big(\sum_{\sum k_i = k} I_{\geq k_1} \cdots I_{\geq k_n}\Big)$$

Meaning of the calculation that

$$D\,\omega(x_1, x_2) = \rho\big(D(x_1x_2) - Dx_1 x_2 - x_1 Dx_2\big) + \omega(Dx_1, x_2) + \omega(x_1, Dx_2)$$

Statement that D is a derivation on the associated graded algebra $\mathrm{gr}\,Q$.

$$D(I_{\geq k}) \subset I_{\geq k} + R_{\geq k+1}$$

A) $D(I^t) \subset I^t + t^{-1}R^t$

Thus $D((I^n)^t) = D((I^t)^n)$

$$\subset (I^t)^n + t^{-1}(I^t)^{n-1}$$

Want $D - t\partial_t : I^t \to t^{-1}R^t$

$$\begin{tikzcd}
 & F^pX^t \arrow[dl, dashed] \arrow[d, "D-t\partial_t"] \\
F^{p-2}X^t \arrow[r, "t^{-1}"] & F^{p-2}X^t
\end{tikzcd}$$

So see if we can make this work.

Details and review. We have

Q alg w. filtration $Q_{\geq k}$ comp alg st. $\bigcup Q_{\geq k} = Q$

form $Q^t = \bigoplus_{k\in\mathbb{Z}} t^k Q_{\geq k} \subset \mathbb{C}[t,t^{-1}]\otimes Q$

graded $T = \mathbb{C}^t$ alg

induced filtrations $(RQ)_{\geq k}$, $X(RQ)_{\geq k}$

also $(IQ^n)_{\geq k}$, $(F^p_{IQ}X(RQ))_{\geq k}$

formulas. $X(RQ)^t = X_T(R_TQ^t) \simeq \Omega_T(Q^t)$

$$(F^p_{IQ}X(RQ))^t = F^p_{I_TQ^t}X_T(R_TQ^t) \simeq F^p\Omega_T(Q^t).$$

grading $Q_* = \bigoplus_n Q_n$ comp $Q_{\geq k} = \bigoplus_{n\geq k} Q_n$, $1 \in Q_0$

$D : \bar{Q} \to Q$ $D = n$ on Q_n. D on Q^t comm. with t^{-1}

get $\hat{D}$ derv on RQ, on RQ^t ..

$L_{\hat{D}}$ on $X(RQ)$, on $X(RQ)^t$

B) Concentrate on the properties of L_D, h_D on $X = X(RQ)$ relative to the bifiltration $F^pX_{\geq k}$:

$$h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$$

$$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$$

Proof of former. Extend D to Q^t to commute with T, then to D on $R_TQ^t = (RQ)^t$, then to L_D, h_D on $X_T(R_TQ^t) = X(RQ)^t$. We know by relative version of 4.?

$$\begin{tikzcd}
L_D, h_D : F^p_{I_TQ^t} X_T(R_TQ^t) \arrow[r] \arrow[d, phantom, "\parallel"] & F^{p-2}_{I_TQ^t} X_T(R_TQ^t) \arrow[d, phantom, "\cup"] \\
(F^pX)^t & (F^{p-2}X)^t
\end{tikzcd}$$

Proof of latter. Consider $L_D - D_t = L_D - t\partial_t$ on $X_T(R_TQ^t) = X(RQ)^t$. Then what happens to $(F^pX)^t$? What happens to I^t. Claim $(L_D - D_t)(I^t) \subset t^{-1}R^t$

Suffices to check for generator of I^t.

$$\begin{tikzcd}
0 \arrow[r] & I^t \arrow[r] & R^t \arrow[r] \arrow[d] & Q^t \arrow[r] & 0 \\
& & R^t/t^{-1}R^t \arrow[d, phantom, "\parallel"] & & \\
0 \arrow[r] & I(\mathrm{gr}\,Q) \arrow[r] & R(\mathrm{gr}\,Q) \arrow[r] & \mathrm{gr}\,Q \arrow[r] & 0 \\
& & \text{here } L_D = D_t \arrow[u] & &
\end{tikzcd}$$

c)

$$\begin{tikzcd}
 & & 0 \arrow[d] & & \\
 & & R^t \arrow[d, "t^{-1}"] & L_D - D_t \arrow[dl, dash] & \\
0 \arrow[r] & I^t \arrow[r] \arrow[d] & R^t \arrow[r] \arrow[d] & Q^t \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & I(grQ) \arrow[r] \arrow[d] & R(grQ) \arrow[r] \arrow[d] & grQ \arrow[r] & 0 \\
 & 0 & 0 & &
\end{tikzcd}$$

$L_D - D_t \ : \ R^t \longrightarrow t^{-1}R^t$ So what next you idiot??

$$L_D - k \ : \ F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$$

$$h_D \ : \ F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$$

$$\begin{tikzcd}
F^p X_{\geq k+1} \arrow[r, phantom, "\subset"] \arrow[d, phantom, "\cap"] & F^{p-2} X_{\geq k+1} \arrow[d, phantom, "\cap"] \\
F^p X_{\geq k} \arrow[r, phantom, "\subset"] \arrow[ur, "1 - k^{-1}L_D"] & F^{p-2} X_{\geq k}
\end{tikzcd}$$

The other point is ~~how to get from $F^pX_{\geq}$~~

$$F^p X_{\geq k} \sim F^p(\Omega Q_{\geq k})$$

$$\left(\chi^p_{\geq k} = X_{\geq k}/F^p X_{\geq k}\right) \sim \Theta(\Omega Q_{\geq k})$$

D)

$$(IQ^n)_{\geq k} = \sum_{\sum k_i = k} (IQ)_{\geq k_1} \cdots (IQ)_{\geq k_n}$$

$$F^{2n}X_{\geq k} = IQ^{n+1}_{\geq k} + \sum_{i+j=n} [IQ^n_{\geq i}, RQ_{\geq j}] \leftrightarrows$$

$$\sum_{i+j=n} \natural \left(IQ^n_{\geq i}\, d(RQ_{\geq j})\right)$$

I want the minimum to say about Joachim's version of Nistor's construct.

Q filtered alg, w. filt $Q_{\geq n}$, linear splitting of filt.

$X = X(RQ)$ $\qquad$ $F^pX = F^p_{IQ}\, X(RQ)$

$X_{\geq n} = (X(RQ))_{\geq n}$ $\qquad$ $F^pX_{\geq n} = (\text{———})_{\geq n}$

D = deriv. on RQ extending degree op. on Q

L_D, h_D on X.

Claims ① $\left(\mathcal{X}^p_{\geq n} = X_{\geq n}/F^pX_{\geq n}\right) \sim \theta(\Omega Q_{\geq n})$

②

$$\begin{tikzcd}
F^pX_{\geq n+1} \arrow[r, "h_D"] & F^{p-2}_{\geq n+1} \\
\cap & \cap \\
F^pX_{\geq n} \arrow[r, "h_D"'] \arrow[ruu, "S_n = 1 - \frac{1}{n}L_D"] & F^{p-2}_{\geq n}
\end{tikzcd}$$

$n > 0$

gives

E)

$$\begin{tikzcd}
X_{\geq n+1} \arrow[r, "S"] \arrow[d, "\iota_n"'] & X_{\geq n+1}[2] \arrow[d, "\iota_n"] \\
X_{\geq n} \arrow[r, "S"'] \arrow[ur, "s_n"] & X_{\geq n}[2]
\end{tikzcd}$$

i.e. $[\iota_n] \in HC^0(X_{\geq n+1}, X_{\geq n}) = HC^0(\Omega_{\geq n+1}, \Omega_{\geq n})$

$[S_n] \in HC^2(X_{\geq n}, X_{\geq n+1}) = HC^2(\Omega_{\geq n}, \Omega_{\geq n+1})$

satisfy $[S_n][\iota_n] = S \in HC^2(\Omega_{\geq n+1}, \Omega_{\geq n+1})$

$[\iota_n][S_n] = S \in HC^2(\Omega_{\geq n}, \Omega_{\geq n})$

What's the next point? [redacted]

Go back over *Joachim's version* of the Nistor construction

$$\begin{tikzcd}
A \arrow[r, shift left, "\iota"] \arrow[r, shift right, "\iota^{\gamma}"'] & Q \arrow[r] & L \otimes B \\
 & q^k \arrow[r, mapsto] & J^k \otimes B
\end{tikzcd}$$

$$A \xrightarrow{i} Q \longrightarrow$$

My map

$$X(RA) \longrightarrow X(S \otimes RB) \longrightarrow S_\natural \otimes X(RB) \longrightarrow J_{\#}^{2m+1} \otimes X(RB)$$

Joachim's map

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] & {} \\
 & X(RQ)_{\geq k} \arrow[u, phantom, "\cup"] \arrow[r] & J_{\#}^k \otimes X(RB)
\end{tikzcd}$$

F)

$$\begin{tikzcd}
X(RA) \arrow[r] \arrow[d] & S_{\natural} \otimes X(RB) \arrow[d] \\
X(RQ) \arrow[d, dash] & {} \\
{} &
\end{tikzcd}$$

I don't know what the next step should be. I guess I can take Joachim's construction

$$\begin{tikzcd}
X(RA) \arrow[r, "i_*"] & X(RQ) \arrow[r] \arrow[d, "\pi_-"] & S_{\natural} \otimes X(RB) \arrow[d] \\
 & \pi_- X(RQ)_{\geq 1} \arrow[r] \arrow[d, "S_1"] & \pi_- S_{\natural, \geq 1} \otimes X(RB) \arrow[d] \\
 & \pi_- X(RQ)_{\geq 3} \arrow[d, "S_3"] & {} \\
 & {} \arrow[d, "S_{2m-1}"] & {} \arrow[d] \\
 & \pi_- X(RQ)_{\geq 2m+1} \arrow[r] \arrow[rd] & \pi_- S_{\natural, \geq 2m+1} \otimes X(RB) \arrow[d, "\delta_1"] \\
 & & J_{\#}^{2m+1}
\end{tikzcd}$$

What do I do?

I need to know

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d, phantom, "\cup"] & S_{\natural} \otimes X(RB) \arrow[d, phantom, "\cup"] \\
X(RQ)_{\geq n} \arrow[r] & S_{\natural, \geq n} \otimes X(RB) \arrow[d] \\
 & {}
\end{tikzcd}$$

6) 7/23 -

Joachim's construction

$Q = QA$ filt $Q_{\geq k}$, grading Q_n

structure on $X = X(RQ)$, $F^pX = F^p_{IQ} X(RQ)$

$X_{\geq k}$, $F^pX_{\geq k}$

Prop: 1) $L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$

2) $L_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$

3) $\gamma - (-1)^k : F^pX_{\geq k} \longrightarrow F^pX_{\geq k+1}$

~~Construction $X(RA) \xrightarrow{k^*} X(RQ)_A$~~

Consequence Put $\mathcal{X}_{\geq k} = (\mathcal{X}^p_{\geq k})$, $\mathcal{X}^p_{\geq k} = X_{\geq k}/F^pX_{\geq k}$

Then

$$\begin{tikzcd}
\gamma_- \mathcal{X}_{\geq 2m+1} \arrow[r, "S"] \arrow[d, "L_k"'] & \gamma_- \mathcal{X}_{\geq k+1}[2] \arrow[d, "L_k"] \\
\gamma \mathcal{X}_{\geq k} \arrow[r, "S"] & \mathcal{X}_{\geq k}[2]
\end{tikzcd}$$

$1 - \frac{1}{2m-1} L_D : \gamma F^pX_{\geq 2m-1}$

$\longrightarrow \gamma F^pX_{\geq 2m} = \gamma_- F^pX_{\geq 2m+1}$

$$\begin{tikzcd}
\gamma_- \mathcal{X}_{\geq 2m+1} \arrow[r, "S"] \arrow[d, "L_m"'] & \gamma_- \mathcal{X}_{\geq 2m+1}[2] \arrow[d, "L_m"] \\
\gamma_- \mathcal{X}_{\geq 2m-1} \arrow[r, "S"] \arrow[ur, "1 - \frac{1}{2m-1} L_D"] & \gamma_- \mathcal{X}_{\geq 2m-1}[2]
\end{tikzcd}$$

$[S_m] \circ \cdots \circ [S_1] \in HC^{2m}(\gamma_- \mathcal{X}, \gamma_- \mathcal{X}_{\geq 2m+1})$

$[S_m] \circ \cdots \circ [S_1] \circ \gamma_- \circ \iota \in HC^{2m}(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 2m+1})$

H) Joachim-Nistor const.

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] \arrow[d, "\gamma_-"] & S_\flat \otimes X(RB) \arrow[d] \\
& \gamma_- X_{\geq 1} \arrow[r] \arrow[d, "P_m(L_D)"] & S^{odd}_{\flat, \geq 1} \otimes X(RB) \arrow[d, "P_m(t\partial_t)"] \\
& \gamma_- X_{\geq 2m+1} \arrow[r] \arrow[dr] & S^{odd}_{\flat, \geq 2m+1} \otimes X(RB) \arrow[d, "e_1"] \\
& & J^{2m+1}_\# \otimes X(RB)
\end{tikzcd}$$

Before this I should give Joachim's map

$$X(RA) \xrightarrow{\gamma_- i} \gamma_- X(RQ) \xrightarrow{P_m(L_D)} \gamma_- X(RQ)_{\geq 2m+1} \xrightarrow{\circledast} J^{2m+1}_\# \otimes X(RB)$$

$$F^p_{IA} X(RA) \longrightarrow \gamma_- F^p X_{\geq 1} \longrightarrow \gamma_- F^{p-2m} X_{\geq 2m+1} \longrightarrow J^{2m+1}_\# \otimes F^p_{IB} X(RB)$$

What remains is the last map $\circledast$, its properties, relation to $X(RQ) \longrightarrow S_\flat \otimes X(RB)$, the map induced by $X \circ R$? applied to

$$Q \xrightarrow[\text{linear}]{t^D} \bigoplus t^n Q_{\geq n} \xrightarrow[\text{hom.}]{} S \otimes B$$

I) 7/24 0530 $\mathcal{X}_{\geq k} = \left(X_{\geq k} / F^p X_{\geq k} \right) \sim Q^b_{\geq k}$

1) $L_D, h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$

2) $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

3) $\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

need $L_D = [\partial, h_D]$, $[L_D, h_D] = 0$

last needed to say $\gamma = (-1)^{L_D}$ commute

$$\begin{tikzcd}
\gamma_- F^p X_{\geq k+1} \arrow[r] & \gamma_- F^{p-2} X_{\geq k+1} \\
\gamma_- F^p X_{\geq k} \arrow[u, phantom, "\cap"] \arrow[r] \arrow[ur, "1 - \frac{1}{k} L_D"] & \gamma_- F^p X_{\geq k} \arrow[u, phantom, "\cap"]
\end{tikzcd}$$

$[\partial_D, \gamma_- h_D] = \gamma_- L_D$ Anyway

$$\begin{tikzcd}
\mathcal{X}_{\geq k+1} \arrow[r, "S"] \arrow[d, "\iota_k"'] & \mathcal{X}_{\geq k+1}[2] \arrow[d, "\iota_k"] \\
\mathcal{X}_{\geq k} \arrow[r, "S"] \arrow[ur, "S_k"] & \mathcal{X}_{\geq k}[2]
\end{tikzcd}$$

so $[S_k]$ etc. etc. etc. etc.

Let's try to define the character

$Ch^0 \in HC^0(A, \gamma_- Q^b_{\geq 1})$

$Ch^0 : X(RA) \longrightarrow X(RQ) \xrightarrow{\gamma_-} \gamma_- X(RQ)$

J) Define Chern character -

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X(RQ)_{\geq 1}$$

$$\gamma_- \iota_* = \frac{1}{2}\left(\iota_* - \iota^{\gamma}_*\right)$$

$$S_{2m-1} \, S_{2m-3} \cdots S_3 \cdot S_1 \; \gamma_- \iota_*$$

define $Ch^{2m} \in HC^{2m}\Big(A, \underbrace{\gamma_- Q^b_{\geq 2m+1}}_{\gamma_-(\Omega Q)_{\geq 2m+1}}\Big)$

$$\gamma_-(\mathcal{X}_Q)_{\geq 2m+1}$$

Nistor notation Q^b = mixed cx assoc. to

So how to say all this. We have maps of supercomplexes

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X(RQ)_{\geq 1}$$

$$\xrightarrow{s_1} \gamma_- X(RQ)_{\geq 3} \longrightarrow \cdots$$

$$\xrightarrow{s_{2m-1}} \gamma_- X(RQ)_{\geq 2m+1}$$

$$F^p_{IA} X(RA) \longrightarrow F^p_{IQ} X(RQ) \longrightarrow \gamma_- F^p X_{\geq 1}$$

$$\xrightarrow{s_1} \gamma_- F^{p-2} X_{\geq 3} \longrightarrow \cdots$$

$$\xrightarrow{s_{2m-1}} \gamma_- F^{p-2m} X_{\geq 2m+1}$$

$$\therefore \left[s_{2m-1} \, s_{2m-3} \cdots s_3 s_1 \gamma_- \iota_*\right] \in HC^{2m}\left(\mathcal{X}_A, \gamma_- \mathcal{X}_{Q, \geq 2m+1}\right)$$

K) Digress to have fuss

Recall $T = \mathbb{C}[t^{-1}] = \mathbb{C}^t$

V filtered v.s. $V_{\geq k}$ ~~$\cup V_{\geq k} = V$~~

$$V^t = \bigoplus t^k V_{\geq k} \subset \mathbb{C}[t,t^{-1}] \otimes V$$

Can identify a filtration on V with a graded $\mathbb{C}^t$ submodule of $\mathbb{C}[t,t^{-1}] \otimes V$ in this way.

I would like to understand abstractly why $L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$ and $\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$.

This means forming $F^p X^t = \bigoplus t^k F^p X_{\geq k}$ which I know is $F^p_{I_T Q^t} X_T(R_T Q^t)$

To start with $D : \bar{Q} \to Q$ deg op. only linear but such that $\gamma = (-1)^D$ is comp. with alg. structure. $\therefore$ $\gamma(IQ) \subset IQ$

You want

to consider L_D on $X^t = X(RQ)^t$. commutes with t^{-1}, T linear. Consider $F^p X^t = F^p_{I^t} X_T(R^t)$

Old proof gives $L_D, h_D : F^p X^t \longrightarrow F^{p-2} X^t$

But we know $D : R^t \longrightarrow R^t$

~~[illegible]~~ $D = D_t$ on Q^t. no.

$Q^t = \bigoplus t^k Q_{\geq k}$

$D - D_t : Q^t \longrightarrow t^{-1} Q^t$

L

Joachim's version of Nistor's construction

$Q = QA$ filt. $Q_{\geq k}$ comp. w. alg st.

v.s. grading $Q = \bigoplus Q_n \longrightarrow$ filt.

D degree op.

superalg $\gamma = (-1)^D$ alg. autom order 2 resp. filt. + grading.

$X = X(RQ)$, $F^pX = F^p_{IQ}\,X(RQ)$.

$X_{\geq k}$, $F^pX_{\geq k}$ induced filt., $\mathcal{X}_{\geq k} = (X_{\geq k}/F^pX_{\geq k})$

Claim: $\mathcal{X}_{\geq k} \sim$ Hodge tower of $\mathbf{F}_{-k}(QA, qA)^b$

$\Omega Q_{\geq k}$ in Nistor's notation.

$$\begin{tikzcd}
F^pX_{\geq k+1} \arrow[r, hook] \arrow[d, phantom, "\cap"] & F^{p-2}X_{\geq k+1} \arrow[d, phantom, "\cap"] \\
F^pX_{\geq k} \arrow[r, hook, "\frac{1}{k}h_D"] \arrow[ur, "\| \frac{1}{k}h_D"', "S_k"] & F^{p-2}X_{\geq k}
\end{tikzcd}$$

~~What I want to~~ [redacted] ~~make to myself is that~~

Claim: $L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$.

Put $S_k = 1 - \frac{1}{k}L_D$ $k \neq 0$. Then

$$X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_- X(RQ) = \gamma_- X(RQ)_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- X(RQ)_{\geq 2} = \gamma_- X(RQ)_{\geq 3}$$

$$\xrightarrow{S_{2m-1}} \gamma_- X(RQ)_{\geq 2m} = \gamma_- X(RQ)_{\geq 2m+1}.$$

M) carries

$$F^p_{IA}X(RA) \longrightarrow F^p X \xrightarrow{\gamma_-} \gamma_- F^p X = \gamma_- F^p X_{\geq 1}$$

$$\xrightarrow{S_1} \gamma_- F^{p-2} X_{\geq 2} = \gamma_- F^{p-2} X_{\geq 3}$$

$$\xrightarrow{S_{2m-1}} \gamma_- F^{p-2m} X_{\geq 2m} = \gamma_- F^{p-2m} X_{\geq 2m+1}.$$

for all p whence we have a map of special towers

$$\mathcal{X}_A \longrightarrow \mathcal{X}_{Q,\geq 2m+1}[2m]$$

i.e. $\quad ch^{2m} \in HC^{2m}(\Omega A, \Omega Q_{\geq 2m+1})$

Nistor uses the map

$$S_n \cdot S_{n-1} \cdot \cdots \cdot S_1 \gamma_- \iota$$

The point is that he constructs

$$[S_k] \in HC^2(\Omega Q_{\geq k}, \Omega Q_{\geq k+1}) \qquad k \geq 1.$$

Then defines

$$ch^{2n} = [S_n] \cdots [S_1][\gamma_- \iota] \in HC^{2n}(\Omega A, \Omega Q_{\geq n+1})$$

Point is that on the image γ_- S_k for k even unnecessary.

N)

Next step – end map

$$Q \xrightarrow{t^D} Q^t \xrightarrow[\text{hom.}]{} L^t \otimes B$$

$$X(RQ) \xrightarrow{t^{L_D}} X(RQ)^t \longrightarrow L^t_\eta \otimes X(RB)$$

The claim is that ~~we have a~~ they homom.

$$Q^t \longrightarrow L^t \otimes B$$

~~which~~ induces

$$\begin{tikzcd}
X^t = X(RQ)^t \arrow[r] & L^t_\eta \otimes X(RB) \\
F^p X^t \arrow[u, phantom, "\cup"] \arrow[r] & L^t_\eta \otimes F^p_{IB}
\end{tikzcd}$$

This amounts to a compatibles map [redacted]

$$F^p X_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB}$$

or

$$\begin{tikzcd}
X_{\geq k} \arrow[r] & J^k_\# \otimes X(RB) \\
F^p X_{\geq k} \arrow[u, phantom, "\cup"] \arrow[r] & J^k_\# \otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

Can define it.

<u>hom</u> $Q^t \longrightarrow L^t \otimes B$ of graded T-alg.

get

$$\begin{tikzcd}
X_T(R_T Q^t) \arrow[r] \arrow[d, "\cong"'] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[d, "\cong"] \\
X(RQ)^t & L^t_\eta \otimes X(RB)
\end{tikzcd}$$

,

$$F^p_{I_T Q^t} X_T(R_T Q^t) \longrightarrow F^p_{I_{L^t}(L^t \otimes B)} X_{L^t}(R_{L^t}(L^t \otimes B))$$

0) Concretely have $Q \longrightarrow L \otimes B$

$Q_{\geq k} \longrightarrow J^k \otimes B$

$$\begin{tikzcd}
\Omega Q \arrow[r] & L \otimes \Omega B \\
\Omega Q_{\geq k} \arrow[u, phantom, "\cup" description] \arrow[r] \arrow[dr] & J^k \otimes \Omega B \arrow[u, phantom, "\cup" description] \arrow[d] \\
 & J^k_{\#} \otimes \Omega B
\end{tikzcd}$$

compatible with b on ΩQ and ΩB.

$$X_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$$

$$F^p X_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB}(RB)$$

~~Yes give this pen some practice~~

~~Yes give this pen some practice~~

What is next? I need

$$Q \xrightarrow{t^0} Q^t$$

Not really to do the Nistor const. All you need is the maps

$$\begin{tikzcd}
X_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RB) \\
F^p X_{\geq k} \arrow[u, phantom, "\cup" description] \arrow[r] & J^k_{\#} \otimes F^p_{IB} X(RB).
\end{tikzcd}$$

Anyway

$$HC^0\left((\Omega Q)_{\geq k}, J^k_{\#} \otimes \Omega B\right).$$

So combine with

$$Ch^{2m} = [S_{2m-1}][S_{2m-3}] \cdots [S_1][\gamma_- \iota_*] \in HC^{2m}(\Omega A, (\Omega Q)_{\geq 2m+1})$$

to get a class in $HC^{2m}(\Omega A, J^{2m+1}_{\#} \otimes \Omega B)$.

P)

Now I want to correlate with my construction

$$\begin{tikzcd}
X(RA) \arrow[r] & X(RQ) \arrow[r] \arrow[d, "\gamma_-"] & S_q \otimes X(RB) \arrow[d, "\pi_-"] \\
& \gamma_- X(RQ) \arrow[r] \arrow[d, "P_m(L_D)"] & S_q^{odd} \otimes X(RB) \arrow[d, "P_m(D_t)"] \\
& \gamma_- X(RQ)_{\geq 2m+1} \arrow[r] & S^{odd}_{q, \geq 2m+1} \otimes X(RB) \arrow[d, "ev_1"] \\
& & J^{2m+1}_{\#} \otimes X(RB)
\end{tikzcd}$$

The basic point I am missing?

$$X(RQ) \xrightarrow{t^{L_D}} \bigoplus_{k \geq 0} t^k X(RQ)_{\geq k} \longrightarrow S_q \otimes X(RB)$$

Your map

$$\underbrace{X(RA) \longrightarrow X(S \otimes RB)} \longrightarrow S_q \otimes X(RB).$$

induced by

$p + tq : A \longrightarrow S \otimes B$

what I need is