1 It's what to go over. Things to do in the next $1\frac{1}{2}$ hours. Getzler.

Let's review the physicists approach. Friedan + Windey. Let's find the basic data. We start with the Dirac operator on $M = \mathbb{R}^n$. The path integral is supposed ~~over loop space LM. an~~ a current on LM.

~~Basic fu~~ Coordinates fns. are x_t^μ $1 \le \mu \le n$ $t \in$ and $\psi_t^\mu = dx_t^\mu$. There is also vector field v with $v x_t^\mu = \dot{x}_t^\mu = \partial_t x_t^\mu$. Thus

$$(d + i(v))\, x_t^\mu = \psi_t^\mu$$

$$(d + i(v))\, \psi_t^\mu = \dot{x}_t^\mu$$

and $$(d + i(v))^2 = \mathcal{L}_v .$$

Fundamental 1-form is

$$\alpha = \int_{S^1} \dot{x}_t^\mu \psi_t^\mu \, dt$$

$$\boxed{(d + i(v))(\alpha) = \int_{S^1} \left(\dot{\psi}_t^\mu \psi_t^\mu + \dot{x}_t^\mu \dot{x}_t^\mu \right) dt}$$

This is the ~~Lagrangian~~ action. How would one know?

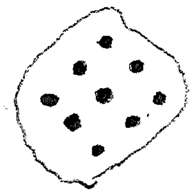
We should start with the commutation relations which

2 Basic idea is that the equations of motion ~~satisfy~~ come from some variational principle associated to $\int L\,dt$

$$0 = \int \delta (\dot{x}_t^\mu)^2 = 2\int \dot{x}_t^\mu \, \delta \dot{x}_t^\mu$$

$$= -2\int \ddot{x}_t^\mu \, \delta x_t \Longrightarrow \ddot{x}_t^\mu = 0.$$

$$\delta \int \dot{\psi}_t^\mu \psi_t = \int \delta\dot{\psi}_t \, \psi_t + \int \dot{\psi}_t \, \delta\psi_t$$



$$0 = \int -\delta\psi_t \, \dot{\psi}_t + \dot{\psi}_t \, \delta\psi_t = 2\int \dot{\psi}_t \, \delta\psi_t$$

$$\Longrightarrow \dot{\psi}_t = 0$$

Presence of a gauge field modifies the action. Gauge field is $A_\mu dx^\mu$

In order to handle this physicists introduce auxiliary fields to describe the ~~quotient bundle~~ coefficient bundle

Auxiliary fields to describe the coefficient

Coefficient bundle $\longrightarrow$ exterior alg. whose endos are Clifford algebra

We have to understand ordinary parallel transport. ODE 0 space dim'l QM.

0-space dim'l QM.

This should be fermionic. ~~Fields ψ~~

Operators ψ^μ generating Clifford alg.
to be classically understood as Grass. vbles.

Equation of motion $(\partial_t + A(t))\psi_t = 0$.

Operators ψ^μ

Wait: The QM $\longrightarrow$ Clifford alg. + quadratic Hamiltonians

Hamiltonian picture γ^μ and $\frac{1}{4}\omega_{\mu\nu}\gamma^\mu\gamma^\nu$

$$\left[\tfrac{1}{4}\omega_{\mu\nu}\gamma^\mu\gamma^\nu, \gamma^\lambda\right] = \tfrac{1}{2}\omega_{\mu\lambda}\gamma^\mu - \tfrac{1}{2}\omega_{\lambda\nu}\gamma^\nu$$

$$= \omega_{\mu\lambda}\gamma^\mu$$

~~Class~~ Log. picture

$$H = \tfrac{1}{4}\omega_{\mu\nu}\gamma^\mu\gamma^\nu \longrightarrow e^{tH} = e^{\frac{1}{4}\omega_{\mu\nu}\gamma^\mu\gamma^\nu}$$

and then

$$\gamma^\mu = e^{tH}\gamma^\mu e^{-tH}$$

satisfies $\dot\gamma^\mu = [H, \gamma^\mu] = -\omega_{\mu\nu}\gamma^\nu$

$$(\partial_t + \omega)\gamma = 0.$$

~~Effectively~~ In the operator picture we have the commutations relations $[\gamma^\mu, \gamma^\nu] = 2\delta$ [illegible] quad. Ham. (dynamics)

4 So ~~we~~ we need to understand these kin + dyn in Lag. path integral picture.

$$\partial^{\mu} \rightsquigarrow \psi_t^{\mu} \quad \text{Grassman field over time line.}$$

Dynamics and Kinematics. There will be something that corresponds to the comm. relations. Lagrangian has to be

$$\int \psi_t^{t} (\partial_t + \omega) \psi_t$$

talking about ω as 1-form in S^1 values in $so(n)$. Lie alg. of SO ~~n = no~~

The first part is intimately related to the ~~across stuff here~~ commutation relations.

~~It stem results from the fact~~

~~It~~ notice that $\int f dg$ is an intrinsically defined 1-form – geometric. On the Grassmann alg it gives a skew form like $\psi^t \omega \psi$. Why does the translate into ~~zeldi~~ commutation relations.

First point $\int f dg$ starts out for functions but then thanks to quad. form extends to vector functions. Then for a vector ... bundle with quad. form

circle

Thus we learn that a v.b. V with connection and quadratic form over time line is needed to write Lag. Next one needs to know how to do the ~~into theory~~ integral. So we have this skew form

$$\int_{S^1} \langle \psi | \nabla \psi \rangle$$

on $\Gamma(I, V)$. ~~We do the pat~~

The path integral is related somehow to the Green's function of the operator ∇.

How to understand

First of all: We have a real vector space with inner product

First of all we need to understand the meaning of the path integral in good cases. What we have is apparently something very subtle!!!!!!!!!!! In other words we have a global skew-form $\int_{S^1} \langle \psi_1 | \nabla \psi_1 \rangle$ and pointwise quadratic forms.

So what occurs is the following.

There is some Lagrangian style stuff ~~in which~~

[illegible] ~~in quadratic forms is~~

6 where an orthogonal transformation is done by C.T. of a skew form.

Composition.

$$\frac{1+X}{1-X}\,\frac{1+Y}{1-Y} = \frac{1+(X+Y+XY)}{1-}$$

$$\frac{\frac{1+x}{1-x}\,\frac{1+y}{1-y}-1}{\frac{1+x}{1-x}\,\frac{1+y}{1-y}+1} = \frac{x+y}{1+}$$

$$\frac{\left(\frac{1+x}{1-x}\,\frac{1+y}{1-y}-1\right)+}{\left(\frac{1+x}{1-x}\,\frac{1+y}{1-y}+1\right)-+} =$$

$$\frac{1+X}{1-X}\,\frac{1+Y}{1-Y} - \frac{1-X}{1-X}\,\frac{1-Y}{1-Y}$$

$$\frac{\frac{1}{1-X}(X+Y)\frac{1}{1-Y}}{\frac{1}{1-X}(1+XY)\frac{1}{1-Y}} = \underbrace{\frac{1}{1-X}(X+Y)}\,\underbrace{\frac{1}{1+xy}\,1-X}$$

$$\frac{1}{1-X}(X+Y)\left(\frac{1}{1+XY} - X\frac{1}{1+YX}\right)$$

$$\frac{1}{1-x}X\frac{1}{1+XY}$$

So

$$\left(-1 + \frac{2}{1-x}\right)\left(-1+\frac{2}{1-y}\right)$$

$$= 1 - 2\left(\frac{1}{1-x}+\frac{1}{1-y}\right) + 4\,\frac{1}{1-x}\,\frac{1}{1-y} = -1 + \frac{2}{1-z}$$

$$\frac{1}{1-z} = 1 - \left(\frac{1}{1-x}+\frac{1}{1-y}\right) + 2\left(\frac{1}{1-x}\,\frac{1}{1-y}\right)$$

$$= 1 - \frac{1}{1-x}\left(1-y+1-x\right)\frac{1}{1-y} + 2\left(\frac{1}{1-x}\,\frac{1}{1-y}\right)$$

$$\frac{1}{1-z} - 1 = \frac{1}{1-x}(x+y)\frac{1}{1-y}$$

$$\shortparallel$$

$$+\frac{z}{1-z} \qquad \frac{1}{z} - 1 = \frac{1-z}{z} = (1-y)\frac{1}{x+y}(1-x)$$

$$\frac{1}{z} = 1 + (1-y)\frac{1}{x+y}(1-x)$$

$$= (1-y)\left\{\frac{1}{1-y}\cdot\frac{1}{1-x} + \frac{1}{x+y}\right\}(1-x)$$

$$= \cancel{(1-y)}\,\cancel{\frac{1}{1-y}}\,\frac{1}{1-x}\left(x+y + (1-x)(1-y)\right)\frac{1}{x+y}(1-x)$$

$$z = \frac{1}{1-x}(x+y)\frac{1}{1+xy}(1-x)$$