Kazhdan

$G \qquad \mathcal{H} = \mathbb{C}_c^\infty(G)$ Hecke alg.

$\mathcal{E}_t(G) = \{$irred. tempered repns.$\}$

Lemma: $f \in \mathcal{H}$ $\quad \operatorname{Tr} \pi(f) = 0$ all $\pi \in \mathcal{E}_t(G)$

then $\int_{\Omega} f \, d\omega = 0$ for any regular conjugacy class.

$G = GL_n F \qquad F$ local field

$\varepsilon : PGL_n G \longrightarrow \mu_n \qquad \varepsilon(g) = e^{\frac{2\pi i}{n} v(\det g)}$

$f \in \mathcal{H}_0 \qquad g \in G'_{ell.} \qquad \lambda_g(f) = \int_{x \in PG} \varepsilon(x) f(x^{-1} g x)\, dx$

$\mathcal{H}_0 = \mathcal{H}(G, K_0)$, $K_0 = GL_n(\mathcal{O})$

space generated by all $\lambda_g \in \mathcal{H}_0^*$ has dim 1.

Let $L \supset F$ unram. $\qquad L^* \subset G_0 \subset G \qquad \theta \in \widehat{F^*/\mathcal{O}^*}$

1) If $\Omega_g \cap L^* = \phi$, then $\lambda_g = 0$.

Assume $g \in L^* \qquad \operatorname{sup} f \subset G_0$

Precise statement: $\exists!$ tempered $\sigma_\theta \in \mathcal{E}_t(G_0)$

1) $V^K \neq 0$

2) $\sigma | F^\times = \theta \operatorname{Id}$

3) For any $x \in G - G_0 \quad \sigma^x \not\simeq \sigma$

Thomas Parker

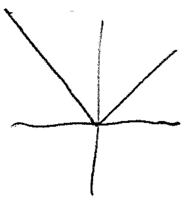
1. super symm. σ-model

$$f : \mathbb{R}^{1,1} \longrightarrow (M, g) \qquad \text{compact}$$

super-symmetric Lagrangian: $$L(f,\psi) = \frac{1}{2} \int_{\mathbb{R}^{1,1}} |\nabla f|^2 + i\, g_{ij}\, \overline{\psi}^i \not{D} \psi^j + \frac{1}{6} R_{ijkl}\, \overline{\psi}^i \psi^j \overline{\psi}^k \psi^l$$

$$\psi^i \in \left(\text{spinors on } \mathbb{R}^{1,1}\right) \otimes f^*(T_M) \qquad n = 1, \dots, n$$

$$S \cong \mathbb{C}^2$$



$$T(\mathbb{R}^{1,1}) = L \oplus L^{-1}$$

$$S = L_{1/2} \oplus L_{-1/2}$$

Quantize $$\begin{cases} \psi^i \longrightarrow \frac{1}{\sqrt{2}} (a_i + a_i^*) \\ \overline{\psi}^i \longrightarrow \frac{i}{\sqrt{2}} (a_i - a_i^*) \end{cases}$$

Witten: If we drop to $\mathbb{R}^1$

Hilbert space $\Omega^*(M)$

$a_i^* = e_i \wedge \qquad \{e^i\}$ basis for 1-forms

$a_i = e_i \lrcorner$

$$L \mapsto H = \frac{1}{2}\left[|\nabla f|^2 + \cancel{i\, g_{ij}\, \overline{\psi}^i \nabla_T \psi^j} + R_{ijkl}\, a_i^* a_j a_k^* a_l \right]$$

almost exactly Bocher formula

$$dd^* + d^*d = \underbrace{\nabla^* \nabla}_{\shortparallel} + \text{curv. term} + \frac{R}{6}$$

$$- \sum_i \nabla_{e_i} \nabla_{e_i} - \nabla_{\nabla_{e_i} e_i}$$

fixed relatively prime -

$$(x,0) = t(m,n) + s(\theta,1) \quad \mod \mathbb{Z}^2$$

I should be able to use t as a parameter

$$\begin{tikzcd}
\overset{t}{\mathbb{R}} \arrow[r] \arrow[d] & \overset{t \bmod \mathbb{Z}}{S^1} \arrow[d] \\
\underset{x=t(m-n\theta)}{S^1} \arrow[r] & \{\text{leaves}\}
\end{tikzcd}$$

(tm,tn) in $\mathbb{T}^2/\mathbb{R}$

$(t(m-n\theta),0)$

$$s = -tn$$
$$x = tm - tn\theta$$
$$= t(m-n\theta)$$

Is this onto the fibre product.

Let $(x,0) \equiv t(m,n) + s(\theta,1) \pmod{\mathbb{Z}^2}$

Then $(x,0)$

$$\begin{tikzcd}
& & \overset{t}{\mathbb{R}} \arrow[d] \\
& & S^1 \arrow[d] & t \bmod \mathbb{Z} \\
\mathbb{R} \arrow[r] & S^1 \arrow[r] & \mathbb{T}^2/\mathbb{R}
\end{tikzcd}$$

$(tm,tn) \bmod \mathbb{R}(\theta,1) + \mathbb{Z}^2$

$\wr$

$(t(m-n\theta),0) \bmod \mathbb{R}(\theta,1) + \mathbb{Z}^2$

$$\mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{T}^2 .$$

$$\begin{tikzcd}
\mathbb{R} \times \mathbb{R} / \mathbb{R}(\theta,1) \times \mathbb{Z}^2 \arrow[d] \\
\mathbb{R} / \mathbb{Z}^2 = \mathbb{R}/\{m-n\theta\}
\end{tikzcd}$$

$$(x,y) \mapsto x - y\theta$$

2. path integrals

$$\text{heat kernel} = \int_{p} e^{-\int L} \qquad K(x,y,t) = \int_{\substack{\text{paths}\\ \text{from } x \text{ to } y\\ \text{in time } t}} e^{-\int L}$$

$$K(x,y,t) = \frac{1}{(2\pi t)^{n/2}} e^{-\frac{|x-y|^2}{2t}} \left[1 + t\frac{R}{6} + \cdots\right] \quad \text{on scalars}$$

(1-1)

$$K_{\nabla^*\nabla} = \frac{1}{(2\pi t)^{n/2}} e^{-\frac{|x-y|^2}{2t}} e^{-\int_0^t \frac{R}{6} + \int_0^t \omega}$$

connection form

$$K_{\nabla^*\nabla + R} = e^{\frac{1}{2}\int |\nabla f|^2 + \int_0^t R/6 + \int_0^t \omega + R_{ijkl} a \cdots a}$$

$$\chi(M) = \int_{p} (-1)^d e^{-\int_0^t L} \qquad d = \text{degree on form}$$

$$= \int (-1)^d e^{-\int_0^t |\nabla_t f|^2 + \text{connection form} + \frac{R}{6} + R_{ijkl} a_i^* a_j a_k^* a_l}$$

now use stationary phase

ignore because same on even + odd

$$= \int_{\substack{\text{pt paths}\\ (= M)}} (-1)^d e^{0 + 0 + \cancel{R}/6}$$

$$= \int_M (-1)^d e^{-tR_{ijkl} a_i^* a_j a_k^* a_l}$$

$$= \mathrm{Pfaff}(R)$$

On $\Omega(M) \otimes E$ get extra term

$$+ F^{\alpha}_{\beta i j} \, c^*_\alpha c_\beta \, a^*_i a_j$$

then formula becomes

$$Pf(R) \cdot ch(E)$$

$\left\{ \begin{array}{l} \text{Alvarez} \\ \text{Friedmann} \end{array} \right.$ Comm. Math. Phys.