1 Point: Suppose we have a even cocycle i.e. a sequence of cochains

**K operation**

$$f_1 = \tau(\varphi)$$
$$f_2 = \tau(\omega)$$

satisfying
$$b' f_{2n} = (1-\lambda) f_{2n+1}$$
$$b f_{2n+1} = \tfrac{1}{n} N f_{2n}$$

This $\tau$ is not a trace on $R$. But we have this $K$ operation.

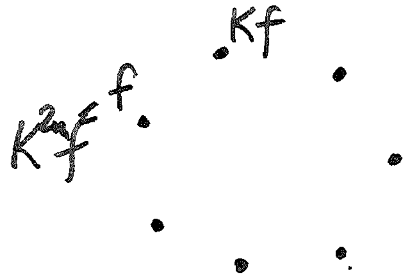
What does one know about $\tau - K\tau$?

If I fix $I^m$, then I can ~~always~~ ~~fix~~ take $f_{2m}, f_{2m+1}$ and say these come from a trace on $I^m$. Then I can look at $\tau - K\tau$. I know that $\tau - K\tau$ is a trace on $I^m/I^{m+1}$

$$\begin{tikzcd} \varphi_{2m+1} \arrow[r, "\lambda-1"] & 0 \\ \varphi_{2m} & \varphi_{2m} \arrow[u, "b'"'] \end{tikzcd}$$

$$\varphi_{2m+1} - \varphi^K_{2m+1} = b\varphi_{2m}$$
$$\varphi_{2m} - \varphi^K_{2m} = (1-\lambda)\varphi_{2m}$$

$$K^{2m} f_{2m+1} = f_{2m+1}$$




$f -$

So you can arrange

Idea I have: I have certain linear functional $\tau$ on $R$ and a basic ~~operator~~ operator $\tau \mapsto K\tau$. I know that $\tau - K\tau = \tau' d$ for some $\tau'$ determined by $\tau$.

$$R \xrightarrow{d} \Omega^1 R_\natural$$

~~This operator d is clear.~~ So what happens? Somehow you would like to do something like KMS. Want $\tau$.

Suppose have $\tau$ on $R$.

get $\quad \tau(a_1^- \cdots a_{2n}^-) = \varphi_{2n}$

want $\quad h_{2n} = \varphi_{2n}$ up to a scalar factor

$$\sum^{\parallel} \lambda^{2n-2i} g_{2n}$$ — as $g_{2n-1} = 0$. OK.

$\tau'(\omega^{n-1} d\rho) = 0.$

One thing you can look for is map

$(\Omega R)_\natural \longrightarrow R_\natural$ Yes

You have $\quad \tau(\omega^n) = \varphi_{2n} \quad \lambda^2$ inv.

$\tau'(-\rho\omega^{n-1} d\rho) = g_{2n}$

so $\quad h_{2n} = n\, g_{2n}.$ wait $\boxed{\text{O.K.}}$

so you want

$\rho\omega^{n-1} d\rho \longmapsto \omega^n$ up to scalars

$\omega^{n-1} d\rho \longmapsto 0$

$$b(\theta_-^n)(a_0, \dots, a_n)$$
$$= [a_0^+ a_1^- \dots a_{n-1}^-, a_n^-] + (-1)^{n+1}[a_0^- \dots a_{n-1}^-, a_n^+]$$

$$(b\varphi_{2n})(a_0, \dots, a_{2n})$$
$$= \tau([a_0^+ a_1^- \dots a_{2n-1}^-, a_{2n}^-])$$

So define a trace on $R = Q^+$ by means of the cochain

$$f_{2n+1} = b\varphi_{2n} \qquad f_{2n} = (1-\lambda)\varphi_{2n}$$

Call this trace $\tau_1$ so that

$$\tau_1(a_0^+ a_1^- \dots a_{2n}^-) = \tau([a_0^+ a_1^- \dots a_{2n-1}^-, a_{2n}^-])$$
$$\tau_1(a_1^- \dots a_{2n}^-) = \tau([a_1^- \dots a_{2n-1}^-, a_{2n}^-])$$

Apparently this defines a new trace on $R$. Is it possible to describe this as

~~things as~~ $\tau(D(a_0^+ a_1^- \dots a_{2n}^-))$

where $D$ is a derivation of $R$, or maybe $\tau' d$ with $\tau'$ a trace on $\Omega^1 R$. Now

$$\tau' d(a_0^+ a_1^- \dots a_{2n}^-) =$$

4 should work out easily.

$$\tau' d(a_0^+ a_1^- a_2^-)$$

$$= \tau' d\left(a_0^+\left((a_1 a_2)^+ - a_1^+ a_2^+\right)\right) \quad ?$$

YES

$\tau'$ will see only the top.

$$g_{2n} = \tau'(-\rho\, \omega^{n-1} d\rho) \quad \cancel{= \varphi_{2n}}$$

$$h_{2n} = \underbrace{\left(\sum_1^n \lambda^{2n-2i}\right) g_{2n}}_{n\, y_{2n}} = \varphi_{2n}$$

Thus I want to define $\tau'$ by requiring $\tau'(-\rho\, \mu_{2n-1}) = \varphi_{2n}$.

$$\tau'\left(\sum -\rho\, \omega^{i-1} d\rho\, \omega^{n-i}\right)$$

~~Think of having $\tau$ on $I^n/[R, I^n] = (I\otimes_R)^n$~~

~~$I^n \longrightarrow \tau - K\tau$ is a trace on $I^n/I^{n+1}$ ?~~

~~$I^n/I^{n+1}$~~

OKAY

Given $\tau$ on $I^n/[I, I^{n-1}] = (I\otimes_R)^n_\sigma$

$$= (J\otimes_Q)^{2n}_{\lambda^2}$$

given $\tau$ on $I/[R, I] = I\otimes_R = (J\otimes_Q)^2$

$$I \subset J^2$$

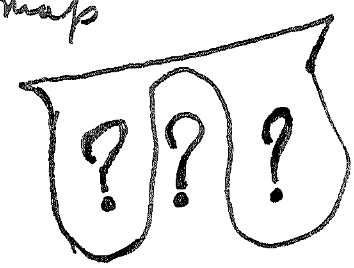
Whatever this map I am after is it disconnects the different levels. You therefore expect to see a some trace trace on $\Omega^{ev}$.

$$R \subset Q$$
$$\Omega^{ev} \subset Q$$

There appears to be a map

$$R_q \longrightarrow \Omega^{ev}_q$$

$$a_0^+ a_1^- \cdots a_{2n}^- \longmapsto$$



Amazing! ~~You~~ have ~~perfect~~

Given a trace on ~~R~~

Important traces on $R$ come from even cyclic cocycles. Coboundaries?

$$\psi_{2m+1} \xrightarrow{\lambda - 1} 0 \qquad \uparrow -b$$

fact that $\psi_{2m}$ cyclic says can take $g_{2m} = d\psi_{2m}$

$\psi_{2m}$

What about ?

$f_{2m+1}$

$$f_{2m} \xrightarrow[N]{\lambda^2\text{-inv.}} 0 \qquad \lambda f_{2m} = -f_{2m}$$

So what about