(E) $$(1-\lambda)b'_\rho = (1-\lambda)\big(b' - (\rho\otimes 1)(1-\lambda)\big)$$

$$b_\rho(1-\lambda) = \big(b - (1-\lambda)(\rho\otimes 1)\big)(1-\lambda)$$

~~Obviously these are equal.~~

The question now is ~~what to~~ whether the constructions so far lead to some kind of S-operator on $\bar{C}^\lambda(A)$.

$$0 \to \bar{C}^\lambda(A) \xrightarrow{N_\lambda} T(\bar{A}) \xrightarrow{1-\lambda} \bar{A}\otimes T(\bar{A}) \xrightarrow{\pi} \bar{C}^\lambda(A) \to 0$$

Could you hope to use this sequence to construct the S operation on the cyclic complex. There's some problems because the derivations

$$b'_\rho = b' - \iota_\rho(1-\lambda)$$

$$b_\rho = b - (1-\lambda)\iota_\rho$$

do not have square zero.

$$(b'_\rho)^2 = -b'\iota_\rho(1-\lambda) - \iota_\rho b(1-\lambda) + \iota_\rho(1-\lambda)\iota_\rho(1-\lambda)$$

$$= -\underbrace{\big(b'\iota_\rho + \iota_\rho b - \iota_\rho(1-\lambda)\iota_\rho\big)}_{\text{curvature ?}}(1-\lambda)$$

(H) 1/17 first day of term.

The problem now is whether you can construct a reasonable S operator on $\bar{C}^\lambda(A)$ associated to ρ.

So we want to understand

What do we really want to understand?

We have A and $\rho : A \to \mathbb{C}$.

Then this tells us how to form

What is it that I really want to do?

Anyway what exactly happens.

$$0 \longrightarrow \bar{C}^\lambda(A)_n \longrightarrow \bar{\Omega}^n \tilde{A} \Longrightarrow \bar{C}^\lambda(A)_{n+1} \longrightarrow 0$$

This is not exact but the deviation is acyclic. What do we actually have

$$\begin{tikzcd}
\bar{C}^\lambda(A)_n \arrow[r, hook] & \Omega^n A \arrow[r] & \bar{C}^\lambda(A)_{n+1} \\
 & \bar{A}^{\otimes n} \arrow[d] & \\
\bar{A}^{\otimes n,\lambda} \arrow[r, hook] \arrow[ur, hook] & A \otimes \bar{A}^{\otimes n} \arrow[r] \arrow[d] & \bar{A}^{\otimes n+1}_\lambda \\
 & \bar{A}^{\otimes n+1} \arrow[ur] \arrow[d] & \\
 & 0 &
\end{tikzcd}$$

I $\quad 0 \longrightarrow \bar{A} \longrightarrow A \otimes \bar{A} \xrightarrow{d} \bar{A}^{\otimes 2} \longrightarrow 0$

$$a\,da \longleftarrow\!\shortmid\; da^2$$

$$-da\,a$$

$$[b, da^2] = -da\,a + a\,da$$

So where are we know. I am interested is trying to construct an S operator on $\bar{C}^{\lambda}(A)$

$\bar{A}^{\otimes n, \lambda}$

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \bar{A}^{\otimes n} & \longrightarrow & \Omega^n A & \longrightarrow & \bar{A}^{\otimes n+1} & \longrightarrow & 0 \\ & & & & & & \downarrow & & \\ & & & & & & \bar{A}^{\otimes n+1}_{\lambda} & & \end{array}$$

Intrinsically what happens is that $\operatorname{Ker} B / \operatorname{Im} B$ is acyclic except in degree 0.

$$0 \longrightarrow \bar{A}^{\otimes n} \longrightarrow \operatorname{Ker} B \longrightarrow (1-\lambda)\bar{A}^{\otimes n+1} \longrightarrow 0$$

$$0 \to (1-\lambda)\bar{A}^{\otimes n} \longrightarrow \operatorname{Ker} B/\operatorname{Im} B \longrightarrow (1-\lambda)\bar{A}^{\otimes n+1} \to 0$$

So what happens?

So what happens?

(J) What we know is that $\operatorname{Ker} B / \operatorname{Im} B$ is B acyclic. Thus we have this splitting $$\bar{\Omega} A = \underset{\substack{\uparrow \\ B\text{-exact}}}{P} \oplus Q$$

$$\begin{tikzcd}[column sep=small]
 M & & \\
 & \operatorname{Ker} B \arrow[ul] & \\
 \operatorname{Im} B \arrow[ur] & & \\
 & 0 \arrow[ul] &
\end{tikzcd}$$

So let's if in the next half hour we can get things done

Let's review the position. I have found ~~an analogue~~ a reduced analogue of the bar construction. I have a twisted version of $\bar{\Omega}\tilde{a}$

~~$\tilde{\Omega}^n \tilde{a}$~~

~~Take possibly nonunital A~~.

$$\begin{tikzcd}
 & & e\bar{\Omega}^n\tilde{a} \arrow[dr, "\sim"] & & \\
0 \arrow[r] & a^{\otimes n} \arrow[r] & \bar{\Omega}^n \tilde{a} \arrow[r] & a^{\otimes n+1} \arrow[r] & 0
\end{tikzcd}$$

intrinsic, but before you argued that from the Hochschild viewpoint $\tilde{a}$ should be thought of as bimodule and thus the splitting

$$\tilde{a} \otimes a^{\otimes n} = (\mathbb{C}e^{\perp} \oplus a) \otimes a^{\otimes n}$$

is more natural.

(K) So how to proceed?

$$\begin{pmatrix} 1 & s \\ & 1 \end{pmatrix}\begin{pmatrix} b & (1-\lambda) \\ & -b' \end{pmatrix}\begin{pmatrix} 1 & \bar{s} \\ & 1 \end{pmatrix}$$

$\cancel{1-b's-sb'}$
$\cancel{-\lambda-cs}$

$$= \begin{pmatrix} 1 & s \\ & 1 \end{pmatrix}\begin{pmatrix} b & -bs+1-\lambda \\ 0 & -b' \end{pmatrix} = \begin{pmatrix} b & \overset{=0}{1-\lambda-bs-sb'} \\ 0 & -b' \end{pmatrix}$$

$$\begin{pmatrix} 1 & s \\ & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ N_\lambda & 0 \end{pmatrix}\begin{pmatrix} 1 & -s \\ & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & s \\ & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ N_\lambda & -N_\lambda s \end{pmatrix} = \begin{pmatrix} sN_\lambda & -sN_\lambda s \\ N_\lambda & -N_\lambda s \end{pmatrix}$$

$$\begin{pmatrix} 1 & \overset{B_0}{\cancel{s(1-\lambda)}} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} b & 1-\lambda \\ & -b' \end{pmatrix}\begin{pmatrix} 1 & -\overset{B_0}{\cancel{s(1-\lambda)}} \\ & 1 \end{pmatrix} = \begin{pmatrix} b & \\ & -b' \end{pmatrix}$$

$$\begin{pmatrix} 1 & \overset{B_0}{\cancel{(1-\lambda)s}} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ N_\lambda & 0 \end{pmatrix}\begin{pmatrix} 1 & \overset{-B_0}{\cancel{(1-\lambda)s}} \\ & 1 \end{pmatrix} = \begin{pmatrix} \overset{B_0}{\cancel{(1-\lambda)s}} N_\lambda & 0 \\ N_\lambda & 0 \end{pmatrix}$$

$$bB + Bb = 0 \tag{26}$$

Remark As $Bb = -bB$ has square zero, the operator $\kappa^{n(n+1)} = 1 - Bb$ on Ω^n is unipotent, and hence κ is a *quasi-unipotent* operator in the terminology of the theory of monodromy. When $Bb \neq 0$ on Ω^n, $1 - Bb$ has infinite order, and hence so does κ. For example, when $n = 1$ we have $Bb(a_0da_1) = d[a_0, a_1]$, so $Bb \neq 0$ unless every commutator is a scalar.

(R) Do we have

$$B \xhookrightarrow{i} A \xrightarrow{e} B$$

$$e(ab) = e(a)b.$$

Yes. Thus ~~the [illegible]~~ $\Gamma(A \xrightarrow{e} B)$ acts on A. Are there any obvious relations?

$$(a_1 i b j a_2) \cdot a = a_1 i b e(a_2 a)$$

~~[illegible]~~

$$= a_1 b\, e(a_2 a)$$

So we find ~~[illegible]~~

$a_1 \otimes b \otimes a_2$ acts as $a \mapsto a_1 b\, e(a_2 a)$

So we get $A \otimes_B A$ acting on A.

$$(a_1 \otimes a_2)\, a = a_1\, e(a_2 a)$$

$$(a_1 \otimes a_2)(a_1' \otimes a_2') = a_1\, \underbrace{e(a_2 a_1') \otimes}\, a_2'$$

~~[illegible]~~ Start then with

$$\mathbb{C} \subset \mathbb{C}[s_1] \qquad s_1^2 = 1$$

$$e(s_1) = 0$$

Do in general. $\mathbb{C} \subset \mathbb{C}[x]/(x^2 - \lambda x - 1) \;\big|\; e(x) = 0$

To get an identity element in $A \otimes_B A$ need some sort of duality. ~~[illegible]~~ assumes A is a finite projective right B-module

~~braid relations.~~