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So what does this sort of thing mean?

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It certainly helps the pen this heat. So let's continue and try to work out the relations. When we take $\vec{C}^{\lambda}(A\oplus\varepsilon A)$ then we get ~~what~~ a quasi-isomorphism

$$\vec{C}^{\lambda}(\mathbb{C}\oplus\varepsilon\mathbb{C}) \longrightarrow \vec{C}^{\lambda}(A\oplus\varepsilon A)$$

This ought to help in some way. I feel I ought to be able to get some mileage out of this. Can I define a map backwards?

$$B(\mathbb{C}\oplus\varepsilon\mathbb{C}) \longrightarrow B(A\oplus\varepsilon A)$$

So far we have a map $B(A) \longrightarrow B(\mathbb{C}\oplus\varepsilon\mathbb{C})$ given by a twisting cochain $B(A)\longrightarrow \mathbb{C}+\varepsilon\mathbb{C}$ consisting of ρ, ω.

So we ask whether this twisting cochain from $B(A)$ to $\mathbb{C}+\varepsilon\mathbb{C}$ can be extended to Bar$(A\oplus\varepsilon A)$

complex (3) is closely related to the spectral sequence [Q, I.5.5], and one can obtain from it the exact sequence

$$0 \to \bar{H}C_{2n+1}A \to I^{n+1}/[I^n, I] \to H_1(R, I^n) \to \bar{H}C_{2n}A \to 0$$

of [Q, I.5.11].

Q Let's go over the twisting cochain

$\mathcal{B}(A) \longrightarrow R \oplus \varepsilon I$

I work in the bigraded DG algebra

$\mathrm{Hom}(\mathcal{B}(A), R \oplus \varepsilon I)$

two differentials δ coming from the diffl b' of $\mathcal{B}(A)$ and d coming from the diffl of $R \oplus \varepsilon I$ $\therefore$ $d(\varepsilon) = 1$.

Let's go over the calculation

$$(d+\delta)(\rho \mathbin{\bar{\oplus}} \varepsilon\omega)$$

$$= \delta\rho \mathbin{\bar{\oplus}} \omega + \varepsilon\delta\omega$$

$$(\rho - \varepsilon\omega)^2 = \rho^2 - \varepsilon(\rho\omega + \omega\rho)$$

$$(d+\delta)(\rho - \varepsilon\omega) + (\rho - \varepsilon\omega)^2$$

$$= \underbrace{\delta\rho + \rho^2}_{-\omega} + \varepsilon(\delta\omega - \rho\omega - \omega\rho) = 0$$

~~So the next point is to consider~~

Next need to extend. ~~the~~ This time maybe I would like to have $R = I = \mathbb{C}$

$$\mathcal{B}(A + \varepsilon A) = \mathbb{C} + \Sigma(A + \varepsilon A) + \Sigma^2(A + \varepsilon A)^{\otimes 2}$$

R   Now our twisting cochain τ has degree -1. It only sees degrees 1, 2 of $\mathcal{B}(A \oplus \varepsilon A)$, so there is only one other component to add to $\rho - \varepsilon\omega$ and that is $\varepsilon\mu$

$$\varepsilon\mu : \Sigma\varepsilon A \longrightarrow \varepsilon\mathbb{C}$$

Let's imagine instead having $A, J \longrightarrow R, I$
Would like to have $A \xrightarrow{\rho} R$ homom. modulo I, so $\omega : A^{\otimes 2} \longrightarrow I$, also you want ρ to carry J into I. Other piece of information should be $\mu : \varepsilon J \longrightarrow \varepsilon I$.

$$\tau = \rho - \varepsilon\omega + \varepsilon\mu$$

$$(d+\delta)\tau = \delta\rho - \omega + \varepsilon\delta\omega - \varepsilon\delta\mu + \mu$$

$$\tau^2 = \rho^2 + \varepsilon(\rho\omega - \omega\rho) + \varepsilon(-\rho\mu + \mu\rho)$$

$$\delta\rho + \rho^2 = \omega - \mu \qquad \delta\omega = \rho\omega - \omega\rho$$

$$-\delta\mu - \rho\mu + \mu\rho = 0$$

$$\boxed{\delta\mu + \rho\mu - \mu\rho = 0}$$

S Let's go over this calculation, a twisting cochain

$$B(A \oplus \varepsilon J) \xrightarrow{\tau} R \oplus \varepsilon I$$

τ should have components

$$\tau = \rho - \varepsilon(\omega + \mu)$$

$$(\delta + d)\tau = \delta\rho + \varepsilon\delta\omega + \varepsilon\delta\mu - \omega - \mu + \varepsilon d\mu$$

$$\tau^2 = \rho^2 + \varepsilon\left(\rho(\omega+\mu) - (\omega+\mu)\rho\right)$$

Thus get

$$\delta\rho + \rho^2 - \omega - \mu = 0$$

You've got it all screwed up

So what is the best way to proceed?

$$\delta\tau = \delta\rho + \varepsilon\delta\omega + \varepsilon\delta\mu$$

$$[d,\tau] = d(\rho - \varepsilon\omega - \varepsilon\mu) + (\rho - \varepsilon\omega - \varepsilon\mu)d$$

Let's defer this calculation until we have some more understanding.

What is the point you would like to understand? Go back to your original program of

$$\begin{tikzcd}
 & C^\lambda(\mathbb{C}) \arrow[r, hook] & C^\lambda(A) \arrow[r] \arrow[d] & \bar{C}^\lambda(A) \arrow[d, equal] & \\
0 \arrow[r] & C^\lambda(\mathbb{C}) \arrow[r] & E \arrow[r] & \bar{C}^\lambda(A) \arrow[r] & 0
\end{tikzcd}$$

exact

T What you want to do is to find E inside $C^\lambda(A)$ in a concrete way. This seems pretty reasonable to expect. Do you have any method to propose. Suppose we consider the link with

I was fishing – trying to recall something which might turn out useful. need other papers. So where can we start? Instead of $C^\lambda(A)$, $\bar{C}^\lambda(A)$ consider $C^\lambda(A \oplus \varepsilon A)$, $\underbrace{\bar{C}^\lambda(A \oplus \varepsilon A)}$.

What is involved is the idea that the Chern-Simons form sits inside something like $C^\lambda(R \oplus \varepsilon I)$. I've forgotten this point. Let's see if I can get somewhere. The point as I recall it is to identify the Chern-character forms

$$HC_{2n-1}(A) \longrightarrow I^n/[I, I^{n-1}]$$

with ~~what you get by diagram~~ the edge homom.

$\boxed{HC_3(A)}$				
	$HC_1(R)$		$[I \otimes_R I]^2_\sigma$	
	$HC_0(R)$	$I/[R,I]$		