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It would appear that there are many choices

$$d^i(a_0, \dots, a_n) = \sum_{i=0}^{n+1} (-1)^i (\dots, a_{i-1}, a_i, \dots)$$

$$d'(\dots) = \sum_{i=1}^{n+1} (-1)^i (\dots, a_{i-1}, a_i, \dots)$$

~~Consider~~ Consider $d_0(a_0, \dots, a_n) = (a_0, a_1, a_2, \dots, a_n)$

~~Fact~~ Fact is that $d' = - \sum_{i=0}^n (-1)^i s_i$

$$s_i(a_0, \dots, a_n) = (a_0, \dots, a_i, a_{i+1}, \dots, a_n)$$

$$d_i s_j = \begin{cases} s_{j-1} d_i & i < j \\ 1 & i = j, j+1 \\ s_j d_{i-1} & i > j+1 \end{cases}$$

$$d_0 s_j = s_{j-1} d_0 \quad 1 \leq j \leq n.$$

$$d_0 \sum_{i=1}^n (-1)^i s_i = \sum (-1)^i d_0 s_{i-1} d_0$$

~~What's~~ What's Grothendieck's argument?

$$\begin{array}{ccccccc}
 A & \rightrightarrows & A \otimes A & \rightrightarrows & A \otimes A \otimes A & & \\
 & & & & X \times X \times X & \otimes X & X \\
 & & & & & \downarrow & \downarrow \\
 & \rightrightarrows & X \times X & \rightrightarrows & X & \rightarrow & X \\
 & \rightrightarrows & & & & & \\
 & \rightrightarrows & & & & &
 \end{array}$$

δ Cone construction

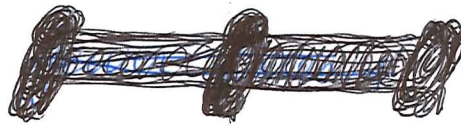
Point is that $d_j s_{j-1} = s_{j-1} d_{j-1}$

so $\sum_{j=0}^{n+1} (-1)^j d_j s_{j-1} = \boxed{\times} d_0 s_{-1} + \sum_{j=1}^{n-1} (-1)^j s_{j-1} d_j$

$\delta s_{-1} = 1 - s_{-1} \delta$

I now want to do the same argument cosimplicially. Thus

$\left(\sum_{i=0}^n (-1)^i s_i \right)$



$M \rightarrow A \otimes M \Rightarrow A \otimes A \otimes M$

I have a cosimplicial thing. In any case you want to use dffl

$\sum_{i=1}^{n+1} (-1)^i s_i : A^{\otimes n+1} \rightarrow A^{\otimes n+2}$

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VOID	VOID	VOID	VOID	VOID	VOID	VOID	VOID						
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BOS	AA	108	U	25APR	1925	BOS	BOS						
LHR	AA	108	U	25APR	1925	LHR	LHR						
LONDON	LHR	AA	109	U	18APR	1130	LONDON						
LONDON	LHR	AA	109	U	18APR	1130	LONDON						
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d

~~$$d_{n+1} \sum_{i=0}^n (-1)^i s_i = \sum_{i=1}^{n+1} (-1)^i d_{n+1} s_i$$~~

$$\sum_{i=0}^n (-1)^i s_i : A^{\otimes n+1} \xrightarrow{\text{deg } n} A^{\otimes n+2}$$

a_0, \dots, a_{n+1}

actually this is $-d'$.

$$d_0 \sum_{i=0}^n (-1)^i s_i = d_0 s_0 + \sum_{i=1}^n (-1)^{i-1} s_{i-1} d_0$$

$$d_0 \left(\sum_{i=0}^n (-1)^i s_i \right) = \pm \left(\sum_{j=0}^{n-1} (-1)^j s_j \right) d_0$$

so I can use either one ~~or the~~

$$d_{n+1} \sum_{i=1}^{n+1} (-1)^i s_i = \left(\sum_{i=1}^n (-1)^i s_i \right) d_n$$

I'm getting confused. So how to handle this?

$$a_0, \dots, a_n$$

$$a_0, 1, a_1$$

$$a_0, a_1, 1, a_2$$

$$A \rightleftharpoons A \otimes A \rightarrow$$

$$a \quad (a, 1)$$

getting nowhere?

$$1, a_n$$

Anyway why am I

I want to seriously work on controlling this homology.

2 I want to consider $C(A) = \bigoplus_{n \geq 0} A^{\otimes n+1}$
 with various ~~of them~~ "differentials". In
 degree n we have the operators

$$(\lambda^i s \lambda^{-i})(a_0, \dots, a_n) = (-1)^i (a_0, \dots, a_{i-1}, a_i, \dots, a_n)$$

for $0 \leq i \leq n+1$.

Basic rule is that $s^2 + \lambda s \lambda^{-1} s = 0$?

$$\textcircled{\otimes} (a_0, \dots, a_{n+1}) \xrightarrow{s} (1, a_0, \dots, a_n)$$

$$\xrightarrow{s} (1, 1, a_0, \dots, a_n)$$

$$\xrightarrow{\lambda s \lambda^{-1}} -(1, 1, a_0, \dots, a_n)$$

~~Bas~~ More generally.

$$\lambda^i s \lambda^{-i} \lambda^j s \lambda^{-j} + \lambda^{j+1} s \lambda^{-j-1} \lambda^i s \lambda^{-i} = 0$$

$$i \leq j$$

Then there are various things I can do.

$$d = \sum_{i=0}^{n+1} \lambda^i s \lambda^{-i} \quad \text{on } A^{\otimes n+1}$$

$$d' = \sum_{i=1}^{n+1} \lambda^i s \lambda^{-i}$$

I think these are differentials $d^2 = 0, d'^2 = 0$.

But also $\sum_{i=1}^n \lambda^i s \lambda^{-i}$

Basically you have $A \xrightarrow{d} A^{\otimes 2} \xrightarrow{d} A^{\otimes 3}$
 and you can tensor on ~~of~~ either side
 with copies of A .

Introduce notation

$$\varepsilon_i(a_0, \dots, a_n) = (-1)^i (a_0, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$$

$$0 \leq i \leq n+1.$$

$$\varepsilon_i \varepsilon_j = -\varepsilon_{j+1} \varepsilon_i$$

$$i \leq j$$

$$\sum_{c \leq i \leq n+1-c'}$$

$$\sum_{j \in [c, n+2-c']} \varepsilon_j \cdot \sum_{i \in [c, n+1-c']}$$

$$= \sum_{c \leq i < j \leq n+2-c'} \varepsilon_j \varepsilon_i + \sum_{c \leq j \leq i \leq n+1-c'} \varepsilon_j \varepsilon_i - \sum_{c \leq j < i+1 \leq n+2-c'} \varepsilon_{i+1} \varepsilon_j$$

Next let us compute the homology

We will keep track of

$$d = \sum_{0 \leq i \leq n+1} \varepsilon_i \quad \text{on } C_n$$

$$\text{and } d' = \sum_{1 \leq i \leq n+1} \varepsilon_i \quad \text{on } C_n$$

$$d = d' + \varepsilon_0$$

Idea is that if we take

$$\mathbb{C} \rightarrow A \rightrightarrows A^{\otimes 2} \rightrightarrows A^{\otimes 3}$$

and tensor with A on the left we get (C, d')

η Now show (C, d') is contractible.

Here we exhibit $b_0(a_0, \dots, a_n) = (a_0 a_1, a_2, \dots, a_n)$

Then $b_0 \varepsilon_i = -1$

$$\begin{aligned} i \geq 2 \quad (b_0 \varepsilon_i)(a_0, \dots, a_n) &= b_0(a_{0i}, a_{i-1}, a_{i+1}, \dots, a_n) (-1)^i \\ &= (a_0 a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots) (-1)^i \\ &= -\varepsilon_{i-1}(a_0 a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots) \end{aligned}$$

$$b_0 \varepsilon_i = -\varepsilon_{i-1} b_0$$

Thus $b_0 \sum_{i=1}^{n+1} \varepsilon_i = -1 - \sum_{i=2}^{n+1} \varepsilon_{i-1} b_0$

d on C_n d' on C_{n-1}

$$\begin{array}{ccccccc} \mathbb{C} & \longrightarrow & A & \xrightarrow{d} & A^{\otimes 2} & \xrightarrow{d} & \dots \\ \downarrow & & \downarrow s & & \downarrow s & & \downarrow \\ A & \longrightarrow & A \otimes A & \xrightarrow{-d'} & A^{\otimes 3} & \xrightarrow{-d'} & \dots \end{array}$$

$$\begin{aligned} \varepsilon_0 \sum_{i=0}^n \varepsilon_i &= - \sum_{i=0}^n \varepsilon_{i+1} \varepsilon_0 \\ &= - \sum_{j=1}^{n+1} \varepsilon_j \varepsilon_0 \end{aligned}$$

$n A^{\otimes n}$ $n A^{\otimes n+1}$

$$\begin{aligned} (\rho b_0 s)(a_0, \dots, a_n) &= \rho b_0(a_0, \dots, a_n) \\ &= \rho(a_0, \dots, a_n) \\ &= \rho(a_0)(a_1, \dots, a_n) \end{aligned}$$

2 What else? If the world
 Then the kernel of a map of injective
 functors will be left exact.

~~What else?~~

Is an exact functor injective?
 Probably not.

But what you want ~~is~~ ~~RF~~
 to know is that $F \rightarrow R^0F$ is the
 localization

$$(R^0F)(X) = \varinjlim \text{Ker}(F(X) \rightarrow F(Z))$$

~~where~~
 $X = Y/Z$

N solid

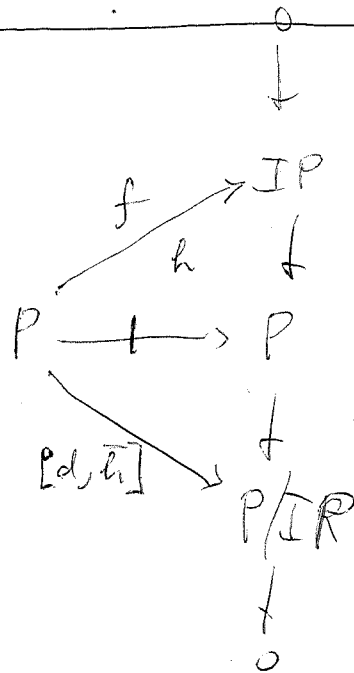
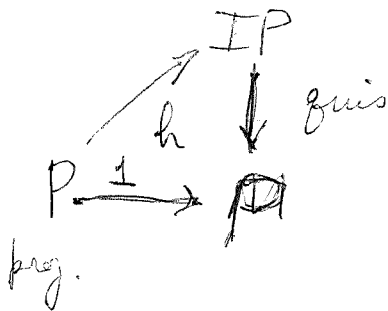
$$\text{Hom}_R(M, N) \xrightarrow{\sim} \text{Hom}_{M_t}(M, N)$$

$$\text{Hom}_R(M, N) \xleftarrow{\sim} \text{Hom}_R(M^\#, N)$$

$$\text{Hom}_{M_t}(M, N) \xleftarrow{\sim} \text{Hom}_R(M, N^\#)$$

~~OKAY~~

~~Wait~~ Wait.



K. What else (???)

Gabriel Quillen embedding of an ^{small} exact cat into an abelian cat.

\mathcal{E} exact cat $\text{add}(\mathcal{E}^{op} ab) = \mathcal{A}$

$\mathcal{S} \subset \mathcal{A}$ full subcat of effaceable functors
 $\forall F \in \mathcal{S} \iff \forall \xi \in F(X) \exists X' \twoheadrightarrow X \ni \xi \in \text{Ker}\{F(X) \rightarrow F(X')\}$.

Check it's a Serre subcat closed under \oplus

$$0 \rightarrow F'(X) \rightarrow F(X) \rightarrow F''(X) \rightarrow 0$$

$$\begin{array}{ccc} & \downarrow & \downarrow \\ & F(X_1) & \rightarrow F''(X_1) \end{array}$$

$$\downarrow$$

What are the effaceable free injectives? ✓

Suppose E injective and effaceable free

Suppose $0 \rightarrow X' \rightarrow X \rightarrow X'' \rightarrow 0$

exact in \mathcal{E} . Then have

$$0 \rightarrow h_{X'} \rightarrow h_X \rightarrow h_{X''} \rightarrow \text{Im } h_{X'/h_X} \rightarrow 0$$

here need X' is $\text{Ker } X \rightarrow X''$ also want

efface. so need pull back

$$0 \rightarrow X' \rightarrow X \rightarrow X'' \rightarrow 0$$

if so then $h_{X''}/\text{Im } h_X$ will be efface.

so we will have

$$0 \rightarrow E(X'') \rightarrow E(X) \rightarrow E(X') \rightarrow 0$$

so E will be an exact functor.

Φ So where are we? Examining.

$$\left(\underbrace{T(\bar{A}^*) * \mathbb{C}[h]}_R \right)_\eta \quad \text{with } -\delta$$

$$\left(R \oplus R \wedge R \oplus R \wedge R \wedge R \oplus \dots \right)_\eta$$

$$= R_\eta \oplus R \oplus R^{\otimes 2} \oplus \dots$$

But a better way to proceed might be

$$T(\bar{A}^*) * \mathbb{C}[h]$$



$$= \mathbb{C}[h] \oplus \mathbb{C}[h] \otimes \bar{A}^* \otimes \mathbb{C}[h] \oplus \dots$$

$$\therefore \left(T(\bar{A}^*) * \mathbb{C}[h] \right)_\eta = \mathbb{C}[h]_\eta \oplus \mathbb{C}[h] \otimes \bar{A}^* \oplus \left(\mathbb{C}[h] \otimes \bar{A}^* \right)_\eta^{\otimes 2} \oplus \dots$$

So what is left?

Basically you need ~~to construct~~ ^{a homotopy} $\mathbb{C}[h]$

Possible idea: Use the ~~additive~~ linear isomorphism

$$\mathbb{C}[h] = \mathcal{L}(\varepsilon, u) = \wedge(\mathbb{C}\varepsilon \otimes S(\mathbb{C}u))$$

$$\delta(h) = h^2$$

$$\begin{array}{ccc} \varepsilon & \xrightarrow{\delta} & u \\ u & \xrightarrow{\delta} & 0 \end{array}$$

Koszul
diff.

$$u \mapsto \varepsilon = du$$

deRham diff.

So what diff to use?

$$\text{Can look at } (A * \mathbb{C}[h], d)_\eta = (A * \mathbb{C}[h], -\delta)_\eta$$

