β It would appear that there are many choices

$$d^{0}(a_0, \dots, a_n) = \sum_{i=0}^{n+1} (-1)^i (\dots, a_{i-1}, 1, a_i, \dots)$$

$$d'(\quad\text{———}\quad) = \sum_{i=1}^{n+1} (-1)^i (\quad, a_{i-1}, 1, a_i, \dots)$$

Consider $d_0(a_0, \dots, a_n) = (a_0 a_1, a_2, \dots, a_n)$

Fact is that $d' = -\sum_{i=0}^{n} (-1)^i s_i$

$$s_i(a_0, \dots, a_n) = (a_0, \dots, a_i, 1, a_{i+1}, \dots, a_n)$$

$$d_i s_j = \begin{cases} s_{j-1} d_i & i < j \\ 1 & i = j, j+1 \\ s_j d_{i-1} & i > j+1 \end{cases}$$

$$d_0 s_j = s_{j-1} d_0 \qquad 1 \le j \le n.$$

$$d_0 \sum_{i=1}^{n} (-1)^i s_i = \sum (-1)^i d_0 s_{i-1} d_0$$

What's <u>Grothendieck's argument</u>?

$$A \rightrightarrows A \otimes A \;\overset{\longrightarrow}{\rightrightarrows}\; A \otimes A \otimes A$$

$$X \times X \times X \qquad \cancel{X} \times X \qquad X \downarrow$$

$$\overset{\longrightarrow}{\rightrightarrows} \quad X \times X \rightleftarrows X \longrightarrow pt$$

8 Cone construction

Point is that $d_j s_{-1} = s_{-1} d_{j-1}$

so $$\sum_{j=0}^{n+1} (-1)^j d_j s_{-1} = d_0 s_{-1} + \sum_{j=1}^{n-1} (-1)^j s_{-1} d_{j-1}$$

$$\partial s_{-1} = 1 - s_{-1}\partial .$$

I now want to do the same argument cosimplicially. Thus

$$\left(\sum_{i=0}^{n} (-1)^i s_i \right)$$

$$M \longrightarrow A\otimes M \rightrightarrows A\otimes A \otimes M$$

I have a cosimplicial thing. In any case you want to use diff

$$\sum_{i=1}^{n+1} (-1)^i s_i \quad : A^{\otimes n+1} \longrightarrow A^{\otimes n+2}$$

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$$\cancel{d_{n+1}\sum_{i=0}^{n+1}(-1)^i s_i = \sum_{i=1}^{n+1}(-1)^i s_{i-1} d_0}$$

$a_0, \dots, a_{n+1}$

$$\sum_{i=0}^{n}(-1)^i s_i \quad : \overset{\deg n}{A^{\otimes n+1}} \longrightarrow A^{\otimes n+2}$$

actually this is $-d'$.

$$d_0 \sum_{i=0}^{n}(-1)^i s_i = d_0 s_0 + \sum_{i=1}^{n}(-1)^{i-1} s_{i-1} d_0$$

$$d_0\left(\sum_{i=0}^{n}(-1)^i s_i\right) = 1 - \left(\sum_{j=0}^{n-1}(-1)^j s_j\right) d_0$$

So I can use either one ~~or the~~

$$d_{n+1}\sum_{i=1}^{n+1}(-1)^i s_i = \left(\sum_{i=1}^{n}(-1)^i s_i\right) d_n .$$

I'm getting confused. So how to handle this?

$a_0, \dots, a_n$

$$A \overset{0}{\underset{1}{\leftleftarrows}} A \otimes A$$

$a \qquad (a, 1)$

$a_0, 1, a_1$

$a_0, a_1, 1, a_2$

n

$1, a_n$

Anyway why am I getting nowhere?

I want to seriously work on controlling this homology.

E I want to consider $C(A) = \bigoplus_{n\geq 0} A^{\otimes n+1}$ with various ~~differen~~ "differentials". In degree n we have the operators

$$(\lambda^i s \lambda^{-i})(a_0, .., a_n) = (-1)^i (a_0, .., a_{i-1}, 1, a_i, .., a_n)$$

for $0 \leq i \leq n+1$.

Basic rule is that $s^2 + \lambda s \lambda^{-1} s = 0$?

$$(a_0, .., a_{n+1}) \xmapsto{s} (1, a_0, .., a_n)$$

$$\xmapsto{s} (1, 1, a_0, .., a_n)$$

$$\xmapsto{\lambda s \lambda^{-1}} -(1, 1, a_0, .., a_n)$$

More generally.

$$\lambda^i s \lambda^{-i} \lambda^j s \lambda^{-j} + \lambda^{j+1} s \lambda^{-j-1} \lambda^i s \lambda^{-i} = 0$$

$$i \leq j$$

Then there are various things I can do.

$$d = \sum_{i=0}^{n+1} \lambda^i s \lambda^{-i} \quad \text{on } A^{\otimes n+1}$$

$$d' = \sum_{i=1}^{n+1} \lambda^i s \lambda^{-i}$$

I think these are differentials $d^2 = 0$, $d'^2 = 0$.

But also $\sum_{i=1}^{n} \lambda^i s \lambda^{-i}$

Basically you have $A \xrightarrow{d} A^{\otimes 2} \xrightarrow{d} A^{\otimes 3}$ and you can tensor on either side with copies of A.

$\int$ Introduce notation

$$\varepsilon_i(a_0, \ldots, a_n) = (-1)^i (a_0, \ldots, a_{i-1}, 1, a_i, \ldots, a_n)$$
$$0 \leq i \leq n+1.$$

$$\varepsilon_i \varepsilon_j = -\varepsilon_{j+1} \varepsilon_i \qquad i \leq j$$

$$\sum_{\substack{\cancel{c \leq i \leq n+1} \\ c \leq i \leq n+1-c'}} \varepsilon_i$$

$$\sum_{j \in [c, n+2-c']} \varepsilon_j \cdot \sum_{i \in [c, n+1-c']} \varepsilon_i$$

$$= \sum_{c \leq i < j \leq n+2-c'} \varepsilon_j \varepsilon_i + \underline{\sum_{c \leq j \leq i \leq n+1-c'} \overset{-\varepsilon_{i+1}\varepsilon_j}{\varepsilon_j \varepsilon_i}}$$

$$- \sum_{c \leq j < i+1 \leq n+2-c'} \varepsilon_{i+1} \varepsilon_j$$

Next let us compute the homology

We will keep track of $d = \sum_{0 \leq i \leq n+1} \varepsilon_i$ on C_n

and $d' = \sum_{1 \leq i \leq n+1} \varepsilon_i$ on C_n $\qquad d = d' + \overset{s}{\overset{\shortparallel}{\varepsilon_0}}$

Idea is that if we take

$$\mathbb{C} \longrightarrow A \rightrightarrows A^{\otimes 2} \overset{\textstyle\rightarrow}{\rightrightarrows} A^{\otimes 3}$$

and tensor with A on the left we get (C, d')

7 Now show (C, d') is contractible.

Here we exhibit $b_0(a_0, \dots, a_n) = (a_0 a_1, a_2, \dots, a_n)$

Then $b_0 \varepsilon_1 = -1$

$i \geq 2$ $\quad (b_0 \varepsilon_i)(a_0, \dots, a_n) = b_0(a_0, \dots, a_{i-1}, 1, a_i, \dots)\,(-1)^i$

$$= (a_0 a_1, a_2, \dots, a_{i-1}, 1, a_i, \dots)\,(-1)^i$$

$$= -\varepsilon_{i-1}(a_0 a_1, a_2, \dots, a_{i-1}, a_i, \dots)$$

$$b_0 \varepsilon_i = -\varepsilon_0 b_0$$

Thus $\quad b_0 \underbrace{\sum_{i=1}^{n+1} \varepsilon_i}_{d' \text{ on } C_n} = -1 - \underbrace{\sum_{i=2}^{n+1} \varepsilon_{i-1}}_{d' \text{ on } C_{n-1}} b_0$.

$$\begin{tikzcd}
\mathbb{C} \arrow[r] \arrow[d] & A \arrow[r, "d"] \arrow[d, "s"] & A^{\otimes 2} \arrow[r, "d"] \arrow[d, "s"] & {} \arrow[d] \\
A \arrow[r] & A \otimes A \arrow[r, "-d'"] & A^{\otimes 3} \arrow[r, "-d'"] & {}
\end{tikzcd}$$

$$\underbrace{\varepsilon_0 \sum_{i=0}^{n} \varepsilon_i}_{\text{on } A^{\otimes n}} = -\sum_{i=0}^{n} \varepsilon_{i+1} \varepsilon_0$$

$$= -\underbrace{\sum_{j=1}^{n+1} \varepsilon_j \cdot \varepsilon_0}_{\text{on } A^{\otimes n+1}}$$

$$(\rho\, b_0\, s)(a_0, \dots, a_n) = \rho\, b_0(1, a_0, \dots, a_n)$$

$$= \rho(a_0, \dots, a_n)$$

$$= \rho(a_0)(a_1, \dots, a_n)$$

λ

What else? If the world

Then the kernel of a map of injective functors will be left exact.

~~What else?~~

Is an exact functor injective?
Probably not.
But what you want ~~is R^0F~~ to know is that $F \mapsto R^0F$ is the localization.

$$(R^0F)(X) = \varinjlim_{X = Y/Z} \operatorname{Ker}\big(F(Y) \to F(Z)\big)$$

N solid

$$\operatorname{Hom}_R(M,N) \xrightarrow{\sim} \operatorname{Hom}_{M_t}(M,N)$$

$$\operatorname{Hom}_R(M,N) \xleftarrow{\sim} \operatorname{Hom}_R(M^\#, N)$$

$$\operatorname{Hom}_{M_t}(M,N) = \operatorname{Hom}_R(M, N^\#)$$

$\boxed{\text{~~OKAY~~}}$ ~~What~~ Wait.

$$\begin{tikzcd} & IP \arrow[d, "\text{quis}"] \\ P \arrow[ur, "h"] \arrow[r, "1"'] & P \end{tikzcd}$$

proj.

$$\begin{tikzcd} & 0 \arrow[d] \\ & IP \arrow[d] \\ P \arrow[ur, "f"] \arrow[r, "1"] \arrow[dr, "{[d,h]}"'] & P \arrow[d] \\ & P/IR \arrow[d] \\ & 0 \end{tikzcd}$$

K What else ???

Gabriel Quillen embedding of an small exact cat into an abelian cat.

$\mathcal{E}$ exact cat $\quad \mathrm{add}(\mathcal{E}^{op}, ab) = \mathcal{A}$

$\mathcal{S} \subset \mathcal{A}$ full subcat of effaceable functors

$\forall F \in \mathcal{S} \iff \forall \xi \in F(X) \ \exists\ X' \twoheadrightarrow X \ \ni$
$\xi \in \mathrm{Ker}\{F(X) \to F(X')\}$.

Check it's a Serre subcat closed under $\oplus$.

$$\begin{tikzcd} 0 \arrow[r] & F'(X) \arrow[r] & F(X) \arrow[r] \arrow[d] & F''(X) \arrow[r] \arrow[d] & 0 \\ & \arrow[r] & F(X_1) \arrow[r] & F''(X_1) & \end{tikzcd}$$

0

✓

~~What~~ are the effaceable free injectives?

Suppose E injective and effaceable free

~~Suppose~~ $0 \to X' \to X \to X'' \to 0$

exact in $\mathcal{E}$. Then have

$$0 \to h_{X'} \to h_X \to h_{X''} \to \mathrm{Im}\ h_{X''}/h_X \to 0$$

here need X' is Ker $X \to X''$ also want

effac. so need pull ~~back~~

$$0 \to X' \to X \to X'' \to 0$$

If so then $h_{X''}/\mathrm{Im}\, h_X$ will be effac.

so we will have

$$0 \to E(X'') \to E(X) \to E(X') \to 0$$

so E will be an exact functor.

Φ So where are we? Examining

$$\big(\underbrace{T(\bar{A}^*)}_{R} * \mathbb{C}[h]\big)_{\natural} \quad \text{with } -\delta$$

$$\big(R \oplus RhR \oplus RhRhR \oplus \cdots\big)_{\natural}$$

$$= R_{\natural} \oplus R \oplus R^{\otimes 2}_{\lambda} \oplus \cdots$$

But a better way to proceed might be

$$T(\bar{A}^*) * \mathbb{C}[h] \qquad \boxtimes$$

$$= \mathbb{C}[h] \oplus \mathbb{C}[h] \otimes \bar{A}^* \otimes \mathbb{C}[h] \oplus \cdots$$

$$\boxed{\therefore \big(T(\bar{A}^*) * \mathbb{C}[h]\big)_{\natural} = \mathbb{C}[h]_{\natural} \oplus \mathbb{C}[h] \otimes \bar{A}^* \oplus \big(\mathbb{C}[h] \otimes \bar{A}^*\big)^{\otimes 2}_{\lambda} \oplus}$$

So what is left?

Basically you need ~~to contract~~ a homotopy on $\mathbb{C}[h]$

Possibly idea: Use the ~~additive~~ linear isomorphism

$$\mathbb{C}[h] = S[\varepsilon, u] = \Lambda \mathbb{C}\varepsilon \otimes S(\mathbb{C}u)$$

$\delta(h) = h^2$ $\qquad \varepsilon \xmapsto{\delta} u$, $\; u \xmapsto{\delta} 0$ Koszul diffl.

$u \longmapsto \varepsilon = du$ deRham diffl.

So what diffl to use?

Can look at $(A * \mathbb{C}[h], d)_{\natural} = (A * \mathbb{C}[h], -\delta)_{\natural}$