

r] A homotopy should be an odd cochain $\varphi \in (\tilde{\Omega}A)^*$ such that $\varphi(b+B) = \varphi$ except in degree zero.

~~So we end up taking the cycle~~ So we end up taking the cycle

$$\sum_{n>0} (-1)^n \frac{\omega^n}{n!}$$

and writing it as a coboundary. But this cycle corresponds to the homomorphism

$$RA \rightarrow \mathbb{C} \text{ induced by } \varphi.$$

The hard part will be to lift E into $\tilde{\Omega}A$

Review it.

$$0 \rightarrow \Lambda \otimes B(\mathbb{C}) \rightarrow E \rightarrow \tilde{\Omega}A \rightarrow 0$$

Maybe you can actually ~~lift~~ E into $\tilde{\Omega}A$. As Λ -module E should be a direct sum of these two, so it should be a question of lifting ~~the~~ $\tilde{\Omega}A \rightarrow \tilde{\Omega}A$ compatibly ~~to~~ with B . Also you want to lift $\Lambda \otimes B(\mathbb{C})$ into $\tilde{\Omega}A$. That's probably very simple, so what approach.

$$R(\tilde{A}) \rightarrow \mathbb{C} \times RA$$

I remember this being hard. First case

$$R(\tilde{\mathbb{C}}) \rightarrow \mathbb{C} \times \mathbb{C}$$

$\tilde{\mathbb{C}} = \mathbb{C}[e]$ where $e = 1$ in \mathbb{C} .

$$\begin{array}{l} \rho(e) = x \\ x \rightarrow 0 \\ \text{any} \end{array} \quad \begin{array}{l} x \rightarrow 1 \\ \tilde{\mathbb{C}} \rightarrow \mathbb{C} \end{array}$$

5) There is a unique lifting \hat{e} to an idempotent made out of $p(e) = x$. Anyway ~~the~~ after lifting the idempotent e you need to lift RA into the centralizer of \hat{e} . The question is whether there is a reasonable way to do this? You can take

$$\hat{e}R(\tilde{A})\hat{e} \longrightarrow RA$$

In order to construct a lifting you need only lift A linearly so that e goes to \hat{e}

$$\begin{array}{ccc} & A & \\ \nearrow & & \searrow \\ R(\tilde{A}) & \longrightarrow & RA \\ \downarrow & & \parallel \\ \hat{e}R(\tilde{A})\hat{e} & \longrightarrow & RA \end{array}$$

$$\begin{array}{ccc} & W^{p(e)} & \\ \nearrow \text{Linear} & \downarrow & \\ A & \longrightarrow & \mathbb{C} \times RA \\ a & \longmapsto & (0, pa) \end{array}$$

$p(e)$ centralizes \hat{e}

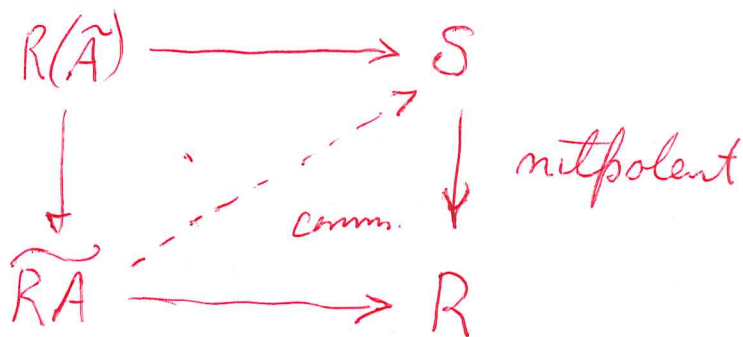
To be checked carefully

$$R(\tilde{A}) \longrightarrow \underbrace{R\mathbb{C} \times RA}_{\tilde{R}A} = \mathbb{C} \times RA$$

~~Ass~~ A hom. $\tilde{R}A \longrightarrow R$ same as an idempotent $e \in R$ and a linear map $p: A \longrightarrow eRe$ such that $p(1_A) = e$

The claim is that there's a quasi-canonical process for lifting:

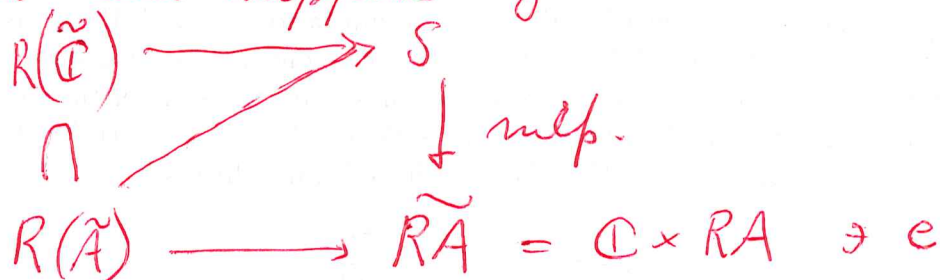
t



How does it work? How does it go?

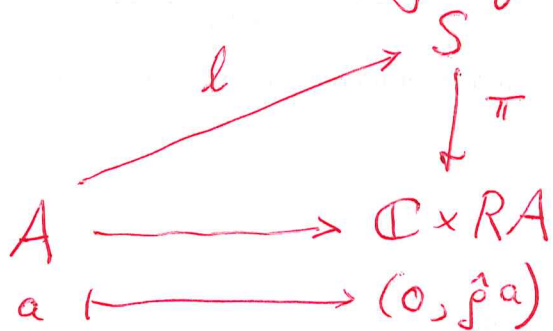
We have the following data: An idempotent $e \in R$ and a $f: A \rightarrow eke$ such that $f(1_A) = e$

Better can suppose given



e is the identity of RA

Start again. Suppose S is a nilpotent extension of $\mathbb{C} \times RA$ and suppose that we are given $l: \mathbb{C} \times A \rightarrow S$ lifting of the subspace $\mathbb{C} \times A \subset \mathbb{C} \times RA$. such that $(1, 0)$ goes to the identity of S .



$e = 1_A$. Then $\pi l(e) = (a, \hat{e})$. Unique poly in $l(e)$ which is idempotent in S , call it $\hat{e} \in S$.

u | so \hat{e} has to go to $(0, 1) \in \mathbb{C} \times \mathbb{R}A$.

Now consider

$$a \mapsto \hat{e}l(a)\hat{e} \in \hat{e}S\hat{e} \subset S$$

$$\begin{aligned} \pi(\hat{e}l(a)\hat{e}) &= (0, 1)(0, \hat{p}a)(0, 1) \\ &= (0, \hat{p}a). \end{aligned}$$

$$\text{so } \hat{e}l(1_A)\hat{e} = \hat{e}l(1_A)$$

so what we find is that we can cut S down to $\hat{e}S\hat{e}$ and also l down to $a \mapsto \hat{e}l(a)\hat{e}$ and we have

$$\begin{array}{ccc} & \hat{e}l\hat{e} \nearrow & \hat{e}S\hat{e} \\ & & \downarrow \\ A & \xrightarrow{\hat{p}} & RA \end{array}$$

The problem is that $\hat{e}l(1_A)\hat{e}$ is not necessarily equal to \hat{e} . This seems to happen already for $A = \mathbb{C}$. $l(1) = \times$

$$\begin{array}{ccc} & l \nearrow & S \\ \mathbb{C} & \longrightarrow & \mathbb{C} \times \mathbb{C} \\ \mathbf{1} & \longmapsto & (0, \mathbf{1}) \end{array}$$