(p) Can we handle the other side.

~~Take~~

In this case you have an inverse system

$$\cdots \longrightarrow I^{\otimes_R 2} \otimes_R M \longrightarrow I \otimes_R M \longrightarrow M$$

I'm thoroughly confused as what to do.

<u>Excision proof:</u> What to do?

Some idea, any ideas. Goodwillie thm:

Where to go next?

What are we thinking about?

Where to?

Let's try something like the following.

Go over excision proof. Recall that besides the spectral sequence it basically amounts to some sort of h-unitality.

$I = I^2$, I flat over $\tilde{I}$ $\Longrightarrow$

$$I \otimes I \otimes I \longrightarrow I \otimes I \longrightarrow I$$

is exact. Reason.

$$\cdots \longrightarrow I \otimes I \otimes \tilde{I} \longrightarrow I \otimes \tilde{I} \longrightarrow I \longrightarrow 0$$

is contractible. Tensor with I on the right, get

$$\longrightarrow I \otimes I \otimes I \longrightarrow I \otimes I \longrightarrow I \otimes_{\tilde{I}} I \longrightarrow 0$$

exact. Better proof: Have free right $\tilde{I}$ resolution of k

$$\longrightarrow I \otimes I \otimes \tilde{I} \longrightarrow I \otimes \tilde{I} \longrightarrow \tilde{I} \longrightarrow k \longrightarrow 0$$

now tensor $\otimes_{\tilde{I}} I$ get

$$\longrightarrow I \otimes I \otimes I \longrightarrow I \otimes I \longrightarrow I \longrightarrow I/I^2 \longrightarrow 0$$

(9) Let's translate this into adic systems
This means we form something like

$$J = tI \oplus t^2 I^2 \oplus \cdots$$

We can try to prove some sort of h-unitality. What does this mean?

$$\to J \otimes_{k[t]} J \to J \to$$

So what might happen?

~~What sort of ...~~

$$\cancel{\to R\otimes I \otimes I \to R \otimes I \to R \to R/I \to 0}$$

Start with the result that $I = I^2$ and
I I flat $\Rightarrow$ I is R-flat.
The main implication of this is that $I \overset{!}{\otimes}_R I = \underset{= I}{I^2}$.
Recall that step. I assume I is h-unital

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & \\
\arrow[r] & I\otimes I \otimes I \arrow[r] & I \otimes I \arrow[r] \arrow[d] & I \arrow[r] \arrow[d] & 0 \\
 & \arrow[r] & R\otimes I \arrow[r] \arrow[d] & I \arrow[r] \arrow[d] & 0 \\
 & \arrow[r] & R/I \otimes I \arrow[r, "0"] & R/I \arrow[r] & 0
\end{tikzcd}$$

yields very nices a free R-resolution of I

$$R \otimes_I B(I) \otimes_I I$$

$$I \overset{!}{\otimes}_I I \simeq I \Rightarrow R \overset{!}{\otimes}_I I \simeq I$$

(r) So what might be the adic version this this result. ~~What~~

I flat over itself

$$\longrightarrow \tilde{I} \otimes I \otimes I \longrightarrow \tilde{I} \otimes I \longrightarrow I$$

$$\longrightarrow \tilde{I} \otimes I \otimes M \longrightarrow \tilde{I} \otimes M \longrightarrow M \longrightarrow 0$$

always standard resolution

If I is right I flat, get

$$\longrightarrow I \otimes I \otimes M \longrightarrow I \otimes M \longrightarrow I \otimes_I M \longrightarrow 0$$

Better

$$\cdots \longrightarrow \tilde{I} \otimes I \longrightarrow \tilde{I} \longrightarrow k \longrightarrow 0$$

exact & I right $\tilde{I}$ flat

$$\Longrightarrow \; \to I^{\otimes 3} \longrightarrow I^{\otimes 2} \longrightarrow I \longrightarrow \underset{\substack{\shortparallel \\ I/I^2}}{I \otimes_{\tilde{I}} \tilde{I}/I} \longrightarrow 0$$

Better would be

$$\operatorname{Tor}_n^{\tilde{I}}(I, k) = \begin{cases} 0 & n > 0 \\ I/I^2 & n = 0. \end{cases} \quad \text{as } I \; \tilde{I}\text{ flat}$$

so wait.

$$0 \longrightarrow I \longrightarrow R \longrightarrow R/I \longrightarrow 0$$

$$\underbrace{\operatorname{Tor}_n^{\tilde{I}}(I, k)}_{\substack{0 \text{ except} \\ \text{for } I/I^2 \\ \text{in degree } 0}} \to \operatorname{Tor}_n^{\tilde{I}}(R, k) \longrightarrow \underbrace{\operatorname{Tor}_n^{\tilde{I}}(R/I, k)}_{R/I \otimes \operatorname{Tor}_n^{\tilde{I}}(k,k)}$$

$$\begin{cases} R/I & \text{for } n=0 \\ R/I \otimes I/I^2 & \text{for } n=1 \end{cases}$$

(5) So therefore what do we find?

$$\begin{tikzcd}
& 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
& I\otimes I\otimes I \arrow[r] \arrow[d] & I\otimes I \arrow[r] \arrow[d] & \cancel{I} \arrow[r] \arrow[d] & 0 \\
& R\otimes I\otimes I \arrow[r] \arrow[d] & R\otimes I \arrow[r] \arrow[d] & \cancel{I} \arrow[r] \arrow[d] & 0 \\
& R/I\otimes I\otimes I \arrow[r] \arrow[d] & R/I\otimes I \arrow[r, "0"] \arrow[d] & 0 \arrow[d] & \\
& 0 & 0 & 0 &
\end{tikzcd}$$

$$0 \longrightarrow I^2 \longrightarrow \mathrm{Tor}_0^{\tilde{R}}(R, I) \longrightarrow R/I \otimes I/I^2 \longrightarrow 0$$

$$0 \longrightarrow I\otimes_{\tilde{I}} I \longrightarrow R\otimes_I I \longrightarrow R/I \otimes_I I \longrightarrow 0$$

$$I\otimes_{\tilde{I}} I \cong I^2, \qquad R/I \otimes_I I \cong R/I\otimes I/I^2$$

Somethings work ok. e.g.

~~$R\otimes I \longrightarrow I$~~ proj over R

$$I\otimes R \longrightarrow I$$

I is projective as left R-module, so you get an $\tilde{I}$!! ~~No!!!!~~

so