Trying to understand cyclic coh. and anomalies.
fermion quantization from the beginning.
I want to understand a fermion Lagrangian so well that I can see the classical limit. So what does this amount to?
Introduce time — time and the appropriate boundary conditions.

Maybe I can start with the Hilbert-Hamiltonian viewpoint and then progress. Take a single fermion a, a^* with the Hamiltonian operator $\omega(a^*a - \frac{1}{2})$. Take the propagator e^{-itH} and try to express it as a path integral à la Feynman. Need intermediate states. The ~~only states~~ are $|0\rangle, |1\rangle$. So a path flips between the two. On the other hand I would like to think of a classical path as being much more continuous.

$$e^{-\int \bar{\psi}(t) A \psi(t)\, dt}$$

should give the Green's functions.

Green's function:

$$\langle 0 | T[a(t)a^*(t')] | 0 \rangle = G(t,t')$$

$$a(t) = e^{iHt} a e^{-iHt} = e^{-i\omega t} a.$$

$$H = \omega a^* a$$
$$[H,a] = -\omega\{a^*,a\}a = -\omega a$$

$$\cancel{a(t) = U(t,0)\, a\, U(0,t) = e^{-iHt}\, a\, e^{iHt}}$$

$$\cancel{a(t)\,\psi = e^{-iHt}\, a\, e^{iHt}\,\psi}.$$

$$\left[\frac{a^2}{2}, \begin{pmatrix} a \\ a^* \end{pmatrix}\right] = \begin{pmatrix} 0 \\ a \end{pmatrix}$$

$$\left[a^* a, \begin{pmatrix} a \\ a^* \end{pmatrix}\right] = \begin{pmatrix} -a \\ a^* \end{pmatrix}$$

$$\left[\frac{a^{*2}}{2}, \begin{pmatrix} a \\ a^* \end{pmatrix}\right] = \begin{pmatrix} -a^* \\ 0 \end{pmatrix}$$

$$\cancel{\left[\frac{a^2}{2}, \begin{pmatrix} a \\ a^* \end{pmatrix}\right] =}$$

$$\left[\frac{a^2}{2}, (a \;\; a^*)\right] = (a \;\; a^*)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\left[a^* a, (a \;\; a^*)\right] = (a \;\; a^*)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left[\frac{a^{*2}}{2}, (a \;\; a^*)\right] = (a \;\; a^*)\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$$

$$e^{\gamma \frac{a^2}{2}}\, (a \;\; a^*)\, e^{-\gamma \frac{a^2}{2}} = (a \;\; a^*)\begin{pmatrix} \phi & \gamma \\ 0 & \phi \end{pmatrix}$$

$$S = T\left\{ e^{-i\int_{-\infty}^{\infty} H(t)\,dt} \right\}$$

Which combinations of a, a^* are destruction operators? $[a, a^*] = 1 > 0.$

$$b = xa + ya^*$$

$$b^* = \bar{x}a^* + \bar{y}a$$

$$[b, b^*] = [xa + ya^*, \bar{x}a^* + \bar{y}a]$$

$$= |x|^2 - |y|^2$$

So we have a 2-diml space.

$$(b \;\; b^*) = (a \;\; a^*)\begin{pmatrix} x & \bar{y} \\ y & \bar{x} \end{pmatrix}$$

$$|x|^2 - |y|^2 = 1.$$

So we get a group of matrices $SU(1,1)$ which is of dimension 3, isom. I think to $SL_2(\mathbb{R})$. Doesn't make much difference

S

a_{out} $\quad$ a_{in}

$$S = e^{iH_0 t} \; U(t,t') \; e^{-iH_0 t'}$$

$$U_0(0,t) \qquad U_0(t',0)$$