Search for missing idea:

Space of connections $\mathcal{A}$. Idea is that $\tau^{\#}(\partial\rho\,\omega^n)$ is a 1-form on $\mathcal{A}$.

Over $\mathcal{A}\times M$ have $pr_2^*(E)$ and taut. conn. $d_{\mathcal{A}} + d_M + \rho$ (assuming E trivial)

Curvature $$(d_{\mathcal{A}} + d_M + \rho)^2 = \underbrace{d_{\cancel{M}}\rho + \rho^2}_{\omega} + d_{\mathcal{A}}\rho$$

$$\int_{M^{2n+1}} \mathrm{tr}(\omega + d_{\mathcal{A}}\rho)^{n+1}/(n+1)! = \int_{M^{2n+1}} \mathrm{tr}(d_{\mathcal{A}}\rho\,\omega^n)/n!$$

There should be <u>various</u> <u>substitutes</u> for $\mathcal{A}$ available. ~~[illegible]~~ Connes operator version; also my idea where connections are handled via NR.

~~Example~~ Fundamental behavior

$$\bullet\quad [\delta+\rho, \omega] = 0$$

$$[\delta+\rho, \partial\rho] = \partial\omega$$

As long as you work with the Grass, ~~and unitary gps.~~ you probably won't see the free loop space. ~~You want the free loop of~~

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Think carefully in low degrees.

$\rho : A \longrightarrow \mathcal{L}(eH) \qquad \rho a = eae$

We assume $\omega(a_1, a_2) = e a_1 (1-e) a_2 e \in \mathcal{L}^1(eH)$

Then we have $f_2(a_0, a_1, a_2) = \mathrm{tr}(\rho(a_0)\, \omega(a_1, a_2))$

$$f_2 = \mathrm{tr}^q (\partial\rho\, \omega)$$

$$\delta f_2 = \mathrm{tr}^q(\partial\omega\,\omega) = \partial\, \mathrm{tr}(\omega^2/2)$$

$$f_2(a_0, a_1, a_2) = \mathrm{tr}(e a_0 e a_1 (1-e) a_2 e)$$

We have $\omega : \bar{A}^{\otimes 2} \longrightarrow \mathcal{L}^1(eH)$ and $[\delta + \rho, \omega] = 0$

Principal bundle P/M for a v.b. E

$$P \times^G \mathbb{C}^n \xrightarrow{\sim} E \qquad G = U_n$$

$R = M_n(\Omega P)$ DGA

$A \in R^1$ conn. form $F = dA + A^2$

$$0 \to \Omega^1 R \to R \otimes R \longrightarrow R \longrightarrow 0$$

$$0 \to \Omega^1_A \otimes \Omega^1_B \longrightarrow \begin{array}{c} A \otimes A \otimes \Omega^1_B \oplus \\ \Omega^1_A \otimes (B \otimes B) \end{array} \longrightarrow \Omega^1(A \otimes B) \longrightarrow 0$$

$$\Omega^1(A \otimes B) \cap (A \otimes A) \otimes (B \otimes B)$$

Model $\mathbb{C}\langle A, dA \rangle$

$M_n(\Omega(P))$

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General question: What can be said about the free loop space of BU and its equivariant cohomology?

$$\operatorname{Hom}_{S^1\text{-man.}}(Y, LX) = \operatorname{Hom}_{man}(Y, X).$$

$$\begin{tikzcd} L(BU) \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{cart}"] & BU^I \sim BU \arrow[d] \\ BU \arrow[r] & BU \times BU \end{tikzcd}$$

~~free loop map~~ $H^*(L(BU)) = H^*(BU) \otimes H^*(U)$

In fact if we think of $L(BU)$ as ~~class~~ bundle with autom. Then $\exists$

$$L(BU) \longrightarrow U$$

But the S^1 action is not obvious.

What we know is the following: We know that the ~~odd~~ even classes on $L(BU)$ are definitely not equivariant.

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ungraded case G acts on H^N; get family param. by **BG**; Get $BG \longrightarrow U$ which we want to transgress to $G \longrightarrow BU$. Nowhere here do we see $L(BU)$.

4] P principal U_N-bundle over M. What can you say about

$$\left(\Omega(P)\otimes M_N\right)^{U_N} \quad ?$$

locally this is

$$\Omega(M)\otimes\left(\underbrace{\Omega(U_N)}\otimes \overset{\mathfrak{g}}{M_N}\right)^{U_N}$$

$$\Omega(M)\otimes\left(\Omega(U_N)\otimes\Lambda\mathfrak{g}^*\otimes\mathfrak{g}\right)^{U_N}$$

$$\Omega(\cancel{M})\otimes\Lambda\mathfrak{g}^*\otimes\mathfrak{g}$$

$$\Omega(P\times P) = \Omega(P)\otimes\Omega(P)$$

$$\begin{array}{c}\Omega(P\times P)\\ \downarrow \Delta^*\\ \Omega(P)\\ \downarrow\\ 0\end{array}$$

$$\begin{array}{ccc} & & \Omega(P)\\ & & \uparrow\\ \Omega(P) & \longleftarrow & \Omega(P\times P)\end{array}$$

principal bundle

$$\mathcal{G} = U_N(A) \qquad \tilde{\mathfrak{g}} = A\otimes M_N$$

$$\downarrow\pi$$

$$\mathcal{G}/G \qquad \Omega(\mathcal{G}, \overset{\mathrm{End}}{(\pi^*E)})^{\mathcal{G}} = \mathrm{Hom}(\Lambda\tilde{\mathfrak{g}}, \mathcal{L}(H)\otimes M_N)^G$$

$G = U_N$ inv.

$$\mathrm{Hom}\left(\Lambda(A\otimes\mathfrak{g})^*, \mathcal{L}(H)\otimes\mathfrak{g}\right)^G$$

5 primitive part ~~[illegible]~~ is exactly

$$\mathrm{Hom}\left(A^{\otimes ?}, \mathcal{L}(H)\right)$$

Can you actually see the non-triviality in the topology?

principal bundle $P \xrightarrow{G} B$

$$0 \to \Omega^1(B) \to \Omega^1(P)^G \to \Omega^0(P, \mathfrak{g}^*)^G \to 0$$

$$\left(\Omega(P)\right)^G = \left(\Omega(P)_{hor} \otimes \Lambda\mathfrak{g}^*\right)^G$$

$$\Omega^1(P)^G = \Omega^1(P)^G_{hor} \otimes \Lambda^0\mathfrak{g}^* \oplus \left(\Omega^0(P)\otimes\mathfrak{g}^*\right)^G$$

$$\left(\Omega^1(P)\otimes\mathfrak{g}\right)^G = \left(\Omega^1(P)_{hor}\otimes\mathfrak{g}\right)^*$$

$$\oplus \left(\Omega^0(P)\otimes\mathfrak{g}^*\otimes\mathfrak{g}\right)^G$$

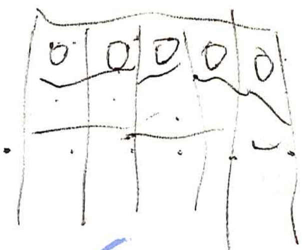
connection

$$\Lambda\mathfrak{g}^* \to \Omega(P)$$

locally $\left(\Omega(P)\otimes M_N\right)^G = \left(\Omega(B)\otimes\Omega(G)\otimes M_N\right)^G$

$$= \Omega(B)\otimes\Lambda\mathfrak{g}^*\otimes M_N$$

6 | I am Studying an algebra, namely



$$\Omega^\bullet(\mathcal{G}, R\otimes M_N)^{G\times G}$$

$$\| $$

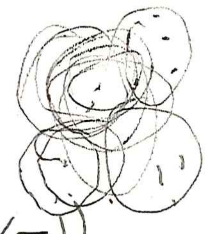
$$\mathrm{Hom}(\Lambda\tilde{\mathfrak{g}}, R\otimes M_N)^G \sim (\Omega(P)\otimes\mathfrak{g})^{\mathfrak{g}}$$

IS $\Omega_{hor}(P)\otimes\Lambda$

So now I am looking at some generalizations OKAY

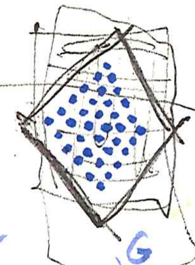
What am I trying to understand?

There are some complications but basically we have on $\mathcal{G}$ an action of $G\ltimes G$. Maybe first of all one should look at



$\Omega(\mathcal{G}, R\otimes M_N)^G$ relative to $\Omega(\mathcal{G}/G, \mathrm{End}(E))_{bas}$

I am proposing to look at Ω



Filtration. In general you have something like

$$0\to\Omega^1(B,\mathrm{End}\,E)\to(\Omega^1(P)\otimes(R\otimes M_N))^G\to\begin{pmatrix}\mathfrak{g}^*\otimes\\ R\otimes M_N\end{pmatrix}\to 0$$

So if we choose a connection, maybe it splits this sequence

$$\mathrm{Hom}(\bar{A},R)\longrightarrow\mathrm{Hom}(A,R)\longrightarrow R\to 0$$

$$\underline{\mathrm{Hom}}(A(\tilde{\mathfrak{g}}/\mathfrak{g}), R\otimes M_N)^G\to\underline{\mathrm{Hom}}(\Lambda\tilde{\mathfrak{g}}, R\otimes M_N)^G$$