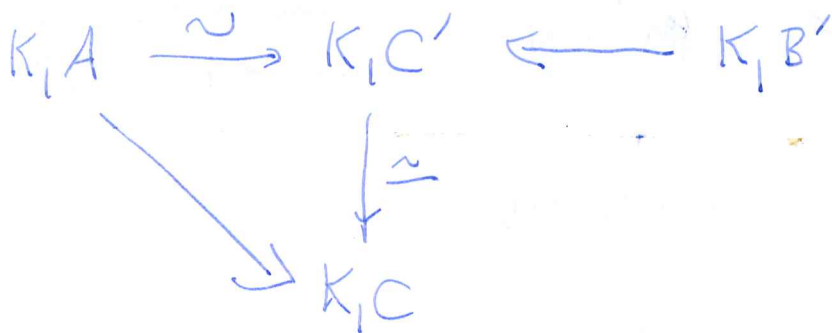
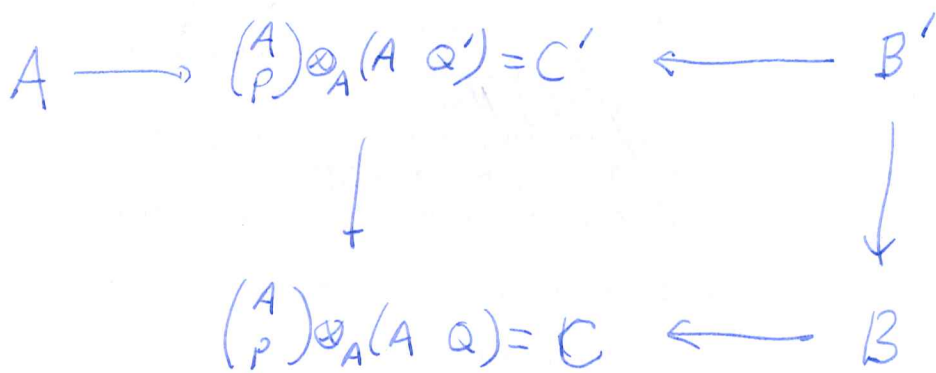
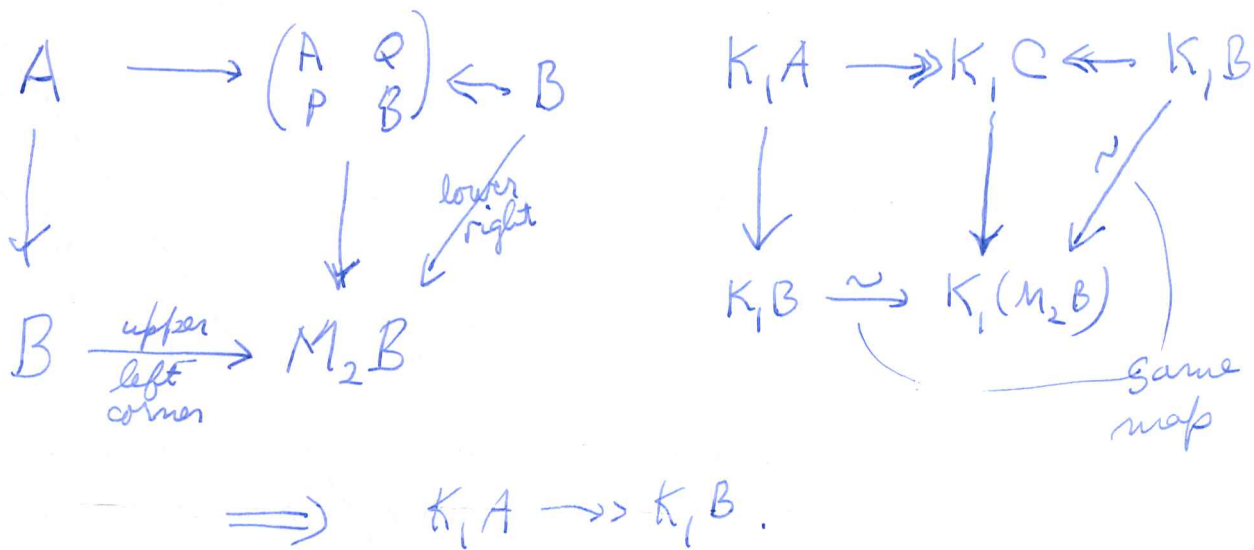


{0} $A \rightarrow B$ ~~map~~ homo + Map



{II} 10/9/95 - 1944 Try Morita invariance for K_2 . First review the steps for ~~things~~ K_1 .

The key point is that if $A \xrightarrow{\phi} B$ is a ring homom. which ~~induces~~ induces a Morita equivalence, then $K_1 A \rightarrow K_1 B$. Why? I'm assuming $A=A^2, B=B^2$. The rest should be Vasenstein's identity. I can suppose $A \hookrightarrow \phi A$, because $A \rightarrow \phi(A)$ will be a split extn, so $GL(A) \rightarrow GL(\phi A)$. Then I have the Morita context

$$\begin{pmatrix} A & AB \\ BA & B \end{pmatrix} \quad \text{+ the rest is Vasenstein.}$$

~~Injectivity step~~ injectivity step

$$P \otimes_A Q = B \quad \updownarrow \text{ is } B \text{ flat.}$$

$$C = \begin{pmatrix} A & Q \\ P & B \end{pmatrix}$$

assume either Q is A -flat or P is A^{op} flat. $\Leftrightarrow P \otimes_A Q = B$ is A right flat

$$Q \text{ } A\text{-flat} \Rightarrow Q = \varinjlim F_\alpha \quad F_\alpha \simeq A^{n_\alpha}$$

$$\parallel$$

$$AQ = \varinjlim AF_\alpha \quad \parallel \quad F_\alpha^* A \quad \parallel \quad \mathbb{Q} \otimes \mathbb{C}$$

$$P \rightarrow \text{Hom}_A(Q, A) \rightarrow \text{Hom}_A(F_\alpha, A)$$

$$C = \begin{pmatrix} A & Q \\ P & B \end{pmatrix} = \varinjlim_\alpha \underbrace{\begin{pmatrix} A & AF_\alpha \\ P & P \otimes_A AF_\alpha \end{pmatrix}}_{C_\alpha}$$

$$K_1 C = \varinjlim_\alpha K_1 C_\alpha \quad \text{need } C_\alpha \text{ idempotent OK. as } \hat{C}_\alpha = \begin{pmatrix} A \\ P \end{pmatrix} \otimes_A (A \ Q)$$

Anyway we have $A \rightarrow C_\alpha \rightarrow M_n(A)$ which gives injectivity.

Actually I'm try to prove Morita invariance of $K_1 A$ for firm rings. So the above gives ~~if~~ $K_* A \hookrightarrow K_* C$ if (P, A, Q) are flat.

\exists so for K_1 you get $K_1 A \xrightarrow{\sim} K_1 C \xleftarrow{\sim} K_1 B$
 when A, B flat on one side. ~~What happens?~~
 What method? ~~You want to choose~~ Suppose
 you want to get from A to B

$$(A, A, \mu)$$

Suppose ~~A, B~~ A, B M. eg + firm. Choose
 $\circ K \rightarrow A' \xrightarrow{f} A \rightarrow 0$ with A' A -flat firm. Then we
 know that $K_1 A' \xrightarrow{\sim} K_1 A$.

$$\begin{array}{ccccc}
 K_1 A' & \longrightarrow & K_1 C' & \xleftarrow{\sim} & K_1 B \\
 \downarrow \cong & & \downarrow \cong & & \parallel \\
 K_1 A & \longrightarrow & K_1 C & \longleftarrow & K_1 B
 \end{array}$$

Start again. ~~Restrict~~ Restrict to firm stuff first.

$$C = \begin{pmatrix} A & Q \\ P & B \end{pmatrix} = \begin{pmatrix} A \\ P \end{pmatrix} \otimes_A (A \ Q) = \begin{pmatrix} Q \\ B \end{pmatrix} \otimes_B (P \ B)$$

comp. firm.

Suppose $A \ Q$ flat (equiv. B is left flat).

$$\text{Then } Q = \varinjlim F_i = \varinjlim A F_i$$

$$\begin{aligned}
 C &= \varinjlim C_i & C_i &= \begin{pmatrix} A \\ P \end{pmatrix} \otimes_A (A \ A F_i) \\
 & & &\rightarrow \begin{pmatrix} A \\ F_i^* A \end{pmatrix} \otimes_A (A \ A F_i) \cong M_{n_i}(A).
 \end{aligned}$$

$\text{Prop. } C \text{ completely firm, } B \text{ left or right flat} \Rightarrow K_* A \xrightarrow{\sim} K_* C. \text{ iso for } K_1.$

Then given ~~C completely~~ C comp. firm, choose

$$\begin{array}{ccccc}
 A' \twoheadrightarrow A & \text{get} & K_* A' & & K_1 C' \xleftarrow{\sim} K_1 B \\
 & & \downarrow \cong & & \downarrow \cong & \parallel \\
 & & K_1 A & & K_1 C \longleftarrow & K_1 B
 \end{array}$$