5) Question is still what to do about B? Actually ~~it is clear~~

$$A * \mathbb{C}[h] = R_A\big(\underbrace{A \oplus A\otimes A[1]}\big)$$

not a DG algebra otherwise would be acyclic.

Relative $X_A(A * \mathbb{C}[h])$, is this closed to $\Omega\tilde{A}$.

Let me consider stupidities and similar things. Go back to

$$\tilde{R}\tilde{A} \longrightarrow RA$$

$$\boxed{\begin{array}{c} R\tilde{A} \longrightarrow R\mathbb{C} \times RA \\ \hline \Omega\tilde{A} \longrightarrow \Omega\mathbb{C} \times \Omega A \end{array}}$$

So the first point is clear.

$$\underbrace{\Omega\tilde{\mathbb{C}}} \longrightarrow \mathbb{C} \times \mathbb{C}$$

$\Omega(\mathbb{C}[\varepsilon])$ basis $1, e \quad de^n \quad ede^n$

Question: Is there a good free DG alg resolution of $A * \mathbb{C}[\varepsilon]$? Yes the cobar on the bar construction

$$\Omega A \subset A * \mathbb{C}[F] = QA \tilde{\otimes} \mathbb{C}[F]$$

$$a_0 da_1 \dots da_n \longmapsto a[F, a_1] \cdots [F, a_n]$$

t) So the first thing of interest might be to construct this resolution

$$0 \leftarrow \bar{C}^\lambda(A) \leftarrow C^\lambda(A) \xleftarrow{+1} B \xleftarrow{+1} B_\sigma^{\otimes 2} \leftarrow$$

For example can you produce a map from $B = \mathrm{Bar}(A)$ to $C^\lambda(A)$ using the identity of A. Dually can you produce a map

$$\boxed{T(A^*)_4 \xrightarrow{-1} T(A^*)}$$

effect of a derivation

Parts of this should be easy.

$$T(\bar{A}^*) \hookrightarrow T(A^* \oplus \mathbb{C}\varepsilon^*) = T(A^*) \oplus T(A^*)\,\varepsilon^*\, T(A^*) \oplus$$

Let's examine this carefully. We have the algebra A with identity. Have bigraded algebra $T(A^*) * \mathbb{C}[h]$ $\quad |h| = 1 \quad |A^*| = 1$.

δ differential $A^* \to A^* \otimes A^*$ transpose of multiplication. Now the question is — what the difficulty with the other differential.

given by the first author [Cu] using the universal extension, the close relation between traces on the universal extension and cocycles with respect to $b+B$.

My work with Cuntz has led to a new approach to cyclic homology using quasi-free algebras. Quasi-free algebras are noncommutative analogues of nonsingular varieties and manifolds. Tubular neighborhoods, connections, geodesic flow make sense for quasi-free algebras.

The exact sequences \mr, \mr were derived in [Q2] by means of spectral sequences.

Map of exact sequences agreeing with the map from cyclic to reduced cyclic homology. (?)

\mb

u)

$ch_4 \longrightarrow$

$\uparrow$

$g_2 \qquad ch_2 \xrightarrow{B}$

$\uparrow b$

$g_1 \qquad ch_0 \qquad g_{-1}$

so if we have $2 \to E$ zero, then $g_1 = g_3 = 0$. so our map consists of g_{-1}

Direction to head. You have this diagram

$$\begin{tikzcd}
 & K \arrow[r, equal] \arrow[d] & K \arrow[d] & & \\
0 \arrow[r] & \tilde{A} \otimes B(A) \arrow[r] \arrow[d] & E \arrow[r] \arrow[d] & \bar{\Omega} A \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & \mathbb{C} \arrow[r] & \Omega A \arrow[r] & \bar{\Omega} A \arrow[r] & 0
\end{tikzcd}$$

E depends on the choice of ρ it seems.

and you have a canonical lifting

$$\begin{tikzcd}
 & K \arrow[d] \\
\bar{\Omega} \tilde{A} \arrow[r] \arrow[dr] & E \arrow[d] \\
 & \Omega A
\end{tikzcd}$$

I know $\bar{\Omega}\tilde{A} \longrightarrow E$ is a heq of mixed cxs. I would like to ~~prove~~ construct ~~an explicit~~ this heq explicitly. The first step would be to map E into $\bar{\Omega}\tilde{A}$. Now K depends only on $A = \mathbb{C}$, so it ought to be easy to map K into $\bar{\Omega}\tilde{A}$. What actually happens for $A = \mathbb{C}$? $\qquad P\bar{\Omega}\tilde{A} = E$.

$\cancel{e\,de^{2n}}$

$$\kappa(e\,de^n) = (-1)^{n-1} de\, e\, de^{n-1} = (-1)^{n-1}(1-e)\, de^n$$

$$\kappa(de^n) = (-1)^{n-1} de^n$$

v) $$\kappa(e\,de^n) = (-1)^n e\,de^n + (-1)^{n-1} de^n$$

$$\kappa\left((e-\tfrac{1}{2})de^n\right) = (-1)^{n-1}\, de\,(e-\tfrac{1}{2})\,de^{n-1}$$
$$= (-1)^{n-1}\left(-e+\tfrac{1}{2}\right) de^n$$

$$\boxed{\begin{aligned} \kappa\left((e-\tfrac{1}{2})\,de^n\right) &= (-1)^n\,(e-\tfrac{1}{2})\,de^n \\ \kappa(de^n) &= (-1)^{n-1}\,de^n \end{aligned}}$$

So $P\tilde{\Omega}\tilde{\mathbb{C}}$ consists of $e,\ de,\ (e-\tfrac{1}{2})de^2,\ de^3$

$$B(e-\tfrac{1}{2})de^{2n} = (2n+1)\,de^{2n+1}$$

$$\cancel{b(de^{2n+1}) = de^{2n}e - e\,de^{2n}}$$

$$b\left((e-\tfrac{1}{2})de^{2n}\right) = -\,(e-\tfrac{1}{2})de^{2n-1}e + e(e-\tfrac{1}{2})de^{2n-1}$$

$$0 \longrightarrow P\tilde{\Omega}\tilde{\mathbb{C}} \longrightarrow P\tilde{\Omega}\tilde{A} \longrightarrow \boxed{} \longrightarrow 0$$

$$0 \to C^\lambda(\mathbb{C}) \to C^\lambda(A) \to C^\lambda(A)/C^\lambda(\mathbb{C}) \to 0$$

$$\bar{C}^\lambda(A)$$

So we come back to the same problem, namely constructing a lifting from $\bar{C}^\lambda(A)$ into $C^\lambda(A)/C^\lambda(\mathbb{C})$. We know this exists But the question is how to construct it

w) A line of attack would be to analyze proofs that ~~$C^\lambda(A) \to C$~~ the surjection $C^\lambda(A)/C^\lambda(\mathbb{C}) \longrightarrow \bar{C}^\lambda(A)$ is a quis. I remember there being several arguments.

~~[illegible]~~ $\boxed{\text{filtration argument.}}$ in Jacek's paper

$\boxed{\text{LQ argument.}}$

$$\begin{array}{c} C^\lambda(A)/C^\lambda(\mathbb{C}) \sim B(\bar{\Omega}\tilde{A})/B(\bar{\Omega}\tilde{\mathbb{C}}) \\ \downarrow \text{quis} \\ B(\Omega A)/B(\mathbb{C}) \\ \| \\ \bar{C}^\lambda(A) \sim B(\bar{\Omega}A) \end{array}$$

Another version goes as follows:

$$\bar{C}^\lambda(A \oplus \varepsilon A) \sim \bar{C}^\lambda(\mathbb{C} + \varepsilon\mathbb{C}) = C^\lambda(\mathbb{C})$$

$\}$ double complex

$\mathrm{Cone}\left(CC(A) \longrightarrow \bar{C}^\lambda(A)\right)$

Relative Lie algebra cohomology

$H_*(gl(A), gl(\mathbb{C}))$ $\qquad$ h reductive in g

~~$\not{D}$~~ $\qquad E \longrightarrow P\bar{\Omega}A \longrightarrow 0$

y) Then $[b', d](v) = b'(v\xi) + d\,\cancel{b'(v)}$
$$= v\,b'(\xi) = v$$

$$[b', d](\xi) = b'(-\xi^2) + d(b'\xi) = 0.$$

New approach. Go back to ~~$A = \mathbb{C}\oplus$~~

$$T(A^*) = T(\bar{A}^*) * \mathbb{C}[\rho] \qquad d\rho + \rho^2 = 0$$
$$d\theta^i + [\rho, \theta^i] = 0$$

Do I know the homology of dual numbers case.

$(T(A^*), d)_\natural$ in an intelligent way.

Can I get from this ~~problem~~ to something I know

~~Start~~ Repeat. Consider $T(A^*) = T(\bar{A}^*) * \mathbb{C}[\rho]$

with $d\rho + \rho^2 = 0$, $d\theta^i + [\rho, \theta^i] = 0$ dual nos.

I want the homology of $\left(T(\bar{A}^*) * \mathbb{C}[\rho]\right)_\natural$ wrt

d. ~~Is there some way to descend.~~

March 7 I want to organize a lecture

standard resolution

$$R = A * \mathbb{C}[\xi] \qquad |\xi| = 1$$

$$\simeq A$$

Important idea. Let R be a free DG algebra. Then ~~there~~ we know there is an S operation on the homology of $\bar{R}_\natural = R/\mathbb{C} + [R, R]$

$$0 \leftarrow \bar{R}_\natural \xleftarrow{\natural} \bar{R} \xleftarrow{\beta} \Omega^1(R)_\natural \longleftarrow \bar{R}_\natural \longleftarrow 0$$

You have to understand this operation theoretically and the idea is Tsygan's — somehow associated to the action of the DG Lie algebra $\mathbb{C}\varepsilon$ $\varepsilon^2 = 0$ on R.