(h ~~[illegible]~~ the construction but perhaps some of these improve if we go to the derived category.

~~So what ask~~

$$j_* j^* M = \varinjlim \mathrm{Hom}_R(I^n, M)$$

So what can we do? Try to handle things.

~~STILL~~ It still may be true on the derived category level. ~~It~~

It's also possible that the good modules have something to do with compact support.

Modules with <u>compact support</u>?

Lecture. <u>Triple factorization property</u>:

$\forall a_1, \dots, a_n \in A \quad \exists\ b_1, \dots, b_n, c, d \quad \ni$

$a_j = b_j c d \quad \forall i \qquad \mathrm{ann}_\ell(cd) = \mathrm{ann}_\ell(c).$

$\Longrightarrow A^2 = A$ and A is left A-flat.

$$\boxed{\sum_j \alpha_{ij} a_j = 0} \qquad a_j = b_j c d \quad \text{as above}$$

$$\sum \alpha_{ij} b_j c d = 0 \implies \sum \alpha_{ij} b_j c = 0.$$

C^* alg have the TFP. $\qquad C_0(X) \qquad$ vanishing at ∞.

A = ideal of cont. fns. on X compact (T_2) vanishing on closed set Z. $\qquad C_0(X-Z)$.

$f_1, \dots, f_n \qquad \sum_j f_j^* f_j = \sum_j |f_j|^2 \qquad z_1, \dots, z_n$ on unit disk in $\mathbb{C}^n$.

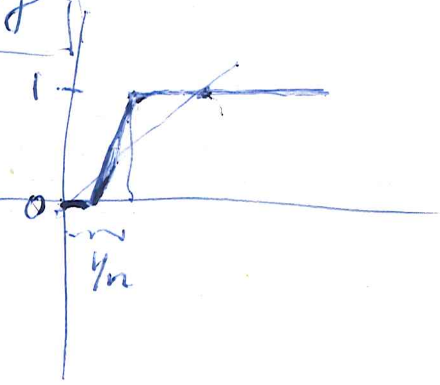
$$f_i = f_i \Big/ \left(\sum |f_j|^2\right)^{1/2}$$

(i $\qquad \dfrac{z_i}{|z|^{2\varepsilon}} |z|^{\varepsilon} |z|^{\varepsilon}$

cont for $2\varepsilon < 1$.

$f_1, \dots, f_n$		$h^2 = \sum_{j=1}^{n} f_j^* f_j$

$$f_j \frac{\phi_n(h)}{h^{1/2}} \to f_j h^{-1/2}$$



$$\cancel{\frac{\phi_n}{h} f_j^* f_j \frac{\phi_n}{h} \le \frac{\phi_n}{h} h^2 \frac{\phi_n}{h} = \phi_n^2}$$

$$\frac{\phi_n - \phi_m}{h^{1/2}} f_j^* f_j \frac{\phi_n - \phi_m}{h^{1/2}} \le (\phi_n - \phi_m)^2 h \xrightarrow[\sup]{} 0$$

$$f_j h^{-1/2} \qquad h^{1/4} h^{1/4}$$

So $I = (x, y)$ $R = k[x, y]$ and what next?

~~So on the end result.~~

Let $A \subset R$ ideal

E injective R-module $\Rightarrow$ ${}_A E = 0$

$$0 \to A \to R \to R/A \to 0$$

$$0 \to \underset{\substack{\| \\ 0}}{\mathrm{Hom}_A(R/A, E)} \to \underset{\substack{\| \\ E}}{\mathrm{Hom}_R(R, E)} \to \mathrm{Hom}_R(A, E) \to 0$$

1]

Question: Given any torsion theory, i.e. Serre subcategory $\mathcal{S}$ closed under $\oplus$'s does $\exists$ flat module F such that

$$F \otimes_R M = 0 \iff M \in \mathcal{S}.$$

$\therefore$ we know this is true for $\xi(R/I)$ and more generally symmetric theories.

Put another way. Given an injective E does $\exists$ a flat module F such that

$$F \otimes_R M = 0 \iff \mathrm{Hom}_R(M, E) = 0$$

How to go the other way. Given F, put

$$E = \mathrm{Hom}_{\mathbb{Z}}(F, \mathbb{Q}/\mathbb{Z})$$

Then

$$\underbrace{\mathrm{Hom}_{\mathbb{Z}}(F \otimes_R M, \mathbb{Q}/\mathbb{Z})}_{\text{exact in } M} = \mathrm{Hom}_R\big(M, \underbrace{\mathrm{Hom}_{\mathbb{Z}}(F, \mathbb{Q}/\mathbb{Z})}_{E}\big) \Longrightarrow E \text{ injective}$$

moreover ~~Hom~~

$$F \otimes_R M = 0 \iff \mathrm{Hom}_{\mathbb{Z}}(F \otimes_R M, \mathbb{Q}/\mathbb{Z}) = 0 \iff \mathrm{Hom}_R(M, E) = 0$$

What are the cyclic torsion modules those left ideals $\mathfrak{a}$ such that $F \otimes_R R/\mathfrak{a} = F/F\mathfrak{a} = 0$.

But now it's probably enough to produce F for the torsion theory generated by a cyclic module.

k] Take a left ideal $\mathfrak{a}$ or

You would like a right flat R-module F such that $F/F\mathfrak{a} = 0$

Wait. Fix a left ideal $A < R$.

Consider injectives E such that $\mathrm{Hom}_R(R/A, E) = 0$ and take the corresponding torsion theory.

What's the problem here? ~~There is~~

$$\begin{pmatrix} A & AR \\ A & R \end{pmatrix}$$

$$\begin{array}{ccc} A\text{-cofirm} & \simeq & AR\text{-cofirm} \\ M & \longmapsto & \mathrm{Hom}_R(R, M) = M \\ \mathrm{Hom}_R(A, N) & \longleftarrow & N \\ A^{op}\text{-firm} & & AR^{op}\text{-firm} \\ F & \longmapsto & F \otimes_A R \\ G \otimes_R A & \longleftarrow & G \end{array}$$

So what I seem to be angling at it the idea that a flat R^{op} module F such that $F = FA$ is automatically a flat R^{op} module such that $F = FAR$.

It seems to be true that the torsion theory defined by $F \otimes_R - = 0$ is always symmetric??

If $F \otimes_R R/A = 0$ i.e. $F \cong F \otimes_R A$