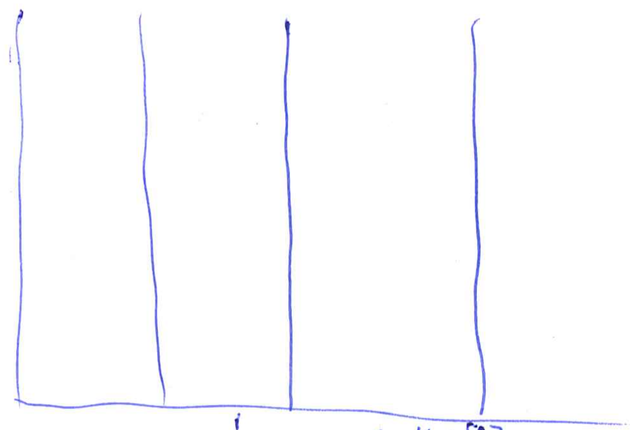
a April 18.

$A \subset R$ left ideal $A^2 = A$.

I want to go over my proof of excision. Let's go back to my old notation. Assume I ideal in R ~~all in everything in the unital context.~~ unital. Form the DG algebra $R \oplus I\varepsilon$ $\varepsilon^2 = 0$ $d(\varepsilon) = 1$.

$$C_\lambda(R/I) \xleftarrow{\text{quis}} C_\lambda(R \oplus \varepsilon I)$$



$C_\lambda(R/I)$ $C_\lambda(R)$ $I \overset{!}{\otimes}_R$ $\Sigma\left(I \overset{!}{\otimes}_R\right)^{[2]}_\sigma$

So the problem is to show that $I \hookrightarrow R$ induces ~~an isomorph~~ quis $I \overset{!}{\otimes}_I \longrightarrow I \overset{!}{\otimes}_R$

more generally $\left[I \overset{!}{\otimes}_I\right]^{(p)} \longrightarrow \left[I \overset{!}{\otimes}_R\right]^{(p)}$ $p \geq 1$.

$\boxed{\text{Idea that}\ \underbrace{}_{\text{assuming } I = I^2}\ I \text{ } I\text{-flat} \Rightarrow I \text{ } R\text{-flat}}$

Easy because for any R-module M we have $I \otimes_I M \xrightarrow{\sim} I \otimes_R M$.

because $x_1 x_2 r \otimes_I m = x_1 \otimes_I x_2 r m = x_1 x_2 \otimes_I r m$.

So does this imply $I \overset{!}{\otimes}_I \longrightarrow I \overset{!}{\otimes}_R$ is a quis. When we calculate $I \overset{!}{\otimes}_I$ or $I \overset{!}{\otimes}_R$ we are using bimodules.

b So we know something about I, as right I, ~~R~~ R module. To calculate $I \overset{!}{\otimes}_I$ you can use any I-bimodule resolution of I which is acyclic for comm. quotient space. functor.

$$\longrightarrow \tilde{I} \otimes I \otimes \tilde{I} \to \tilde{I} \otimes \tilde{I} \longrightarrow \tilde{I} \to 0$$

standard bimod. resolution. tensor with $I \otimes_I -$

$$\xrightarrow{b'} I \otimes I \otimes \tilde{I} \xrightarrow{b'} I \otimes \tilde{I} \xrightarrow{b'} I \to 0$$

One point is that ~~$I \otimes I$~~

$$\mathrm{Tor}_n^{\tilde{I} \otimes \tilde{I}}(\cancel{\tilde{I}}, {}_I M \otimes N_I) = \mathrm{Tor}_n^{\tilde{I}}(\cancel{N}, M).$$

Put another way ${}_I M \otimes N_I$ is acyclic for $- \otimes_I$ if $\mathrm{Tor}_n^{\tilde{I}}(N, M) = 0$ for $n > 0$.

What do we know?

We are assuming $I = I^2$ and I is I-flat.

Then we have ~~the resolution~~

$$\mathrm{Tor}_*^{\tilde{I}}(\mathbb{C}, I) = 0$$

i.e. we have a resolution

$$\cdots \xrightarrow{b'} I \otimes I \otimes I \xrightarrow{b'} I \otimes I \xrightarrow{b'} I \longrightarrow 0$$

If we assume now that I is I flat, then ~~it is R flat~~ the above resolution can be use to compute $I \overset{!}{\otimes}_I$, also for R.

Next point. Wodzicki's converse

Suppose $A \subset R$ left ideal say $\ni A^2 = A$.

Then A flat $/A \iff A$ flat over R.

~~$\Leftarrow$~~ $\Rightarrow$ easy ~~$A \otimes_A M$~~ $M \otimes_A A = M \otimes_R A$

g

$$\begin{tikzcd} N \arrow[r] \arrow[d] & R^s \arrow[dl] \\ A & \end{tikzcd}$$

Let's try to show certain ~~modules are~~ algebras are flat. Let's use linear equations maybe.

A is A flat iff given $a_{ij} x_j = 0$ in A

$\exists$ a'_{jk} and x'_k in A such that $x_j = a'_{jk} x'_k$

and $a_{ij} a'_{jk} = 0$.

$$\begin{tikzcd} \tilde{A}^p \arrow[r, "\cdot a"] \arrow[dr, "0"'] \arrow[rr, bend left, "0"] & \tilde{A}^q \arrow[r, "\cdot a'"] \arrow[d, "\cdot x"] & A^r \arrow[dl, "\cdot x'"] \\ & A & \end{tikzcd}$$

So there exists a triple factorization property which is better than flatness.

$R = \mathcal{L}(H)$ $\quad J$ left ideal in R

claim J flat /R. Proof: Can assume f.g. $J = \sum_{i=1}^{n} R x_i$

First case $n=1$. $\quad Rx \cong R/\mathrm{ann}(x)$

General case

$$R^n \twoheadrightarrow J \subset R$$

h $J \subset R$ ~~to~~ show J flat

$$\begin{tikzcd} R^p \arrow[r] & R^q \arrow[d] \\ & J \end{tikzcd} \qquad \begin{tikzcd} R \arrow[r, "\cdot x"] & R \arrow[r] \arrow[d, "\cdot z"] & {} \\ & J \\ & \cap \\ & R \end{tikzcd}$$

Suppose someone were to give you ~~a linear relations~~ a single linear relation in a C^* alg.

$$a_i x_i = 0.$$

$$\begin{tikzcd} A \arrow[r, "\cdot a_i"] \arrow[dr, bend right, "0"'] & A^n \arrow[d, "\cdot x_i"] \\ & J \end{tikzcd}$$

Ask yourself what you might try to say about a ~~colle~~ family of elements $x_i \in J$.

For example given a bunch of operators on Hilbert space $T_1, \dots, T_n$ we can form $\left(T_i^* T_i\right)^{1/2}$. Any finitely generated left ideal is principal in $\mathcal{L}(H) = R$.

$$\begin{tikzcd} R^n \arrow[r, "\cdot a_i"] \arrow[d, shift left, "y_j"] & R \\ R \arrow[u, shift left, "x_i"] \end{tikzcd}$$

$$\boxed{\begin{array}{l} R = \cancel{\mathrm{Hom}}\,\mathcal{L}(H, H) \\ R^n = \mathcal{L}(H^n, H) \end{array}}$$

Suppose you consider ~~Schwartz~~ Schwartz fns. functions on circle vanishing to ∞ order at ∞.

$$a_{ij} x_j = 0 \qquad x_j = x_j 1$$

$n \qquad \mathrm{Tor}^A_n(k, A) = 0 \qquad n \geq 0.$

A flat as $\tilde{A}$-module ($- \otimes_{\tilde A} A$ exact functor)

$\Rightarrow$ (A is h-unital $\iff$ $A = A^2$)

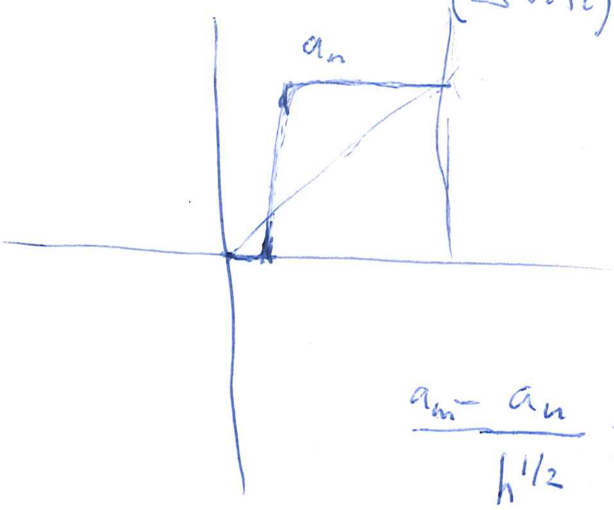
$A = A^2$ + A flat (left or right) over $\tilde A$ $\not\Rightarrow$ A h-unital.

stronger condition $\mathbb{Z} = \tilde{A}/A$ is flat.

A C^* alg.

$f_1, \dots, f_n \in A$

$$f_i = \frac{f_i}{(\sum_i f_i^* f_i)^{1/4}} (\quad)^{1/8} (\quad)^{1/8}$$



$$h^2 = \sum f_i^* f_i$$

$$\frac{a_m - a_n}{h^{1/2}} f_i^* f_i \frac{a_m - a_n}{h^{1/2}} \leq \frac{a_n - a_m}{h^{1/2}} h^2 \frac{a_n - a_m}{h^{1/2}} \to 0$$

$\Rightarrow f_i \frac{a_m}{h^{1/2}}$ Cauchy

Triple factorization: Given $f_1, \dots, f_n$ $\exists$

$f_i = g_i c d$ where $\mathrm{ann}(cd) = \mathrm{ann}(c)$

$\sum_i g_i f_i = 0 \Rightarrow (\sum g_i g_i) cd = 0$

$\Rightarrow \sum g_i g_i c = 0.$

$\therefore f_i = (\underline{g_i c}) d \nearrow$

variation maps.

$$\Lambda \mathfrak{g}_x^* \longrightarrow \Omega$$

$$dX + X^2 = 0 \qquad d\dot{X} + [X, \dot{X}] = 0.$$

$$\Lambda \mathfrak{g}_x^* \longrightarrow \Omega^1_{\Lambda \mathfrak{g}_x^*} = \Lambda \mathfrak{g}_x^* \otimes \mathfrak{g}_{\dot{x}}^*$$

What should the formula be? But even so dually it corresponds to some map $\Lambda \mathfrak{g} \otimes \mathfrak{g} \longrightarrow \Lambda \mathfrak{g}$, and then when we apply invariant theory we should get

$$\boxed{HH(A) \longrightarrow HC(A).}$$

Idea: Think of all the constructions you can make with a Lie algebra and translate them into cyclic theory if you can. Example

$\Lambda \mathfrak{g} \otimes S\mathfrak{g}$ — Lie homology of S of the adjoint representation

$\Lambda \mathfrak{g} \otimes \Lambda \mathfrak{g}$ — ——— Λ ———

$S\mathfrak{g} \otimes \Lambda \mathfrak{g}$ — de Rham complex of $\mathfrak{g}^*$ together with b from the Poisson ~~structure~~ (Lie for the adjoint action?)

$\Lambda \mathfrak{g} \otimes S\mathfrak{g}$ — dual of $W(\mathfrak{g})$ — $\Lambda(\mathfrak{g}[\varepsilon])$. $U(\mathfrak{g}[\varepsilon])$

Is there a de Rham type diffl on $U(\mathfrak{g}) \otimes \Lambda \mathfrak{g}$

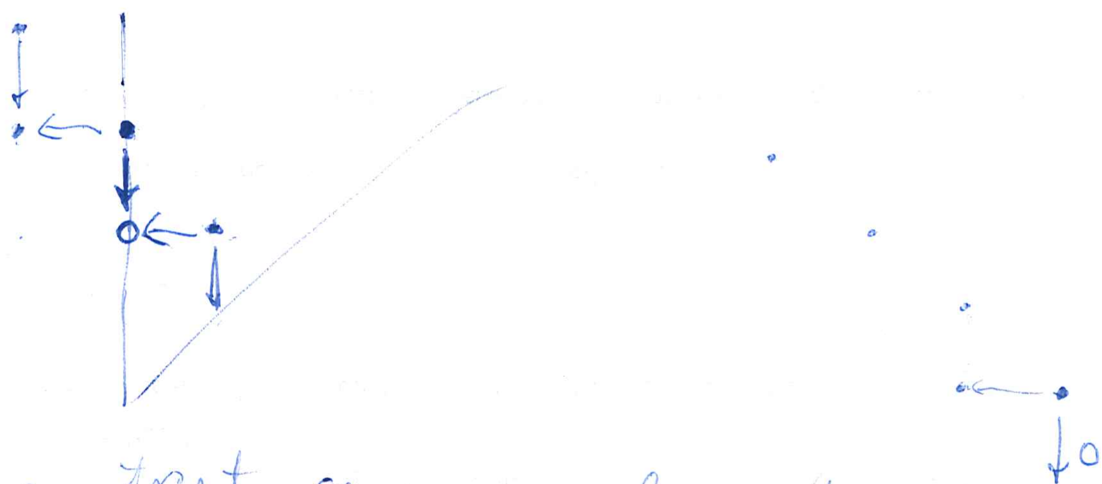
i.e. a derivation $U(\mathfrak{g}) \longrightarrow U(\mathfrak{g}) \otimes \mathfrak{g}$

2 What is going on?

Take a vector space V form $T(V)$ and ask for a differential — Leibniz

Let's consider a mixed complex and try to understand when the cyclic homology is divisible. Want to make each

$$\cdot \longrightarrow HC_n \xrightarrow{S} HC_{n-2} \xrightarrow{B=0} HH_{n-1} \xrightarrow{I} HC_{n-1}$$



As a start can you show that d induces zero on $HC(A)$. In other words that $d: C_\lambda(A) \longrightarrow C_\lambda(A)$ is zero on b-homology.

Is it true for the other complexes

$$C_\lambda(A) \longleftarrow (C, b) \xleftarrow{1-\lambda} (C, b')$$

Look at $A \xleftarrow{b'} A\otimes A \xleftarrow{b'} A\otimes A\otimes A$

is $d = [b', ?]$ $\quad 1 = [b', -d']$

$\cancel{[b', -d]}$ $d = [b', -d'd]$

$d = [b', dd']$

S So what about d'?

$d' \uparrow \downarrow b$

$\textcircled{d'} \overset{\lambda}{\uparrow} \downarrow b$

Does d' induce a map on Hochschild homology. No because you can ~~map~~ project to Ω

The image of d' lies in the degenerate subcomplex so we have the contraction in this case.

[illegible]?

$$\boxed{[b, d] = 1 - K}$$

$$[b, dd'] = (1-K)d'$$

$$[b, \tfrac{1}{1-K} d\, d'] = d'$$

What is K? $\lambda - sc$

Can you find this ~~$c\lambda^{-1}(\lambda - sc) = 1 - c$~~
ought

So the question is this:

I $d : C_\lambda(A) \longrightarrow C_\lambda(A)$ homotopic to zero wrt b? I.e. is there an h of degree $+2$ such that $[b, h] = d$. I checked this is true for (C, b', d) and (C, b, d')

$$[b', d'd] = -d$$

$$[b, \tfrac{1}{1-K} d\, d'] = d'$$