$\langle 5.$ 10/21 ~~Not quite correct~~.

Not quite enough – you're missing: M is a firm $A \otimes A^{op}$ module.

1) $\Rightarrow$ 2) clear. 2) 2') same and mean that M is a unitary module over $\tilde{A} \otimes \tilde{A}^{op}$ firm wrt $A \otimes A^{op}$. We know this is the same as a firm $A \otimes A^{op}$ module which is 3).

2) $\Rightarrow$ 1) Ass. $A \otimes_A M \otimes_A A \xrightarrow{\sim} M$. Then $M = AMA \subseteq AM \subseteq M$ so $AM = M$ and $M = MA$. Then $AM = M \Rightarrow A \otimes_A M$ is firm wrt $A \Rightarrow A \otimes_A M \otimes_A A$ also firm wrt $A \Rightarrow M$ A-firm etc.

Actually ~~If~~ Let $\underset{\text{ideal}}{A \subset B}$ and M be A-firm.

$A \otimes_A M \xrightarrow{\sim} M$ $\quad 0 \to A \to B \to B/A \to 0$

~~so~~ .

Let A be an ideal in B and M a B-mod.

Then $A \otimes_B M \xrightarrow{\sim} M \Rightarrow B \otimes_B M \xrightarrow{\sim} M$

Proof: $A \otimes_B M \xrightarrow{\sim} M \Rightarrow - \otimes_B M$ inv. A^{op}-nil isos.

But $0 \to A \to B \to B/A \to 0$ and $(B/A)A = 0$.

so $A \otimes_B M \xrightarrow{\sim} B \otimes_B M$. ~~Conversely~~

So now where are we?

Conversely $B \otimes_B M \xrightarrow{\sim} M$

$\Rightarrow A \otimes_B B \otimes_B M \xrightarrow{\sim} A \otimes_B M$

Remember $M \otimes_A (A \otimes A)$ and $(A \otimes A) \otimes_A M$

I am confused again. I know that if M is flat firm A-mod, $+ M \to\!\!\!\to A$, and A is h-unital, then $[M \otimes_A]^{(*)}$ yields the good HH & HC.

$<\tau$ At this point I have some understanding of the invariant approach to HH and HC.

Does this simplify ~~the proceeding for~~ my previous proof for Morita invariance?

Now I know that if I take $M(A)$ and choose any firm flat bimod M with surj $M \xrightarrow{f} A$ that $[M\otimes_A]^{(*)}$ has type ind. of the choice of A, M.

Assume A flat on both sides, take $B = P\otimes_A Q$ Mor eq to A, assume B h-unital. Then $P\overset{L}{\otimes}_A A \overset{L}{\otimes}_A Q \xrightarrow{\sim} B$.

$$\begin{array}{c} \| \\ P\overset{L}{\otimes}_A Q \end{array}$$

$Q\overset{L}{\underset{B}{\otimes}} B \overset{L}{\underset{B}{\otimes}} P \to A$ (OK as ${}_BP$, Q_B flat).

I want to take $M = Q\otimes P$, but this is not a flat bimodule over A.

Compare $\quad M\otimes_A(A\otimes A) \longrightarrow M\otimes_A$

$$\begin{array}{ccc} A\otimes Q\otimes P\otimes A & \longrightarrow & Q\otimes P \\ \wr & & \wr \\ [P\otimes A\otimes Q\otimes\,]^{(*)} & & [B\otimes]^{(*)} \end{array}$$

I think one has the same ~~situation~~ namely a surjection $M \longrightarrow Q\otimes P$ with M flat bimod

Then compare $[M\otimes_A]^{(*)}$ with $[(Q\otimes P)\otimes_A]^{(*)} = [B\otimes]^{(*)}$.

~~We need $Q\otimes P$~~

What we know is that ~~when~~ if M is a firm flat bimod over A mapping onto A, then $[M\otimes_A]^{(*)}$ gives the good HH and HC.

I am confused again, because you've forgotten what you ~~need~~ want to prove, namely, ~~that when~~ now that you have an intrinsic defn. of HH, HC for M, you need to see it agrees with $HH(A)$ $HC(A)$ for A h-unital

$< U$ So where are we?

$$\longrightarrow M\otimes_A M \longrightarrow M \longrightarrow A$$

This is a resolution when $M = A\otimes A$.
Can I use it to compute HH

enough to know $M\overset{L}{\otimes}_A M\otimes_A \cdots \xrightarrow{\sim} M\otimes_A M\otimes_A \cdots$
and $A\overset{L}{\otimes}_A M \xrightarrow{\sim} M$

$$M\overset{L}{\otimes}_A = M\otimes_A \quad ?$$

$$M\overset{L}{\otimes}_A = A\overset{L}{\otimes}_A M\overset{L}{\otimes}_A = M\overset{L}{\otimes}_A A\overset{L}{\otimes}_A$$

$$= (A\otimes A)\overset{L}{\otimes}_A A\overset{L}{\otimes}_A$$

So the way we can proceed might be to take $P_A \twoheadrightarrow A$, ${}_AQ \twoheadrightarrow A$ firm flat, then take $Q\otimes P \twoheadrightarrow A\otimes A$

I don't understand what I am trying to do really. You know that $M \twoheadrightarrow A$ with M firm flat A-bimod and A h-unital leads to $\to [M\otimes_A]^{(2)} \to M\otimes_A \simeq A\overset{L}{\otimes}_A$ some canonical isom.

and this gives independence of $[M\otimes_A]^{(*)}$ up to quis from the choice of M. What's missing? I guess it's the assertion that a hom. of h-unital rings $A\to B$ which is a Meq induces $HH_*(A) \xrightarrow{\sim} HH_*(B)$.

$<\phi$ Also maybe I'd like a better quis between $[M\otimes_A]^{(*)}$ and $[A\otimes]^{(*)}$. Suggests.

$$\begin{tikzcd} M\otimes A \arrow[r] \arrow[d] & A\otimes A \\ M & \end{tikzcd}$$

If A is h-unital are things like $A\otimes P$ acyclic for $\overset{L}{\otimes}_A$.

$$\cancel{M \longmapsto M\otimes_A (A\otimes P)\otimes_A = P\otimes_A M}$$

For which M is $M\overset{L}{\otimes}_A = M\otimes_A$? OK!

take $Q\otimes P$.

$$(Q\otimes P)\overset{L}{\otimes}_A = (Q\otimes P)\otimes_{-\tau} B\otimes_{\tau} = P\otimes_{-\tau} B\otimes_{\tau} Q$$
$$= P\overset{L}{\otimes}_A Q .$$

So there is nothing to this. ~~It's too hard~~ [???]

Let's try a little on homology of dialgebras. In the above stuff you replace $M \twoheadrightarrow A$ by $M\otimes A \longrightarrow A$, then you get a cyclic object $[(M\otimes_A A)\otimes_A]^{(*)} = [M\otimes]^{(*)}$ It's as if M is being considered as a ring. In fact we are using the coord system $M\otimes A \longrightarrow A$ such the ring is $B = P\otimes_A Q = M\otimes_A A = M$. This depends only on M as left A-module.

So what do I want to do?

I would like to understand dialg homology.

<X

Guess suppose we have $M \twoheadrightarrow A$ a dialg, really an A-bimodule M mapping ~~onto~~ A.

In any case what happens. Leave ~~this~~ until later. Go back to your review of more for HH. ~~YES!~~ The argument based on knowing that if $u: A \to B$ ind. Meq. then it induces qis $[A\otimes]^{(*)} \to [B\otimes]^{(*)}$, hence $HC(A) \xrightarrow{\sim} HC(B)$. Then I wanted to reduce to this case. ~~So I start~~ Problem: If A B are h-unital Meq then $A * B$ not nec. h-unital. This is OK if one is flat on either side. So the arg. proceeds by choosing $A' \twoheadrightarrow A$ $B' \twoheadrightarrow B$ Meq with A', B' both l+r flat

Then

$$\begin{tikzcd} HC(A') \arrow[r, "\cong"] \arrow[d, "\cong"'] \arrow[dr] & HC(A' * B') \arrow[d] & HC(B') \arrow[l, "\cong"'] \arrow[d, "\cong"] \\ HC(A) & HC(A' * B) & HC(B) \arrow[l] \end{tikzcd}$$

Diagram of Meq h-unital rings

$$\begin{tikzcd} A' \arrow[r] \arrow[d] & A' * B' \arrow[d] \arrow[dr] & & B' \arrow[ll, Leftarrow] \arrow[d] \\ A \arrow[r] & A * B' & A' * B & B \arrow[l] \end{tikzcd}$$

$$\begin{tikzcd} A' \arrow[r] \arrow[d] \arrow[dr] & A' * B' \arrow[d] & B' \arrow[l] \arrow[d] \\ A \arrow[r] & A' * B & B \arrow[l] \end{tikzcd}$$

<u>That certainly was a good idea</u>

Now that I have understood the intrinsic approach to HC based on a choice of $M \twoheadrightarrow A$ I want to aim for a K version.

$\langle\psi$ Can I find an intrinsic K_1 construction? What ideas might be available? Hanlon?

Your idea is to define some sort of K_1 group starting from $M \twoheadrightarrow A$, then show independent of M, then evaluate in the case $Q\otimes P$. I think I need to understand things a little better, maybe on the Lie alg level. So what am I planning to do??

To get cyclic homology you will form $[M\overset{L}{\otimes}_A]^{(*)}_\lambda$ probably. ~~This suggests that~~

Recall excision proof.

Wodzicki $\qquad 0 \longrightarrow A \longrightarrow B \longrightarrow B/A \longrightarrow 0$

$$C^\lambda(R) \qquad F_pB = \begin{cases} 0 & p<0 \\ A & p=0 \\ B & p\geq 1 \end{cases}$$

$$gr\, B = A \oplus B/A$$

$$gr\, C_\lambda(B) = C_\lambda(A\oplus B/A)$$

$$= C_\lambda(A) \oplus \big((B/A)\otimes B \otimes\big)$$

$$\oplus \quad \big((B/A)\otimes B \otimes (B/A)\otimes B \otimes\big)_\lambda$$

$$\sim C_\lambda(A) \oplus B/A \oplus (B/A)^{\otimes 2}_\lambda \oplus \cdots \qquad B\sim k$$

$$C_\lambda(B)/C_\lambda(A) \longrightarrow C_\lambda(B/A) \qquad \text{quis.}$$

My "BRST" version goes as follows

DGA $\qquad B \oplus A[1]$

$$C_\lambda(B\oplus A[1]) = C_\lambda(B) \oplus A[1]\otimes B \oplus [A[1]\otimes B\otimes]^{(2)}_\lambda \oplus \cdots$$

$$\Sigma(A\overset{L}{\otimes}_B) \oplus \Sigma^2[A\overset{L}{\otimes}_{B-B}]^{(2)} \oplus$$

< w So what have I got?

I'm ending up with a complex made of things like $[M \overset{L}{\otimes}_A]_\lambda^{(n+1)}$. Is there a simple answer? If $M = Q \otimes P$, then $[M \otimes_A]_\lambda^{(n)} = B_\lambda^{\otimes n}$, linked to $gl(B)$. ~~Start with~~ !!!

At the moment maybe I should look at $Q \times P$. B operates on the inside, A on the outside. What happens is I want to consider

$(q, p) \in Q^{k,l} \times P^{l,k}$ $\qquad pq \in B^{ll} \quad qp \in A^{kk}$

~~My idea would be to form.~~

Interpretation of (p, q) as factorization

$$B^l \xrightarrow{q} Q^k \xrightarrow{p} B^l \qquad B^{op} \text{ maps}$$

$$P^l \xrightarrow{q} A^k \xrightarrow{p} P^l \qquad A^{op} \text{ maps.}$$

Roughly I take an "compact" operator on a free B^{op}-module and factor it through (free A^{op} module) $\otimes_A Q$. The question is whether I can work in these terms. Formulate things more carefully. ~~Can we see things more simply?~~ I want to get away from sets if possible. Somehow I'm trying to compare "small" autos.

Suppose $(p, q) = (p, aq')$, $(pa, q') = (p', q')$ YES!!

$$qp = a(q'p) \qquad q'p' = (q'p)a$$

So we find that if (p, q) (p', q') are linked as above, then qp and $q'p'$ are linked as aa' and $a'a$. so that $1 - qp$ $1 - q'p'$ conj. mod $E(A)$.