R

$$\rho a_0 \left(\rho(a_1 a_2) - \rho a_1 \rho a_2 \right)$$

$$- \frac{1}{2}\left(\rho(a_0 a_1 a_2) - \rho(a_0 a_1) \rho a_2 \right)$$

$$= \rho(a_1 a_2)\left(\rho(a_0) + \frac{1}{2}\rho(a_2) \right)$$

$$- \frac{1}{2}\rho(a_1 a_2 a_3) - \rho a_0 \rho a_1 \rho a_2$$

$$\rho a_0 \left(\rho(a_1 a_2) - \rho a_1 \rho a_2 \right)$$

$$- \frac{1}{2}\rho(a_0 a_1 a_2) + \frac{1}{2}\rho(a_0 a_1)\rho a_2$$

$$= \rho a_0 \, \rho(a_1 a_2) + \frac{1}{2}\rho(a_0 a_1)\rho a_2$$

$$- \frac{1}{2}\rho(a_0 a_1 a_2) - \rho a_0 \rho a_1 \rho a_2$$

Back to V. Jones, review. Start with

$B \underset{\text{subalg}}{\hookrightarrow} A \xrightarrow{\rho} B$, ρ a B-bimodule map

Assume $\exists\, x_i, y_i \in A$ $\quad i = 1, \ldots, n \ni$

$$\rho(a x_i) y_i = a$$
$$\forall a$$
$$x_i \rho(y_i a) = a$$

depends upon $x_i \otimes y_i \in A \otimes_B A$.

suffices that $a(x_i \otimes y_i) = (x_i \otimes y_i) a$ and that $\rho(x_i) y_i = 1 = x_i \rho(y_i)$. Since

$$\rho(a x_i) y_i = \rho(x_i) y_i a = a.$$

D] Converse also true because

$$A \otimes_B A \xrightarrow{\sim} \mathrm{Hom}_{B^{op}}(A, A)$$

$$(a_1, a_2) \longmapsto \big(\alpha \mapsto (a_1, a_2) * \alpha = a_1 \rho(a_2 \alpha)\big)$$

$$(f(x_i), y_i) \longleftarrow\!\!| \; f$$

$$\longmapsto \big(\alpha \mapsto f(x_i)\rho(y_i \alpha)\big)$$

$$f(x_i)\rho(y_i\alpha) = f(x_i \rho(y_i \alpha)) = f(\alpha)$$

$$(a_1, a_2) \longmapsto \big(\alpha \mapsto a_1 \rho(a_2 \alpha)\big)$$

$$\longleftarrow\!\!| \quad (a_1 \rho(a_2 x_i), y_i) = (a_1, \rho(a_2 x_i) y_i)$$

$$(a x_i, y_i) * \alpha = a x_i \rho(y_i \alpha) = a\alpha$$

$$(x_i, y_i a) * \alpha = x_i \rho(y_i a \alpha) = a\alpha$$

lwg6 stlawrence.maths:quillen Job: tish1 Date: Sat Nov 7 16:49:56 1992

lwg6 stlawrence.maths:quillen Job: tish1 Date: Sat Nov 7 16:49:56 1992
lwg6 stlawrence.maths:quillen Job: tish1 Date: Sat Nov 7 16:49:56 1992
lwg6 stlawrence.maths:quillen Job: tish1 Date: Sat Nov 7 16:49:56 1992

lwg6 / LaserWriter II NT

Sat Nov 7 16:49:56 1992

tish1

stlawrence.maths:quillen

E So given $B \underset{\text{subalg}}{\subset} A \xrightarrow[\substack{B\text{-bimod}\\ \text{maps}}]{\rho} B$

$$\left(\begin{array}{ll} (x_i, y_i) \in A \otimes_B A & \text{central} \\ x_i \rho(y_i) = \rho(x_i) y_i = 1. & \end{array}\right.$$

Next set $R = A \otimes_B A$

$$(a_1, a_2)(a_3, a_4) = (a_1, \rho(a_2 a_3), a_4)$$

$$A \longrightarrow A \otimes_B A \longrightarrow A$$

$$a \longmapsto (ax_i, y_i) = (x_i, y_i a)$$

$$(ax_i, y_i)(a_1, a_2) = (ax_i \rho(y_i a_1), a_2) = (aa_1, a_2)$$

need central element $(x_i, 1, y_i) \in R \otimes_A R$

What I need are examples. So how far to go? B field. Then you classified things.

$$A \otimes_B A \longrightarrow$$

Separable alg. $A \otimes A$

Suppose $B = \mathbb{C}$ and A is separable. Then

$$\begin{tikzcd}
0 \arrow[r] & (\Omega^1 A)^\natural \arrow[r] \arrow[d] & (A \otimes A)^\natural \arrow[r] \arrow[d] & A^\natural \arrow[r] \arrow[d, "\cong"] & 0 \\
0 \arrow[r] & {[A, A]} \arrow[r] & A \arrow[r] & A_\natural \arrow[r] & 0
\end{tikzcd}$$

$$a_1 \otimes a_2 \mapsto a_2 a_1$$

So a central element (x_i, y_i) determined by $y_i x_i$

$(x_i, y_i) \in A \otimes_B A$

$\boxed{F}$ Let $(x_i, y_i) \in A \otimes A$ be the canonical ~~standard elem~~ central element $\ni$ $y_i x_i = 1$.

Then $(x_i a, y_i)$, $(x_i, a y_i)$ central with inv. a $\therefore$ equal.

$$(x_i w, y_i) = (x_i, w y_i)$$

need a ρ $\ni$ $\rho(x_i w) y_i = 1$

$w x_i \rho(y_i) = 1.$

$w x_i \rho(y_i a) = a \implies w x_i$ basis $\implies w$ invertible.

Anyway $\rho(a) = tr(w^{-1} a)$

$$\rho(x_i w) y_i = tr(w^{-1} x_i w) y_i = tr(x_i) y_i = 1.$$

$$x_i w \rho(y_i) = x_i w \, tr(w^{-1} y_i)$$

~~Kadison requires a separating element~~ ~~$x_i \otimes y_i$~~ ~~central~~

~~and so forth~~ an invertible scalar

Kadison notation: central element $f = \tau x_i \otimes y_i \ni$, and $E = \rho$ such that $\tau x_i y_i = 1$

He wants a $B \subset A \xrightarrow{\rho} B$ such that $\boxed{\rho(1) = 1.}$ and central element $(x_i, y_i) \in A \otimes_B A$ $\ni$

$$\rho(x_i) y_i = x_i \rho(y_i) = 1.$$

and finally $x_i y_i = \tau^{-1}$

It involves the extra conditions that $\rho(1) = 1$ and $x_i y_i =$ invertible scalar $\implies A$ sep. extn of B.

G Feb 21

What is puzzling is the scalar τ

~~the role~~

$$(x_i \otimes y_i) \in A \otimes_B A \quad \text{central}$$

$$\rho(x_i) y_i = x_i \rho(y_i) = 1.$$

~~where~~ these are my conditions, the conditions I work with, then Kadison adds two further ones: $\rho(1) = 1$ and $x_i y_i = \tau^{-1}$ where τ is an invertible scalar.

~~Braid group~~

From these conditions he gets idempotents

~~From these conditions he gets zilch.~~

Back to ~~the~~ GNS. Sets $\rho = 1$.

$$\begin{array}{c} A \\ \iota \uparrow \downarrow \rho \\ B \end{array} \qquad A \oplus A \otimes B \otimes A \longrightarrow \mathrm{Hom}_{B^\circ}(A, A)$$

$$(a_1 \otimes b \otimes a_2)(\alpha) = a_1 i\, b\, \rho(a_2 \alpha)$$
$$= a_1 b \rho(a_2 \alpha)$$
$$=$$

so descends via $A \otimes_B B \otimes A \longrightarrow A \otimes_B A$

$$a_1 \otimes b \otimes a_2 \longmapsto a_1 b \otimes a_2$$

~~So what~~ So there is a canonical idempotent $(1,1)$ in $A \otimes_B A$ $\qquad (1,1) * a = \rho(a)$

~~So the~~ The ρ on $A \otimes_B A$ he uses is not the multiplication μ, since

$$\mu(x_i, y_i) = x_i y_i = \tau^{-1}$$

H] I have a construction ~~starting~~ from

$$B \underset{\text{subalg}}{\subset} A \xrightarrow[\text{bimod}]{\rho} B$$

such that $\exists$ A-central element $(x_i, y_i) \in A \otimes_B A$

$\ni \quad \rho(x_i)y_i = x_i\rho(y_i) = 1.$

This gives ~~an alg structure on $R = A \otimes_B A$~~ similar structure

$$A \underset{\text{subalg}}{\hookrightarrow} A \otimes_B A \xrightarrow[\text{bimod}]{\mu} A$$

allowing one to iterate.

Kadison imposes extra conditions namely

$\rho(1) = 1 \qquad x_i y_i = \tau^{-1} \qquad \tau$ inv. in k

Suppose B, A sep algebras over k. Then

$$\begin{array}{ccc} A \otimes^B A & \xrightarrow{\sim} & A \otimes_B A \\ \| & & \\ \{ a_i' \otimes a_i'' \mid a_i' b \otimes a_i'' = a_i' \otimes b a_i'' \} & & \end{array}$$

What sort of games to play?

Can I classify central elements of $A \otimes_B A$

Yes.

$$\begin{array}{ccc} (A \otimes^B A)^A & \xrightarrow{\sim} & (A \otimes_B A)^A \\ \cap & \nearrow & \\ (A \otimes A)^A & & \end{array}$$

and $(A \otimes^B A)^A = ((A \otimes A)^A)^B \simeq A^B$

so the point is that if $(x_i, y_i) \in A \otimes A$ is the unique central element $\ni$ $y_i x_i = 1$, then

I] one has $A \overset{B}{\otimes} A \longrightarrow A^B$

$$(A \otimes_B A)^{A} \xrightarrow{\sim} (A \otimes_B A)_{A} = A/[B,A]$$

$$\simeq A^B$$

so if A, B are both separable the central elements of $A \otimes_B A$ are described 1-1 by $\omega \in A^B = \{a \,/\, [a,B] = 0\}$ via $(x_i \omega, \otimes y_i) \in A \otimes_B A$, where $\sum x_i \otimes y_i \in (A \otimes A)^A$ is the canonical central elt.

Let's go back to HPT and Laplacean methods. Consider then $[d,h] = 1-e$ where $h^2 = eh = he = 0$, and $\theta \ni [d,\theta] = \theta^2$.

$$[d-\theta, h] = 1-e-[\theta,h] \quad \cancel{= (1-\theta h)(1-e)(1-h\theta)}$$

The basic calculation is

$$[d-\theta, h\frac{1}{1-\theta h}] = 1 - \underbrace{\frac{1}{1-h\theta} e \frac{1}{1-\theta h}}_{\tilde{e}} \qquad \tilde{e}^2 = 0.$$

Can I understand this by "Laplacean" methods, whatever this means? The point of view to take might be to break things into blocks relative to e. Thus we have

$$d = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \qquad h = \begin{pmatrix} 0 & 0 \\ 0 & h \end{pmatrix} \qquad e = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\theta = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} \qquad [d,\theta] = \left[\begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}, \begin{pmatrix} & \\ & \end{pmatrix} \right]$$

I Start with the idea of a good module 1630

Quotient category:

$$\mathrm{Hom}_{S^{-1}\mathcal{A}}(X, Y) = \varinjlim_{\{(X', Y') \mid X' \subset X,\ Y' \subset Y,\ X/X' \in S,\ Y' \in S\}} \mathrm{Hom}_{\mathcal{A}}(X', Y/Y')$$

In the case of a general torsion theory

$$\begin{array}{ccc} X' & & \\ \cap & \searrow & Y \\ X & & \downarrow \\ & & Y/Y' \end{array}$$

you can ~~define~~ construct the 'good' functor?

~~Concept~~ Idea is maybe this,

So why not list the main ideas. I think I really want to look at excision for the next week before seeing Joachim.

I think the idea is that

What Joachim has apparently done is to use our X-complex picture of ~~cyclic~~ periodic cyclic homology to prove excision for periodic cyclic cohomology. So there should be a nice way to proceed. ~~Also~~ Let's work entirely noncountably or at least try to.

4] Where to start? Maybe with some version of excision, I mean, some proof of ~~the~~ Wodzicki's excision theorem.

Another idea is the Grothendieck idea of formulating things for a map.

The point is to consider homomorphisms, say surjective to begin with.

$$0 \longrightarrow J \longrightarrow A \xrightarrow{f} B \longrightarrow 0$$

Say f nice when excision holds, i.e. inclusion

$$\hat{\Omega}_{nu} J \longrightarrow \operatorname{Ker}\{\hat{\Omega}_{nu} A \twoheadrightarrow \hat{\Omega}_{nu} B\}$$

is a quis. Here $\Omega_{nu} A$ means the nonunital DG algebra generated by A equipped with b, B.

So how to proceed?

So basically we have a functor

$$A \longmapsto \hat{\Omega}_{nu} A. \qquad \text{Call this } F(A)$$

from algebras to supercomplexes, and then we can ask about maps.

Consider now a composition

$$A \twoheadrightarrow B \twoheadrightarrow C$$

$$\begin{tikzcd}
0 \arrow[r] & I \arrow[r] \arrow[d, equal] & J \arrow[r] \arrow[d] & K \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & I \arrow[r] & A \arrow[r, two heads] \arrow[d, two heads] & B \arrow[r] \arrow[d] & 0 \\
 & & C \arrow[r, equal] & C &
\end{tikzcd}$$

V] What's the plan? Have two maps

$$F(A) \xrightarrow{f} F(B) \xrightarrow{g} F(C)$$

which leads to a triangle

~~Fibre~~

$$\begin{tikzcd} \mathrm{Fib}(f) \arrow[r] & \mathrm{Fib}(gf) \arrow[dl] \\ \mathrm{Fib}(g) \arrow[u, dashed] & \end{tikzcd}$$

More precisely

$$\begin{tikzcd} \mathrm{Fib}(F(A) \to F(B)) \arrow[r] & \mathrm{Fib}(F(A) \to F(C)) \arrow[dl] \\ \mathrm{Fib}(F(B) \to F(C)) \arrow[u, dashed] & \end{tikzcd}$$

We also have ~~can~~ candidates for these, namely

$$\begin{tikzcd} F(I) \arrow[r] & F(J) \arrow[dl] \\ F(K) \arrow[u] & \end{tikzcd}$$

have Δ.

Now the key step if I recall it is to prove that I, J are good, which then gives quis

$$F(I) \xrightarrow{\sim} \mathrm{Fib}(F(A) \to F(B))$$
$$F(J) \xrightarrow{\sim} \mathrm{Fib}(F(A) \to F(C))$$
$$F(I) \xrightarrow{\sim} \mathrm{Fib}(F(J) \to F(K))$$

WJ What ~~is~~ are the main steps? . . . . .
What are the assumptions??

May 23. Alice is 32. ~~So what?~~

Anyway ~~so~~ I need to understand excision in periodic cyclic(co)homology. How is this going to work? Main idea. You have

~~$0 \to B \to A \to B$~~

$$0 \to J \to A \to B \to 0$$

you choose

$$0 \to I \to R \to A \to 0$$

~~and~~ where R is quasi-free, then one has

$$0 \to I \to K \to J \to 0$$

$$0 \to K \to R \to B \to 0$$

Best is to look at

$$R \xrightarrow{f} A \xrightarrow{g} B$$

$$\begin{array}{ccccccccc} 0 & \to & \mathrm{Ker}(f) & \to & \mathrm{Ker}(gf) & \to & \mathrm{Ker}(g) & \to & 0 \\ & & \| & & \| & & \| & & \\ & & I & & K & & J & & \end{array}$$

~~So logically there is a result~~ So we need

$$F(I) \xrightarrow{\text{quis}} \mathrm{Fib}\big(F(R) \to F(A)\big)$$

$$F(K) \xrightarrow{\text{quis}} \mathrm{Fib}\big(F(R) \to F(B)\big)$$

$$F(I) \xrightarrow{\text{quis}} \mathrm{Fib}\big(F(K) \to F(J)\big)$$

X| Put another way, what we have is ortogonal?

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r, dash] & I \arrow[r] \arrow[d, equal] & K \arrow[r] \arrow[d] & J \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & I \arrow[r] & R \arrow[r] \arrow[d] & A \arrow[r] \arrow[d] & 0 \\
 & & B \arrow[r, equal] \arrow[d] & B \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

Apparently there are only 4 triangles here and we want to prove that if three of them are distinguished, then so is the 4th.

In any case the key appears to use the fact that $I, K \subset R$ quasi-free $\Longrightarrow$ ~~they are~~ I, K have nice properties.

Now the next point must be the model to use for periodic cyclic homology. I ~~also~~ would like to know what happens

~~So let's examine~~

Everything we do depends on C_λ. The reason I have difficulty is that I don't ~~under~~ know everything about this. ~~If I take~~ Consider

$$C_\lambda(R \oplus I\varepsilon) = C_\lambda(R) \oplus \left(\varepsilon I \overset{!}{\otimes}_R\right) \oplus \left[\varepsilon I \overset{!}{\otimes}_R\right]^{(2)}_{\lambda} \oplus \cdots$$

~~Then I would like a~~ Now there should be an S operator on this complex. And there

Y] should also be another degree -2 operator which somehow ~~replaces~~ carries

$$\left[I \overset{!}{\otimes}_R\right]_\sigma^{(p)} \longrightarrow \left[I \overset{!}{\otimes}_R\right]_\sigma^{(p-1)} \quad ?$$

So what in fact is the key point? Somehow you want to ~~simplify to able~~ bring in the I-adic filtration.

What do I know? You have

$$\underbrace{\mathrm{Fibre}\left(X(R) \longrightarrow X(R/I^n)\right)}_{\text{wrong?}}$$

This is the old viewpoint that I have difficulty implementing. ~~whe~~

It comes down to the old problem of the S operator on $G_\lambda(R \oplus \varepsilon I)$.

Go on and finish the good module category!

New approach: go over results valid when $A \subset R$ unital, A (left) ideal $\ni A^2 = A$.

~~R/A is R flat $\Leftrightarrow \forall a \; \exists a' \ni$~~

R/A is R projective $\Longleftrightarrow$ $A = Re$ some idem e

$\Longleftrightarrow \exists e \in A \ni ae = a \quad \forall a.$

R/A is R flat $\Longleftrightarrow \forall a \; \exists a' \ni aa' = a.$

$\boxed{7}$ Suppose $P \twoheadrightarrow A$ ~~[illegible]~~

Let's go back to Morita equivalence.

~~[illegible]~~ Given $A = A^2$. I can choose

$$\begin{pmatrix} A & Q \\ P & P\otimes_A Q \end{pmatrix} \qquad Q \otimes P \twoheadrightarrow A.$$

~~choose P flat A^op~~ Choose $P \twoheadrightarrow A$ map of A^{op} modules with P flat + A-good. Similarly $Q \twoheadrightarrow A$ on left. Then have $Q\otimes P \twoheadrightarrow A \otimes A \twoheadrightarrow A$. Then we get a M. equiv. with $B = P \otimes_A Q$ $\quad (B = B \otimes_B B)$.

$$\begin{array}{ccc} A\text{-gmod} & \underset{Q\otimes_B -}{\overset{P\otimes_A -}{\rightleftarrows}} & B\text{-gmod} \\ \Psi & & \\ Q & \longleftrightarrow & B \end{array}$$

$$\begin{array}{ccc} \text{gmod-}A & \underset{-\otimes_B P}{\overset{-\otimes_A Q}{\rightleftarrows}} & \text{gmod-}B \\ \Psi & & \\ P & & B \end{array}$$

The conclusion is then that B is both left and right flat over itself. Now B should be local if A is.

In fact what happens if I start with the maximal ideal is a nondiscrete valuation ring.