So what I would like to do is to see if I can build the complex up as a Postnikov system.

$$\begin{tikzcd}
 & \arrow[r] & 0 \arrow[r] & B_0 \arrow[r] \arrow[d] & U_{-1} \arrow[r] & U_{-2} \arrow[r] & \cdots \\
 & & 0 \arrow[r] & U_0 \arrow[r] \arrow[d] & U_{-1} \arrow[r] & U_{-2} \arrow[r] & {} \\
\arrow[r] & U_2 \arrow[r] & U_1 \arrow[r] & U_0/Z_0 \arrow[r] & 0 \arrow[r] & 0 \arrow[r] & 0 \arrow[r] & {}
\end{tikzcd}$$

What would you really like to do?

You would like to understand to what extent one can ~~...~~ play finite band games. One way to proceed is to place yourself in a situation of canonical resolutions. For example consider R comm. ~~and~~ with a Koszul type bimodule resolution

$$R \otimes \Lambda V \otimes R \longrightarrow R$$

Then any module has a canonical free resolution

$$R \otimes \Lambda V \otimes M \longrightarrow M$$

Apply this to your complex U.

$$R \otimes \Lambda V \otimes U \xrightarrow{\text{quis}} U$$

So what? You get a kind of Cartan Eilenberg resolution.

I think I am asking the wrong questions.?

What questions are good?

Is a quis of complexes of projectives a heq? Equiv. is an ~~com~~ acyclic complex of proj. contractible? Yes, because

$$0 \longrightarrow \underline{Z_n} \longrightarrow U_n \longrightarrow U_{n-1} \longrightarrow \cdots \longrightarrow U_m \longrightarrow B_{m-1} \to 0$$

$\therefore Z_n$ proj, $\quad 0 \longrightarrow Z_n \longrightarrow U_n \longrightarrow Z_{n-1} \to 0$ splits.

$$\begin{pmatrix} 1-e_0 & -e_1 & -e_2 & -e_3 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$= I - \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} (e_0 \ e_1 \ e_2 \cdots)$$

4 ~~Cart~~ Cartan - Eilenberg resol.

$$0 \longrightarrow B_nU \longrightarrow Z_nU \longrightarrow H_nU \longrightarrow 0$$

~~Start~~ Choose proj resolutions of H_nU B_nU for all n. Then fit them together to get a proj res. of Z_nU. Then use

$$0 \longrightarrow Z_nU \longrightarrow U_n \longrightarrow B_{n-1}U \longrightarrow 0$$

to ~~fit~~ combine the resol. of Z_nU $B_{n-1}U$ to a res of U_n

Get double complex with an

$$\begin{array}{ccccc}
\longleftarrow & V_{p1} & \longleftarrow & V_{p+1,1} & \longleftarrow \\
 & \downarrow & & \downarrow & \\
\longleftarrow & V_{p0} & \longleftarrow & V_{p+1,0} & \longleftarrow \\
 & \downarrow & & \downarrow & \\
\longleftarrow & U_p & \longleftarrow & U_{p+1} & \longleftarrow
\end{array}$$

$$\begin{pmatrix} 1-e_0 & -e_1 & -e_2 & -e_3 \\ -d & e_0 & e_1 & e_2 \\ & d & 1-e_0 & -e_1 \\ & & -d & e_0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ d & 1-e_0 & -e_1 & -e_2 \\ & -d & e_0 & e_1 \\ & & d & 1-e_0 \end{pmatrix}$$

5

I want to understand ~~everything~~ whether there's ~~an obstruction to a~~ a finiteness obstruction in the case of supercomplexes.

Consider then $U \underset{i}{\overset{j}{\rightleftarrows}} T \qquad 1 - ji = [d, h]$

$$e_n = i h^{n+1} j$$

$$[d, e_{n+1}] = i \sum_{k=0}^{n} (-1)^k h^k (1 - ji) h^{n-k} j$$

$$= - \sum_{k=0}^{n} (-1)^k e_k e_{n-k} + \overbrace{\sum_{k=0}^{n} (-1)^k}^{\begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}} e_n$$

$$[d, e_0] = 0$$

$$[d, e_1] = e_0 - e_0^2$$

$$[d, e_2] = -e_0 e_1 + e_1 e_0$$

$$[d, e_3] = -e_0 e_2 + e_1^2 - e_2 e_0 + e_2$$

But then you try to construct

$$T \oplus T[1] \oplus T[2] \oplus \cdots$$

differential

$$\left[\begin{pmatrix} d & 1-e_0 & -e_1 & -e_2 \\ & -d & e_0 & e_1 \\ & & d & 1-e_0 \\ & & & -d \end{pmatrix}, \begin{pmatrix} 0 & & & \\ 1 & 0 & & \\ & 1 & 0 & \\ & & 1 & 0 \end{pmatrix} \right]$$

$$[d, h] = 1 - ij$$

6 I would like to go back to ?

What do you want?

There's a circle of ideas Andrew has developed which ~~should~~ might be very important.

One idea: homotopy analogs of natural questions in analyses, ~~the~~ I mean analysis going into K-theory.

homotopy idempotents (projections)

homotopy nilpotent operators.

I think I was beginning to think about these in connection with HPT.

Where do I start?

You try for a counterexample.

We have

$$T \oplus T[1] \oplus T[2] \oplus \cdots$$

Yes

$$\begin{tikzcd}
{} \arrow[d] & & \\
T_1 \arrow[d, "d"'] & {} \arrow[l] \arrow[d] & \\
T_0 \arrow[d, "d"'] & T_0 \arrow[l, "1-e_0"] \arrow[d, "-d"'] & {} \arrow[l, "e_0"] \\
T_{-1} \arrow[d] & T_{-1} \arrow[l, "1-e_0"] \arrow[d] & {} \arrow[l, "e_0"] \\
{} & {} &
\end{tikzcd}$$

vertically you have periodicity

horizontally also some periodicity

Actually a divisible S-module.

There are various questions to be taken up. Various stages of Andrew's theory.

It might be wothwhile to have the infinite ~~cyclic~~ cyclic covering idea.

I would like some sort of counterexample

Idea: Look for an "infinite" supercomplex which has a nuclear deformation of 1, but is not heq a finite proj supercomplex. Possible example might arise from an inf. cyclic covering. Specifically ~~take~~ ~~$R[z,z^{-1}]$~~ something over $R[z,z^{-1}]$

Suppose start with inf. supercx.

The game will be to find an "infinite" supercomplex having a nuclear dif. of 1

~~Anyway what~~

Review what I know about refining a homotopy idempotent to an A^∞ idemp.

$$T: \quad e_0 - e_0^2 = [d, e_1]$$

$$\left[\begin{pmatrix} d & 1-e_0 & -e_1 \\ & -d & e_0 \\ & & d \end{pmatrix}, \begin{pmatrix} & & \\ 1 & & \\ & 1 & \end{pmatrix} \right]$$

$$= \begin{pmatrix} 1-e_0 & -e_1 & 0 \\ -d & e_0 & 0 \\ & d & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ d & 1-e_0 & -e_1 \\ & -d & e_0 \end{pmatrix} = \begin{pmatrix} 1-e_0 & -e_1 & 0 \\ & 1 & -e_1 \\ & & e_0 \end{pmatrix}$$

8

$$= I - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} e_0 & e_1 & e_2 \end{pmatrix} - \begin{pmatrix} -e_2 \\ +e_1 \\ 1-e_0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

The logic as I remember is that

$$\left[\begin{pmatrix} d & 1-e_0 \\ & -d \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right] = \begin{pmatrix} 1-e_0 & 0 \\ 0 & 1-e_0 \end{pmatrix}$$

$$= I - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} e_0 & e_1 \end{pmatrix} - \begin{pmatrix} -e_1 \\ e_0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} d & 1-e_0 \\ & -d \end{pmatrix} \begin{pmatrix} -e_1 \\ e_0 \end{pmatrix} = \begin{pmatrix} -de_1 + (1-e_0)e_0 \\ -de_0 \end{pmatrix} \overset{?}{=} \begin{pmatrix} +e_1 d \\ -e_0 d \end{pmatrix} = \begin{pmatrix} -e_1 \\ e_0 \end{pmatrix} (-d)$$

Then you want to find e_2 such that

$$\begin{pmatrix} e_0 & e_1 & e_2 \end{pmatrix} \begin{pmatrix} d & 1-e_0 & -e_1 \\ & -d & e_0 \\ & & d \end{pmatrix} = \begin{pmatrix} e_0 d & e_0(1-e_0) - e_1 d & -e_0 e_1 + e_1 e_0 + e_2 d \end{pmatrix}$$

$$\overset{?}{=} \begin{pmatrix} de_0 & de_1 & de_2 \end{pmatrix}$$

need to solve $[d, e_2] = -e_0 e_1 + e_1 e_0$

The answer is that you have to modify e_1 before you can solve this.

$$[d, [e_0, e_1]] = [e_0, e_0 - e_0^2] = 0.$$

e_1 itself satisfies $[d, e_1] = e_0 - e_0^2$