3)

$$\widetilde{HP}_*(A) = H\left(\mathrm{Hom}\left(X(I\mathbb{C}^\infty), X(TA/IA^\infty)\right)\right)$$

$\forall k \; \exists l \ni \quad \varphi_k : X(I\mathbb{C}^l) \to X(TA/IA^k)$

compatible

$$0 \to K^\infty \to T \to T/K^\infty \to 0$$

big problem: to show this induces an exact sequence of X-complexes. This is ~~other~~ easy for maps $I\mathbb{C}^\infty$ into.

tool: variables in high power of ideal then spread them out.

$$X(I\mathbb{C}^\infty) = I\mathbb{C}^\infty \oplus \Omega^1(I\mathbb{C}^\infty)_\natural$$

$$\begin{array}{cccc} & x_n & y_n & \\ d1d1, & 1d1d1, (d1)^3, & 1(d1)^3, & \dots \end{array}$$

so maps $I\mathbb{C}^\infty \to \hat{T}A$

$x_n \mapsto a_n$

get sequences/null sequences in $\hat{T}A$

Note: $X(I\mathbb{C}^\infty)$ is inverse system with inj. maps

$X(\hat{T}A)$ —————— surj maps.

4)

$$0 \to J \to A \to B \to 0$$

$$0 \to \hat{T}(A,J) \to \hat{T}A \to \hat{T}B \to 0$$

for simplicity leave out X

$$\hat{T}(A,J) \underset{\substack{\text{easy?} \\ \text{Goodwillie}}}{\sim} \hat{T}(A,J^\infty) \underset{\substack{\text{spreading out} \\ \text{variables}}}{\sim} \hat{T}J^\infty$$

$\hat{T}(A,J^\infty) = T(A,J^\infty)/I(A,J^\infty)^\infty$

(actually for maps from $I\mathbb{C}^\infty$ into this)

$\forall k, k' \ \exists \mathcal{C}$

$$\begin{tikzcd} I\mathbb{C}^l \arrow[r] & T(A,J^k)/I(A,J^k)^{k'} \\ I\mathbb{C}^{l'} \arrow[u, "\rho"] \arrow[d] & T(A,J^r)/I(A,J^r)^{k'} \\ T(J)/\ldots \arrow[ur] & \end{tikzcd}$$

$r > k$

If K ideal in R q-free, $\exists$ conn.

$$\psi : \Omega^1(K^\infty) \to \Omega^2(K^\infty)$$

$$\psi(\omega x) - \psi(\omega)x = \omega dx$$

again $\forall l \ \exists k$ and maps

$$\psi_k : \Omega^1(K^k) \to \Omega^2(K^l)$$

no compatibility for different k.

construction uses a Wodzicki type argument

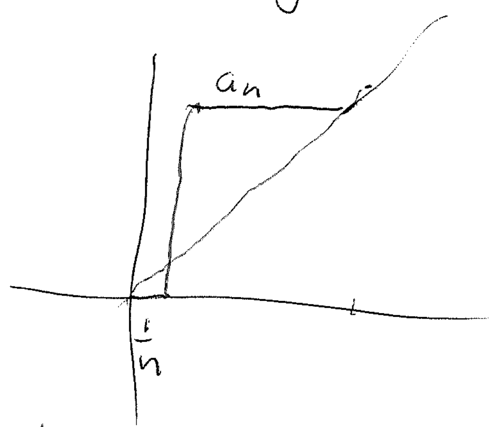
Proof get from R

$$\psi : \Omega^1(R, K^\infty) \to \Omega^2(R, K^\infty)$$

now have to modify so as to get $\Omega^1(K^k) \to \Omega^2(K^l)$

5) $|a_i|^2 = a_i^* a_i$

Given $0 \leq g^2 \leq h^2$ to show $g h^{1/2}$ $\exists$



$$\frac{a_n}{h^{1/2}} g^2 \frac{a_n}{h^{1/2}} \leq \frac{a_n}{h^{1/2}} h^2 \frac{a_n}{h^{1/2}} \longrightarrow h$$

$g \frac{a_n - a_m}{\sqrt{h}}$ Cauchy because

$$\frac{a_n - a_m}{\sqrt{h}} g g \frac{a_n - a_m}{\sqrt{h}} \leq \frac{a_n - a_m}{\sqrt{h}} h^2 \frac{a_n - a_m}{\sqrt{h}} \longrightarrow 0$$

$h^2 = \sum a_i^* a_i$ p431

$g^2 = a_i^* a_i$

6 98. hotel Central | dinner 35 on train | 7.20 bus Bonn | 1.20 milk | 1.00 phone Joachim | 2.00 phone home | 1.00 phone to Hotel Central

50 dinner | 2 × 5. lunchs | 110. Eurocheque Haus Sedlmeyer | 8.00 Euro check charge at | 5.00 phone call home 14.00 grapes | 8.60 lunch McDon

A Hilbert space in a C^*-alg $\mathcal{A}$ with identity I is by definition a closed subspace $H \subset \mathcal{A}$ s.t. $\psi, \psi' \in H \Rightarrow \psi^* \psi' \in \mathbb{C} I$.

The support of H is the projection I_H spanned by the range projections of the $\psi \in H$ (probably all $\psi \psi^*$ where $|\psi| = 1$ and $\psi \in H$).

$\mathcal{O}_H$ is the unique C^*-algebra containing H as a Hilbert space with support I.

$\exists$ canonical identification between *autos of $\mathcal{O}_H$ ~~mapping~~ H onto itself and $U(H)$.

Suppose G compact $\subset U(H)$, $\dim H < \infty$. Then G acts on $\mathcal{O}_H$. Let $\mathcal{O}_G$ be $(\mathcal{O}_H)^G$ fixpt. subalg.

$\mathcal{O}_H$ has a canonical "inner" endomorphism σ defined by $\psi B = \sigma(B)\psi$ for $\psi \in H$, $B \in \mathcal{O}_H$

Cuntz description: $\mathcal{O}(H)$ has canonical "inner" endo $\varphi \ni$ $\varphi(x) T = T x$ for $T \in H$, $x \in \mathcal{O}(H)$

Formula $\varphi(x) = \sum S_k x S_k^*$, S_k orth basis for H.

$M = M_n(\mathbb{C}) = \mathcal{L}(H)$ is embedded in $\mathcal{O}(H)$ as HH^*

have $\boxed{\mathcal{O}(H) = M \otimes \varphi(\mathcal{O}(H))}$

$\forall \; U \in \mathcal{O}(H)^{\text{unitary}}$ get endos

$$\lambda_U : \mathcal{O}(H) \xrightarrow{\sim} \mathcal{O}(UH) \subset \mathcal{O}(H)$$

$$\rho_U : \mathcal{O}(H) \xrightarrow{\sim} \mathcal{O}(HU) \subset \mathcal{O}(H)$$

char by $\quad \lambda_U(T) = UT \qquad T \in H$

$$\rho_U(T) = TU \qquad T \in H.$$

Also M^∞ = union of $M^{\otimes k} = M \otimes \cdots \otimes M \simeq M\varphi(M) \cdots \varphi^k(M)$

is a canonical UHF subalg of $\mathcal{O}(H)$

8) So spend some time on excision.

Excision in HP

I can't understand anything really without the spreading variables argument. This is what Joachim refers to as the Wodzicki part of the argument. It is somehow the basic step. The setting seems to be an arbitrary extension

$$0 \to J \to A \to B \to 0$$

where J is nice, i.e. approximately unital

$$0 \to \hat{T}(A,J) \to \hat{T}A \to \hat{T}B \to 0$$

For simplicity leave out X. $\hat{T}A = \{T(A)/I(A)^n\}$ computes $HP(A)$. Then

$$\hat{T}(A,J) \underset{\text{Goodwillie}}{\sim} \hat{T}(A,J^\infty) \underset{\substack{\text{spreading out}\\ \text{variables}}}{\sim} \hat{T}J^\infty$$

$$\hat{T}(A,J^\infty) = T(A,J^\infty)/I(A,J^\infty)^\infty$$

9) 18.54 IC 512 arrive Bonn 29.07

So what happens at this point.

Simplest case: J right flat and $J = J^2$.

This is the excision situation. ~~[illegible]~~ The important thing to understand must be Wodzicki excision

Go back to $J \subset R$

$$J \otimes_R M \xrightarrow{\sim} M$$

What might be the ideal situation?

$$0 \to J \to R \to R/J \to 0$$

$$0 \to n\mathbb{Z} \to \mathbb{Z} \to \mathbb{Z}/n\mathbb{Z} \to 0$$

$$0 \to (n\text{-torsion}) \to \mathbb{Z}\text{-mod} \to\!\!\to \mathbb{Z}[\tfrac{1}{n}]\text{-mod}$$

~~[illegible]~~ In the ideal situation one has a K-theory exact ~~[illegible]~~ sequence

$$K'_*(\mathbb{Z}/n\mathbb{Z}) \to K_*(\mathbb{Z}) \to K_*(\mathbb{Z}[\tfrac{1}{n}])$$

So what happens. ~~$K_*(\mathbb{Z}) \to K_*(\mathbb{Z}[\tfrac{1}{n}])$~~

(I-good) $\subset$ R-mod

What kind of ~~[illegible]~~ adjoint functors.

$$\mathbb{Z}\text{-mod} \underset{\text{right adjoint}}{\overset{\text{loc.}}{\rightleftarrows}} \mathbb{Z}[\tfrac{1}{n}]\text{-mod}$$

$$\mathrm{Hom}_{S^{-1}R}(S^{-1}M, N) = \mathrm{Hom}_R(M, N)$$

10)

$$
\begin{tikzcd}[column sep=huge]
\mathbb{Z}\text{-unmod} \arrow[r, shift left=2, "\mathbb{Z}[\frac{1}{n}] \otimes_{\mathbb{Z}} -"] & \mathbb{Z}[\frac{1}{n}]\text{-mod} \arrow[l] \\
\end{tikzcd}
$$

$$\xrightarrow[\quad]{} \qquad \mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}[\tfrac{1}{n}], -)$$

$$
\begin{tikzcd}[column sep=huge]
R\text{-unmod} \arrow[r, shift left=2] & R\text{-}g_I\text{-mod} \arrow[l, shift left=2] \arrow[l, shift left=6, "I^{\partial} \otimes_R -"']
\end{tikzcd}
$$

$$\mathrm{Hom}_R(N, M) \xleftrightarrow{\sim} \mathrm{Hom}_{Good}(N, I^{\partial} \otimes_R M)$$

So I learn from this example that the arrows are funny.

~~A H-unital $\Rightarrow$ $HH(A,A) = 0$~~

$R/I \qquad R \leftarrow I$

$$[I \overset{!}{\otimes}_R]^{(3)}_{\sigma}$$

$$[I \overset{!}{\otimes}_R]^{(2)}_{\sigma}$$

$C^{\lambda}(R/I) \quad | \quad C^{\lambda}(R) \qquad I \overset{!}{\otimes}_R$

So what we get when I flat ~~as a bi~~ as right module say is

$$I \overset{!}{\otimes}_R I \sim I^2 \qquad \underbrace{I \overset{!}{\otimes}_R \cdots \overset{!}{\otimes}_R I}_{p \text{ factors}} \sim I^p$$

So what I find is contributions

$$H_*\left(I^p \overset{!}{\otimes}_R\right)_{\sigma}$$

If you in addition assume $I = I^2$, then probably get $I \overset{!}{\otimes}_R = I \overset{!}{\otimes}_I$

11) June 2

What is the K-theory of $\mathcal{O}_n$?

Extension

$$\longrightarrow \mathcal{J}_E \longrightarrow \mathcal{O}_E \longrightarrow 0$$

So where are we going? Is there any direction to follow. It would be nice to be able to handle excision in periodic cyclic homology. What is the key then? You need a way to relate $HP(I)$ to $HP(R)$ and $HP(R/I)$. In any case you have some sort of relative theory, namely the fibre of $HP(R) \longrightarrow HP(R/I)$

---

12) exact sequence of A-bimods exact since $A = AeA$

$$0 \longrightarrow K \longrightarrow Ae \otimes_B eA \xrightarrow{\mu} A \longrightarrow 0$$

Claim $K \cdot A = 0$: $k = \sum_i a_i' e \otimes_B e a_i'' \xmapsto{\mu} \sum_i a_i' e e a_i'' = 0$

To show $kA = 0$ enough $k a_1 e a_2 = 0$

$$\Big(\sum_i a_i' e \otimes_B \underbrace{e a_i''\Big) a_1 e}_{\in B} e a_2 = \sum_i \underbrace{a_i' e e a_i''}_{=0} a_1 e \otimes_B e a_2$$

$$K \otimes_A M \longrightarrow Ae \otimes_B eA \otimes_A M \xrightarrow{\sim} A \otimes_A M \longrightarrow 0$$

$A \otimes_A M \xrightarrow{\cong} M$

Assume M good

$K \otimes_A M = K \otimes_A AM = 0$

12) So what do I do next.

R quasi free, I ideal

Suppose we understand good modules for I. Does this help with periodic cyclic homology? Can you get a long exact sequence in some reasonable way?

Roughly the good I-modules might be linked to some ~~kind~~ of Cuntz-Krieger algebra which has a long exact sequence associated. Proceed vaguely: $\mathcal{O}_{R,I}$ makes I invertible in some way.

Good I-modules = $\mathcal{O}_{R,I}$-modules

~~Good I~~

Somehow I have to separate the bimodule situation A, E from the ideal situation R, I where $I \subset R$. Is there some intuition from alg. geometry? Cartier divisor. Blowing up.

Consider R R/I

Relate I-good modules to R-modules and R/I^∞ modules?

Examples $n\mathbb{Z} \subset \mathbb{Z}$

$\mathbb{Z}[\frac{1}{n}]$-^{u}mod $\mathbb{Z}$-umods

13) You don't know what to expect. Anyway, try to find some way to ~~think~~. What happens in the case of $n\mathbb{Z}$? You have

$$\begin{tikzcd}(n\text{-torsion}) \arrow[r] & (\mathbb{Z}\text{-mod}) \arrow[r, two heads] & (\mathbb{Z}[\tfrac{1}{n}]\text{-mod})\end{tikzcd}$$

and you have ~~two functors~~ two adjoint to base change relative to $\mathbb{Z} \to \mathbb{Z}[\frac{1}{n}]$

$$\begin{tikzcd}(\mathbb{Z}\text{-mod}) \arrow[r, shift left=2, "\mathbb{Z}[\frac{1}{n}]\otimes_{\mathbb{Z}} -"] \arrow[r, shift right=2, "\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}[\frac{1}{n}],\, -)"'] & (\mathbb{Z}[\tfrac{1}{n}]\text{-mod}) \arrow[l, hook]\end{tikzcd}$$

From the viewpt. of HP, how should I be thinking of all this? Good modules might be quasi-free irrelevant

I^∞ $\qquad R \qquad$ R/I^∞

Try to accomplish something before falling asleep: ~~I need to consider~~

I would like to see if there is any relation between Joachim's excision and good module. Test situation R quasi-free. Then any ideal I is projective as right R-module. Recall proof.

$$0 \longrightarrow \underset{\text{proj } R\text{-bimodule}}{\Omega^1 R} \longrightarrow R\otimes R \longrightarrow R \longrightarrow 0$$

$$0 \longrightarrow \Omega^1 R \otimes_R M \longrightarrow R\otimes M \longrightarrow M \longrightarrow 0$$

14) shows any R-module M has proj. dim ≤ 1. Then $0 \to I \to R \to R/I \to 0$ and R/I has proj dim $1 \implies I$ projective as R module.

Then I know that good ~~R~~ solid modules = I-good R-modules: $I \otimes_R M \xrightarrow{\sim} M$ form an abelian category. (more generally for I R^{op}-flat).

~~Is there something else I can do?~~

Now I would like to get ~~[illegible]~~ a good understanding

What's the ~~[illegible]~~ link between

$I \otimes_R M \xrightarrow{\sim} M$ closed under ~~[illegible]~~ arb. $\varinjlim$s

and $N \xrightarrow{\sim} \mathrm{Hom}_R(I, N)$ closed under arb. $\varprojlim$s

$$\mathrm{Hom}_R(I \otimes_R M, N) = \mathrm{Hom}_R(M, \mathrm{Hom}_R(I, N))$$

Wait

$$\begin{tikzcd} N \arrow[r] & \mathrm{Hom}_R(I, N) \arrow[r, shift left] \arrow[r, shift right] \arrow[dr] \arrow[dr, shift left] & \mathrm{Hom}_R(I, \mathrm{Hom}_R(I, N)) \arrow[d, equal] \\ & & \mathrm{Hom}_R(I \otimes_R I, N) \end{tikzcd}$$

$$R \leftarrow I \leftarrow I \otimes_R I \leftarrow I \otimes_R I \otimes_R I \leftarrow$$

So ~~[illegible]~~ assume $R \hookleftarrow I \hookleftarrow I^2 \hookleftarrow I^3 \hookleftarrow I^4 \hookleftarrow \cdots$

Then what about $\varinjlim \mathrm{Hom}(I^n, N)$

Is this the localization functor? This would be reasonable if I flat maybe

15) ~~Would~~ What happens with usual torsion theories. Suppose for example I take the inverse system of ideals I^n. Then I get maybe the Serre subcategory of I torsion modules. Check: Ask that $\forall m \in M \quad \exists n \ni I^n m = 0$. But this doesn't work with extensions.

Would not be closed under $\varinjlim$'s.

expenses to/for Heidelberg
132
98
118
50 meals
10
35
4 phone
437
-350
87

But these are all side issues and the main point should be whether excision holds, i.e. whether it would help to understand excision.

So what is the overall hoped for picture? You have $I \subset R \longrightarrow R/I$, and you would like some sort of excision result.

need some solt of excision result. How might this go???

At the moment it's not clear that ~~excision~~ good modules will be of any use in excision.

e.g. take $R = \bigoplus_{n \geq 0} V^{\otimes n}$ where V has dim r and $I = \bigoplus_{n \geq 1} V^{\otimes n}$. Then good module for I are the samex as $\mathcal{O}_r$ modules and the K-theory for this is something like $\mathbb{Z}/(r-1)\mathbb{Z}$

So it doesn't seem to be worthwhile to pursue good modules from the viewpoint of excision.

Other reasons are

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