AEROACOUSTIC ANALYSIS USING A HYBRID FINITE ELEMENT METHOD

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ABSTRACT

This paper describes a novel 3D analysis of tone noise radiated from turbofan inlets. This analysis combines a standard finite element discretisation of the acoustic field in the axial and radial coordinates with a Fourier spectral representation in the circumferential direction. The boundary conditions at the farfield, fan duct and acoustic liners are treated using the same spectral representation. The resulting equations are solved iteratively using an axisymmetric approximation of the inlet geometry to form an effective preconditioner. The analysis method is far less expensive than a conventional 3D finite element method because relatively few circumferential Fourier modes need to be retained for an accurate field representation.

NOMENCLATURE

$x,\ r,\ \theta$	cylindrical coordinates
$\xi,\ \eta$	coordinates of the canonical finite element
$\rho,\ c$	gas density and speed of sound
$\rho_\infty,\ c_\infty$	freestream gas density and speed of sound
γ	specific heat ratio of the gas
ϕ	(steady mean) flow potential function
$\hat{\phi}$	linear (acoustic) potential function
$\beta,\ \hat{\beta}$	boundary term of steady (and acoustic) weak formulation
$\omega,\ \kappa$	frequency and circumferential mode number of noise source
Z	impedance value of the acoustic liner model
$m,\ M$	circumferential mode number and its maximal value in the spectral representation $(-M \leq m \leq +M)$
N_n	finite element shape function
$\mathbf{\Phi}_0,\ \mathbf{\Phi}$	vector of axisymmetric (and spectral) nodal steady potential values
$\hat{\mathbf{\Phi}}_0,\ \hat{\mathbf{\Phi}}$	vector of axisymmetric (and spectral) nodal linear potential values
$\mathbf{r}_0,\ \mathbf{r}$	vector of axisymmetric (and spectral) nodal steady residual values
$\mathbf{K}_0,\ \mathbf{K}$	Jacobian of axisymmetric (and spectral) discrete mean flow problem
$\hat{\mathbf{L}}_0,\ \hat{\mathbf{L}}$	volume operator of axisymmetric (and spectral) discrete acoustic linear system
$\hat{\mathbf{F}}_0,\ \hat{\mathbf{F}}$	boundary operator of axisymmetric (and spectral) discrete acoustic linear system

INTRODUCTION

The noise generated by the high-bypass turbofan engines in widespread use in modern civil aircraft is an increasing concern for airports and airlines. During takeoff and landing, an important component is the tone noise from the fan, generated by the rotation of shocks attached to fan blades operating at tip speeds exceeding sonic velocity ("buzz-saw" noise) and/or by rotor-stator interaction (blade passing frequency noise). The ability to model the generation and propagation of tone noise as well as the mechanisms through which it can be attenuated (*e.g.* acoustic liners) is therefore very important and has received considerable attention from research.

The majority of methods dedicated to high-bypass engine acoustics follow either the time-domain or the frequency-domain approach. Time-domain methods [Özyörük and Long, 1996, Stanescu et. al., 1999] have become attractive with low cost computing power and are able to model directly multi-frequency sources and nonlinear interactions but generally have a drawback in the treatment of the frequency-dependent lining material. Frequency-domain methods [McAlpine and Fisher, 2002, Özyörük et. al., 2004] are much faster than time-domain methods and treat acoustic liners in a natural way and will continue to be developed and used for the foreseeable future.

Another important aspect of aeroacoustic analysis is the fluid model employed to treat unsteadiness and this can range from the velocity potential equation [Astley, 1983, Eversman et. al., 1985] to the full Navier–Stokes equations [Rangwalla and Rai, 1993, Rumsey et. al., 1998]. Whereas modelling nonlinear wave interactions using the Euler or Navier–Stokes equations is particularly important for understanding the generation of tone noise, the potential model is perfectly adequate for the propagation of tone noise in the nearfield at the subsonic conditions during takeoff and landing [Rumsey et. al., 1998].

This paper introduces a novel 3D method of analysis of the tone noise radiated from asymmetric turbofan inlets based on the frequency-domain potential model. The novelty of the method is to combine a standard finite element (FE) discretisation of the acoustic field in the axial and radial direction with a Fourier spectral representation in the circumferential coordinate. The use of pseudospectral methods for the solution of partial differential equations is not new and the applications are already very diverse [Fornberg, 1996]. The Fourier spectral representation is such a method [Trefethen, 2000] and shares the advantages of the class; the most important for the inlet radiation application is the ability to deal easily with discontinuous coefficients, *e.g.* lining with circumferentially discontinuous properties. This property explains the benefit brought by the method introduced in this paper; because relatively few Fourier modes need be retained for an accurate field representation in the circumferential coordinate, the method involves many fewer discrete unknowns than a conventional 3D FE analysis. Even in the case of a circumferential discountinuity, although the number of circumferential Fourier modes in the acoustic solution is greater than in the smooth variation case, it is still much smaller than the number of nodes required by the conventional 3D FE model to represent the solution in the circumferential direction, therefore the cost of the spectral solution cost is again substantially lower.

AEROACOUSTIC MODEL

It is common practice to consider the low Mach number external airflow during aircraft landing and take-off to be uniform, steady and parallel to the engine axis. Since this implies that the flow is irrotational and homentropic, the velocity field is represented as the gradient of a potential function ϕ. The values of density ρ and speed of sound c are obtained from the non-dimensionalisation

$$(\rho / \rho_\infty)^{\gamma-1} = (c / c_\infty)^2 = 1 - (\gamma-1)\,(q - q_\infty) / c_\infty^2,$$

where $q=\frac{1}{2}|\nabla\phi|^2$ and $\rho_\infty, c_\infty, q_\infty$ are the freestream values.

A computational domain V typical for inlet aeroacoustic applications is depicted in Fig. 1 and its boundaries ∂V are the fan face, the inlet wall (possibly incorporating acoustic lining) and the farfield boundary. Conservation of mass within the domain V gives the nonlinear potential equation

$$\nabla \cdot (\rho\,\nabla\phi) = 0,$$

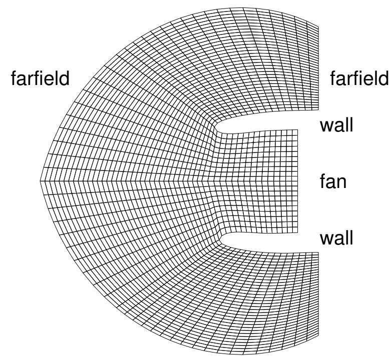


Figure 1: Axial section through a 3D computational domain and mesh around an asymmetric engine inlet geometry showing the boundaries at the fan face, inlet wall and farfield. (The mesh shown is much coarser than a mesh required by a realistic aeroacoustic calculation.)

with Dirichlet boundary conditions on the farfield boundary and Neumann boundary conditions $\partial\phi/\partial n = \beta$ on the inlet solid surface ($\beta = 0$) and fan face boundary (β prescribes the massflow through the engine). The weak formulation of the steady equations is obtained by integrating by parts the p.d.e. and using the boundary conditions:

$$\int_V \rho\, \nabla\phi \cdot \nabla w \,\mathrm{d}V - \int_{\partial V} \beta\, w\, \mathrm{d}S = 0, \qquad \forall w \in H_0^1(V). \tag{1}$$

Tone noise is represented as a harmonic perturbation of frequency ω and small amplitude $\hat{\phi}$ superposed on the steady mean flow field ϕ. Linearising the mass conservation equation, the weak formulation of the acoustic problem is given by:

$$\int_V \rho\, \nabla\hat{\phi} \cdot \nabla w - \frac{\rho}{c^2}\left(\nabla\phi\cdot\nabla\hat{\phi} + \mathrm{i}\,\omega\,\hat{\phi}\right)\left(\nabla\phi\cdot\nabla w - \mathrm{i}\,\omega\, w\right)\,\mathrm{d}V - \int_{\partial V} \hat{\beta}\, w\, \mathrm{d}S = 0, \tag{2}$$

where the harmonic component of the mass flux along the normal $\mathbf{n}$ is the boundary term

$$\hat{\beta} = \rho\left(1 - \frac{1}{c^2}\left(\nabla\phi\cdot\nabla\hat{\phi} + \mathrm{i}\,\omega\hat{\phi}\right)\right)\nabla\hat{\phi}\cdot\mathbf{n}.$$

The boundary conditions of engine inlet aeroacoustics are all represented by $\hat{\beta}$. First, uniform local flow within a cylindrical duct of constant area is assumed at the fan face and, using the radial eigenmodes of duct acoustics, the field $\hat{\phi}$ is decomposed into a sum of incident and radiated modes [Parrett and Eversman, 1986]. The modal amplitudes of the incident modes are specified from the fan CFD data and those of the radiated are unknown and are part of the sought solution. $\hat{\beta}$ is in that case a function of the incident mode prescribed to model the presence of the downstream fan as the source of noise. Then, ray theory is used at the farfield to determine the angle at which the acoustic waves cross the boundary and establish an expression for $\hat{\beta}$ which minimises the reflection of acoustic waves back into the computational domain. Finally, $\hat{\beta} = 0$ at the solid inlet surface but, if acoustic liners are present, additional modelling relates the unsteady normal velocity to the acoustic pressure through a linear boundary equation involving the non-dimensional complex valued liner impedance Z and yields a modified boundary integral [Eversman, 2001]:

$$\int_{\partial V_{\text{liner}}} \hat{\beta}\, w\, \mathrm{d}S = \int_{\partial V_{\text{liner}}} \frac{\mathrm{i}}{\omega Z}\,\frac{\rho}{c}\left(\nabla\phi\cdot\nabla\hat{\phi} + \mathrm{i}\,\omega\,\hat{\phi}\right)\left(\nabla\phi\cdot\nabla w - \mathrm{i}\,\omega\, w\right)\,\mathrm{d}S.$$

DISCRETISATION

To introduce the notations and concepts for the 3D discretisation, a discussion on the simpler axisymmetric case is necessary. If the inlet is axisymmetric, the acoustic problem is 2D in the axial x and radial r coordinates. Following a standard FE approach, the potential field is interpolated within each element using the unknown nodal values ϕ_n and shape functions N_n defined on the canonical element of coordinates $(\xi,\ \eta)$. In an iso-parametric approximation, the coordinates are interpolated in the same manner as the field values and the FE approximation is represented by the nodal summations:

$$\begin{aligned} x(\xi,\eta) &= \sum_n x_n\, N_n(\xi,\eta), \\ r(\xi,\eta) &= \sum_n r_n\, N_n(\xi,\eta), \\ \phi(\xi,\eta) &= \sum_n \phi_n\, N_n(\xi,\eta). \end{aligned}$$

Using this, the weak problem (1) is discretised to the system of steady flow equations

$$\mathbf{r}_0\,(\mathbf{\Phi}_0)\ =\ 0,$$

in which $\mathbf{\Phi}_0$ is the vector of axisymmetric steady potential unknowns defined at the nodes of the computational mesh and $\mathbf{r}_0$ is the corresponding vector of discrete residuals. $\mathbf{r}_0$ is nonlinear in $\mathbf{\Phi}_0$ and the system is solved using the Newton method with the iterate at step k obtained from

$$\mathbf{K}_0\Delta\mathbf{\Phi}_0^{(k)}\ =\ -\,\mathbf{r}_0(\mathbf{\Phi}_0^{(k)}),$$

where $\mathbf{K}_0$ is the Jacobian $\partial\mathbf{r}_0/\partial\mathbf{\Phi}_0$. This system of equations is solved using a direct sparse solution.

The general form of the acoustic potential is $\hat{\phi}(x,r,\theta)\ \exp(\mathrm{i}\omega t)$, where the circumferential mode number κ is an input to the problem, defined by the condition at the fan boundary. Then, using the FE approximation

$$\hat{\phi}(\xi,\eta,\theta) = \sum_n \hat{\phi}_n\, N_n(\xi,\eta)\ \exp\,(\mathrm{i}\kappa\theta),$$

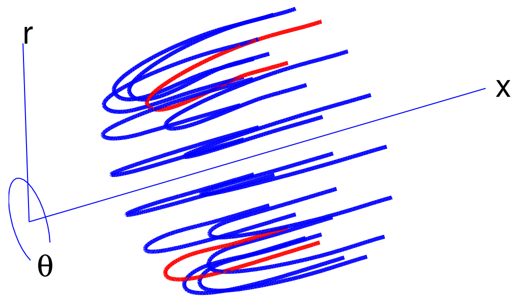
for the acoustic field, the weak problem (2) yields the linear system of equations

$$(\hat{\mathbf{L}}_0 + \hat{\mathbf{F}}_0)\ \hat{\mathbf{\Phi}}_0 = \hat{\mathbf{f}}_0,$$

having the vector of nodal linear potential axisymmetric values $\hat{\mathbf{\Phi}}_0$ as unknown. The operator $\hat{\mathbf{L}}_0$ comes from the discretisation of the volume integral and has the form $\hat{\mathbf{L}}_0 = -\omega^2\mathbf{M}_0 + \mathrm{i}\omega\mathbf{C}_0 + \mathbf{K}_0 + \kappa^2\mathbf{K}_\theta$, where the first three terms are standard in any discretisation of Helmholtz type problems and the fourth is a contribution arising from the circumferential field variation. ($\mathbf{K}_0$ is the same matrix as the Newton update Jacobian.) $\hat{\mathbf{F}}_0$ is an operator which represents the non-reflective conditions at both the fan face and the farfield boundaries and the forcing term $\hat{\mathbf{f}}_0$ is a function of the incident duct mode from the modal boundary condition on the fan face. The discrete axisymmetric acoustic problem is also solved using a direct sparse solver.

Having discussed the axisymmetric case, the key to the new 3D solution method is a spectral Fourier representation of the geometry and field in the circumferential direction. Assume the starting point for the analysis is a 3D Computer Aided Design representation of the studied engine inlet which is “sampled” at a number θ stations such that a series of axial cuts are obtained as in Fig. 2(a). The axial and radial coordinates of the highlight in each section is discrete Fourier transformed in the θ direction in order to obtain the circumferential mean highlight coordinates as well as the variation about that mean as described by M significant circumferential modes of non-axisymmetry. The same Fourier transform is also applied to any series of corresponding nodes on the different axial cuts. If the axial cuts are equally spaced in θ, the discrete Fourier transform can be performed using the FFT algorithm [Trefethen, 2000, FFTW, 2003]. As a result of these operations, the 3D inlet is represented by a series of nodes and their circumferential modal coordinates.

The next step is to set a computational 3D mesh for the given inlet. First, a 2D mesh is built around the circumferential mean of the inlet geometry, Fig. 2(b); the axial and radial coordinates of this mesh, x_0 and r_0, define the circumferential mean values of the 3D mesh coordinates. The 3D mesh is completed by interpolating the modes of asymmetry of the inlet geometry on the 2D axisymmetric



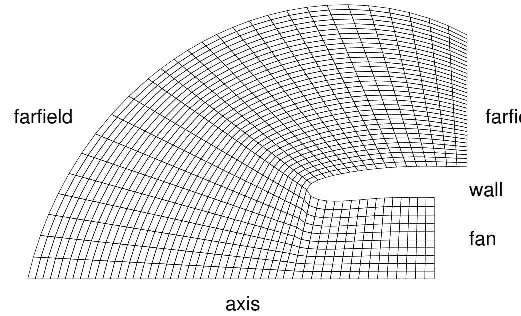


Figure 2: Asymmetric inlet geometry described by a series of axial sections at a number of equally spaced θ stations (a). A very coarse quadrilateral mesh fitted around the inlet axisymmetric mode (b).

mean mesh, from the pre-computed values on the inlet boundary to zero on the rest of the boundary. The result is a series of modes of nodal coordinate asymmetry, denoted x_m and r_m. Lastly, using these along with the axisymmetric mean x_0 and r_0, the coordinates of the 3D mesh within a cell are represented as:

$$\begin{aligned} x(\xi,\eta,\theta) &= \sum_m \sum_n x_{mn}\, N_n(\xi,\eta)\ \exp{(\mathrm{i}m\theta)}; \\ r(\xi,\eta,\theta) &= \sum_m \sum_n r_{mn}\, N_n(\xi,\eta)\ \exp{(\mathrm{i}m\theta)}. \end{aligned} \tag{3}$$

The same shape functions N_n as in the axisymmetric case are used to interpolate in the $(\xi,\,\eta)$ coordinates but the summations are over nodes as well as over circumferential modes $-M \le m \le +M$. This hybrid representation combines standard FE interpolation in the axial and radial coordinates with a Fourier spectral approximation in the circumferential.

Similar to the coordinate representation, the mean flow potential and acoustic potential are interpolated within each element as

$$\begin{aligned} \phi(\xi,\eta,\theta) &= \sum_m \sum_n \phi_{mn}\, N_n(\xi,\eta)\ \exp{(\mathrm{i}m\theta)}; \\ \hat{\phi}(\xi,\eta,\theta) &= \sum_m \sum_n \hat{\phi}_{mn}\, N_n(\xi,\eta)\ \exp{(\mathrm{i}m\theta + \mathrm{i}\kappa\theta)}. \end{aligned} \tag{4}$$

The difference is that whereas the coordinate modes x_{mn} and r_{mn} are input to the analysis, the potential modes ϕ_{mn} and $\hat{\phi}_{mn}$ are the unknowns. Notice that because the spectrum of the acoustic solution is centered around κ rather than 0, the axisymmetric acoustic mode is characterised by the non-zero circumferential mode number $m{+}\kappa{=}\kappa$.

Using the coordinate and potential field spectral representations (3) and (4), the weak formulation of the mean flow problem (1) gives the system of equations

$$\mathbf{r}\left(\mathbf{\Phi}\right) \;=\; 0, \tag{5}$$

whose solution is grouped by circumferential mode within the vector $\mathbf{\Phi} = (\mathbf{\Phi}_{-M},\,\ldots\,,\mathbf{\Phi}_0,\,\ldots\,,\mathbf{\Phi}_{+M})$. As in the axisymmetric case, the spectral 3D steady flow problem is solved by a Newton iteration with the update at step k defined by

$$\mathbf{K}\,\Delta\mathbf{\Phi}^{(k)} = -\,\mathbf{r}\,(\mathbf{\Phi}^{(k)}),$$

and solved using the Conjugate Gradient (CG) method [Barrett et. al., 1994] as the Jacobian $\mathbf{K} = \partial\mathbf{r}/\partial\mathbf{\Phi}$ is symmetric and positive definite. The update equation system is preconditioned with the block diagonal matrix

$$\mathrm{diag}\,\Big(\,\mathbf{K}_0 + M^2\mathbf{K}_\theta,\,\ldots\,,\mathbf{K}_0,\,\ldots\,,\mathbf{K}_0 + M^2\mathbf{K}_\theta\,\Big)\,,$$

an asymptotic approximation of $\mathbf{K}$ in the limit of axisymmetry whereby the asymmetry of the coordinates is regarded as a small perturbation to an otherwise axisymmetric geometry. Preconditioning is equivalent to a series of inexpensive direct 2D solutions, one for each circumferential mode.

Similarly, the weak formulation (2) yields the discrete acoustic problem

$$\left(\hat{\mathbf{L}} + \hat{\mathbf{F}}\right) \hat{\mathbf{\Phi}} = \hat{\mathbf{f}}, \tag{6}$$

with the acoustic solution also grouped by the modal number: $\hat{\mathbf{\Phi}} = (\hat{\mathbf{\Phi}}_{-M}, \ldots, \hat{\mathbf{\Phi}}_0, \ldots, \hat{\mathbf{\Phi}}_{+M})$. The forcing term has the form $\hat{\mathbf{f}} = (0, \ldots, \hat{\mathbf{f}}_0, \ldots, 0)$ which reflects the fact that an acoustic perturbation is only prescribed at the fan face which is necessarily axisymmetric, *i.e.* $m=0$. However, because the matrix $\hat{\mathbf{L}}$ couples all the circumferential modes in the solution $\hat{\mathbf{\Phi}}$, the axisymmetric mode $\hat{\mathbf{\Phi}}_0$ of the spectral solution $\hat{\mathbf{\Phi}}$ differs from the acoustic solution of the axisymmetric problem.

The farfield boundary can be chosen to be axisymmetric, leaving the inlet surface the only asymmetric boundary. Then, if there are no acoustic liners, because $\beta = \hat{\beta} = 0$ on the inlet wall, the boundary operator $\hat{\mathbf{F}}$ is block diagonal, with block $\hat{\mathbf{F}}_m$ representing the appropriate boundary condition for acoustic mode $m+\kappa$. However, the acoustic liner boundary condition couples the modes of the solution $\hat{\mathbf{\Phi}}$ and the diagonality of $\hat{\mathbf{F}}$ is broken.

Although the matrix $\hat{\mathbf{L}}$ is Hermitian, the boundary operator $\hat{\mathbf{F}}$ is not, so the acoustic problem is solved using an iterative solver appropriate for non-Hermitian systems. The Quasi-Minimal Residual (QMR) [Freund and Nachtingal, 1991, Barrett et. al., 1994] algorithm has been used in this work and, though not guaranteed to converge, it has been found to have a very good performance on the inlet acoustic problem. The QMR iterations are preconditioned using

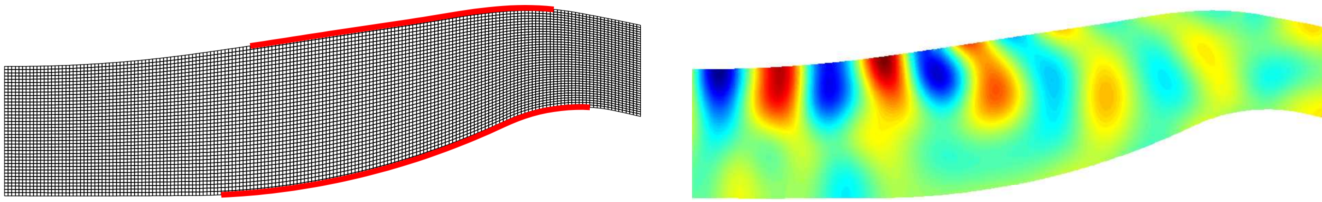
$$\text{diag}\left(\hat{\mathbf{L}}_{-M} + \hat{\mathbf{F}}_{-M}, \ldots, \hat{\mathbf{L}}_0 + \hat{\mathbf{F}}_0, \cdots, \hat{\mathbf{L}}_M + \hat{\mathbf{F}}_M\right),$$

which is again an asymptotic approximation of the system matrix $\hat{\mathbf{L}} + \hat{\mathbf{F}}$ in the limit of axisymmetry, where the asymmetry of both the coordinates and the solution is regarded as a small perturbation to an otherwise axisymmetric geometry and flow. This preconditioner then depends only on the axisymmetric coordinates and steady potential values.

RESIDUAL EVALUATION

The iterative solution algorithms CG and QMR do not require the expensive storage of the system matrices but rather the corresponding residuals, *i.e.* matrix/vector products of the type $\mathbf{K}\mathbf{\Phi}$ or $\hat{\mathbf{L}}\hat{\mathbf{\Phi}}$. Both residuals are discretisations of the volume integrals in the respective weak formulations (1) and (2) and are computed in the same manner which involves a transformation from the modal Fourier domain to the domain of θ dependence as well as the transformation back to the modal representation. Thus, the volume integrals are evaluated over each element using Gauss quadrature so, for any Gauss point coordinates (ξ, η), the coordinates x, r and potential values ϕ, $\hat{\phi}$ are first computed along with their gradients with respect to (x, r, θ) at a number M_θ of "virtual" equally spaced points in the circumferential direction. This is achieved by applying the inverse FFT to the respective circumferential modes of coordinates and potentials already interpolated at (ξ, η) using the shape functions N_n. To minimise aliasing errors while keeping computational cost at a low level, M_θ is the smallest power of 2 which is larger than 4 times the largest Fourier mode number in the coordinate and potential modal representation. Then, using the values at the "virtual" circumferential points, the integrands in the weak formulations are computed and then integrated in (ξ, η) through Gauss quadrature. Finally, the volume integral is completed in θ by applying the FFT to the residual values computed at the "virtual" points and the residuals are obtained in the required spectral representation.

A slight variation of this procedure is followed in the case of acoustic liners which are circumferentially non-uniform (*e.g.* spliced liners). In that case, the impedance Z is a value of θ which could



(a) (b)

Figure 3: Coarse mesh discretisation of the aft-duct of a high-bypass turbofan (a) showing the location of the acoustic liner in red. The acoustic pressure solution (b) excited by the duct eigenmode of radial order 0 and circumferential mode number 20 input at the fan outlet guide vane on the left.

transmitted \ incident	0	1	2
0	0.8674	0.4191	1.2729
1	3.7296	4.2327	6.0781
2	0.6121	0.1451	1.6085

Table 1: Table of errors (measured in %) of the intensity values in transmitted radial modes 0, 1 and 2 ouput by the new method relative to the corresponding values from a commercial aeroacoustics software. The duct eigenmodes of radial order 0, 1 and 2 were in turn incident at the fan boundary; because the duct is not cylidrical, each input radial mode also energizes all other radial modes inside the duct.

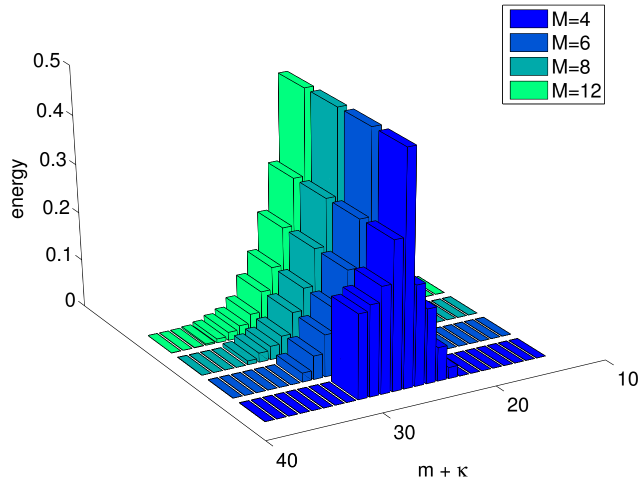
be specified at a number of non-uniformly spaced circumferential locations and/or could vary non-continuously. The FFT algorithm and its inverse must then be replaced by a computationally slower implementation of the discrete Fourier transform on non-uniformly spaced points.

RESULTS

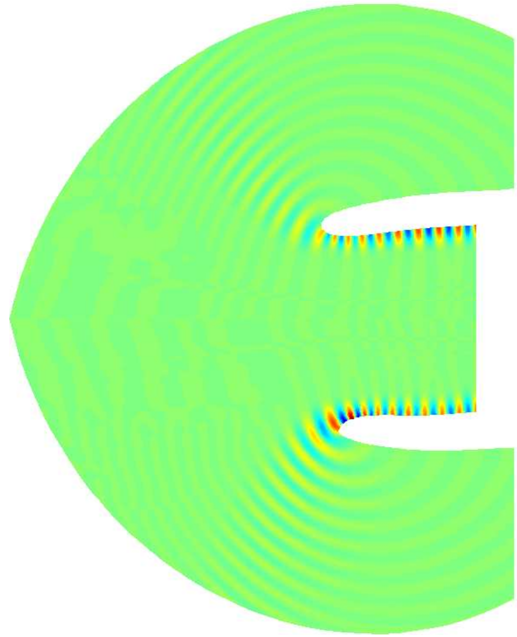
The new computer code has been validated on a number of testcases. Some of these testcases have analytic solutions against which the numerical solutions can be directly compared, *e.g.* a pulsating/vibrating sphere in stagnant gas or a constant area duct with uniform flow. In all cases, the numerical solution converged to the analytic in the limit of a fine computational mesh.

Another interesting validation test compared the solution obtained using the new method with the result from the proprietary aeroacoustics software ACTRAN [ACTRAN, 2003]. Validation against ACTRAN is particularly relevant as the boundary condition used to model the acoustic liner treatment is common [Eversman, 2001]. Two axisymmetric geometries were considered in this test, the first a cylindrical duct and the second an annular aft duct of a modern high-bypass turbofan, Fig. 3. Both ducts contained mean flow and had acoustic liners. The new method used 9-node bi-quadratic quadrilateral elements while ACTRAN used 8-node serendipity quadrilaterals; apart from the mid element nodes, the computatational meshes used by the two codes were the same. Also, the mean flow fields used by both the new code and ACTRAN were identical at the common mesh nodes while the new code used the mean value of the neighbouring nodes at the mid element nodes. Keeping the freequency at 1,600 Hz and the circumferential mode number at 20, the duct eigenmodes of radial orders 0, 1 and 2 were in turn input at the inflow boundary and the modal pressure intensities [ACTRAN, 2003] transmitted in the radial orders 0, 1 and 2 were computed at the outflow. The modal intensity values proved to be in very good agreement and they were no more than 3% off each other in the case of the cylindrical duct and 6% in the case of the aft duct, Table 1.

To demonstrate the spectral 3D method on a realistic case, the acoustic field propagated from a real engine inlet was computed. The inlet geometry is asymmetric with a scarfing angle of 5° (the angle with which the plane of the inlet front is tilted with respect to the engine axis). It was found



(a)



(b)

Figure 4: The variation of the "energy" distribution across mode numbers m with the total number M of circumferential modes used to represent the acoustic solution (a); there is no practical difference between the distribution at $M = 8$ and $M = 12$. An axial section depiction of the acoustic pressure solution obtained using $M = 8$ (b).

this geometry can be accurately described by only five circumferential Fourier modes in the spectral coordinate representation (3), *i.e.* $-2 \leq m \leq +2$. The steady mean flow Mach number is 0.3 at the freestream and 0.4 at the fan. Considering the first blade passing frequency excitation and 26 fan blades, the circumferential mode number is $\kappa = 26$ and the reduced frequency (based on engine radius and freestream speed of sound) is $\omega R/c_\infty = 30$. Using bi-quadratic quadrilateral elements and a minimum of 8 nodes per axial wavelength, the axisymmetric mesh has 14,000 nodes. Although only 5 circumferential Fourier modes describe the geometry, the number of modes required to accurately represent the acoustic field is greater; in order to determine this number, a series of calculations were performed using $M = 4$, 6, 8 and 12 modes. The distribution of "energy" (the RMS of the solution vector in each mode) across modes was computed, Fig. 4(a). Using a too low number of Fourier modes ($M = 4$), "energy" is "trapped" within the modelled spectrum at a loss of solution accuracy but using a too high number of modes ($M = 12$), the "energy" content of the extreme modes in the spectrum is too low to affect the acoustic solution. It was thus found that at $M = 8$ modes the "energy" spectrum is already converged, and using the Fourier modes $-8 \leq m \leq +8$ is practically sufficient to obtain the solution with satisfying accuracy; the acoustic field is depicted in Fig. 4(b). The number $M = 8$ means a total of 0.24 million unknowns so the advantage of the spectral method becomes clear; the equivalent acoustic calculation using a standard 3D FE formulation with 8 nodes per wavelength in the circumferential direction require approximately 3 million unknowns.

Finally, a series of calculations were performed to illustrate the convergence properties of QMR. Keeping the axisymmetric circumferential mean of the same inlet geometry as above but scaling the modes of asymmetry, configurations with scarfing angles of 1° (almost axisymmetric), 2° and 5° (original geometry) were obtained. Figure 5 depicts the QMR convergence histories for the acoustic problems in these three cases and shows clearly how the QMR convergence rate is fastest when the deviation from axisymmetry is smallest. This is explained by the fact that the preconditioner, based on an asymptotic axisymmetric approximation, is most effective when the degree of asymmetry is smallest.

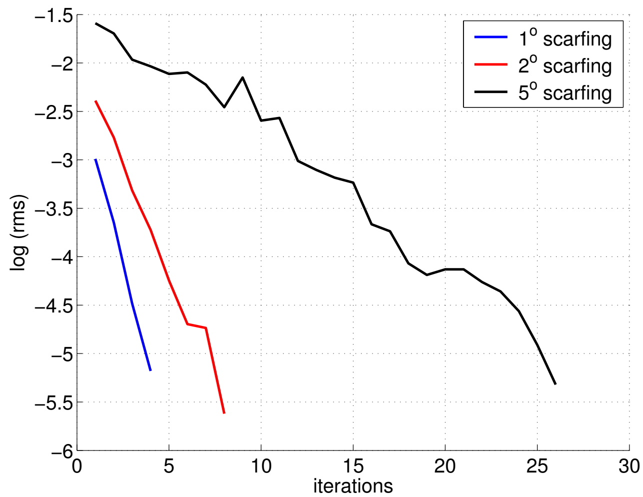


Figure 5: The convergence histories of the QMR method for the inlet with 1°, 2° and 5° scarfing. The lower the degree of asymmetry, the more efficient the preconditioner is and the faster the convergence.

CONCLUSION

This paper has introduced a new method for the analysis of the tone noise radiation from 3D turbofan inlets, combining a standard FE discretisation of the acoustic field in the axial and radial coordinates with a Fourier spectral representation in the circumferential direction. The main features of the method have been discussed, starting from the flow modelling assumptions, continuing with the Fourier spectral discretisation and ending with the axisymmetric preconditioner used to solve the acoustic problem iteratively. Numerical examples of practical relevance have illustrated the method features, in particular how few Fourier modes need to be retained in the solution, and demonstrated the effectiveness of the preconditioner.

Although the proposed method is particularly targeting the niche application of inlet aeroacoustics, it can employed without modification in any situation where the degree of axial asymmetry is relatively low. In any such case, because a relatively small number of circumferential Fourier modes are adequate for an accurate field representation, the method is far less costly than the conventional 3D FE approach. The method is also suited to cope with boundary conditions involving circumferentially varying or discontinuous parameters (*e.g.* acoustic liner impedance).

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