Stabilization of a Linearized Navier-Stokes Solver for Turbomachinery Aeroelasticity

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Abstract. The linear analysis of turbomachinery aeroelasticity relies on the assumption of small level of unsteadiness and requires the solution of both the nonlinear steady and the linear unsteady flow equations. The objective of the analysis is to compute a complex flow solution which represents the amplitude and phase of the unsteady flow for the frequency of unsteadiness of interest. The solution procedure of the linear harmonic Euler/Navier-Stokes solver often consists of a preconditioned fixed-point iteration which in some circumstances may become numerically unstable. The paper summarizes the use of the Recursive Projection Method and the Generalized Minimum Residuals algorithm to provide stabilization and presents a realistic application of both approaches.

1 Introduction

Blade flutter and forced response of turbomachinery rotors [1] are aeroelastic phenomena which may both lead to dramatic mechanical failures if not properly accounted for in the design of the engine. Flutter occurs if the working fluid feeds energy into the vibrating blades, while forced response vibrations are due to an external source of excitation such as the wakes shed by an upstream blade-row. The estimate of both the mean energy flux between fluid and structure in the flutter case and the unsteady forces acting on the blades in the forced response problem requires knowledge of the unsteady flow field. A number of methods have been developed to carry out the analysis of turbomachinery aeroelasticity [9], ranging from uncoupled linearized potential flow solvers [4] to fully-coupled nonlinear three-dimensional unsteady viscous methods [5] and within this range the uncoupled linear harmonic Euler and Navier-Stokes (NS) methods have proved to be a successful compromise between accuracy and cost. This approach relies on the fact that the level of unsteadiness in turbomachinery flows is small and hence views it as a small perturbation of a space-periodic mean steady flow. The unsteady flow field can be linearized about the steady one and due to linearity can be decomposed into a sum of harmonic terms, each of which can be computed independently. The analysis considers a single blade passage rather than the whole blade-row thanks to the cyclic periodicity of both the steady and unsteady flows, thus leading to a great reduction of computational costs. The periodic boundary condition of the linear harmonic equations introduces a phase-shift between the two periodic boundaries, known as *Inter-Blade Phase Angle* (IBPA). The small amplitude of the unsteady aerodynamic forces with respect to the mechanical forces often allows one to neglect the aerodynamic coupling of structural modes and the investigation can be carried out considering one mode at a time. The complete analysis [2] consists of two phases: *a*) calculation of the nonlinear steady flow field about which the linearization is performed and *b*) solution of the linear harmonic equations.

The *HYDRA* suite of parallel codes developed at the *Oxford University Computing Laboratory* includes both a nonlinear (*hyd*) [6] and a linear harmonic (*hydlin*) [3] solver of the inviscid and viscous equations and the solution procedure of both codes is a preconditioned fixed-point iteration. Usually the linear code converges without difficulty, but numerical instabilities have been encountered in situations in which the steady flow calculation itself failed to converge to a steady state but instead finished in a low-level limit cycle, often related to some physical phenomenon such as corner stalls. The main objectives of this paper are to: 1) investigate the numerical instabilities of *hydlin* and 2) demonstrate its stabilization achieved by means of two methods: the *Recursive Projection Method* (RPM) and the *Generalized Minimal Residuals* (GMRES) algorithm.

2 Linear equations

The discrete linear harmonic Euler and NS equations [2] can be viewed as a complex linear system $Ax = b$ of dimension k equal to the product of the number of grid nodes and flow variables. The matrix A depends on the sensitivity of the nonlinear residuals to flow perturbations and the right-hand-side vector b is due to incoming perturbations through the inflow or outflow boundary in the forced response case and to the harmonic deformation of the grid in the flutter problem. The unknown complex vector x represents the amplitude and phase of the unsteady flow for the frequency of unsteadiness of interest. The linear solver *hydlin* can be regarded as the fixed-point iteration:

$$x_{n+1} = (I - M^{-1}A)x_n + M^{-1}b \qquad (1)$$

in which M^{-1} is the preconditioning operator resulting from the Runge-Kutta time-marching algorithm, the Jacobi preconditioner and the multigrid scheme [3]. It should be noted that M^{-1} depends on several numerical parameters such as the number of iterations on each grid level and neither M^{-1} nor A are built explicitly, as *hydlin* only uses the matrix-vector products $M^{-1}Ax$. Linear stability analysis of (1) shows that necessary condition for its convergence is that all the eigenvalues of $M^{-1}A$ lie in the unit disc centred at $(1, 0)$ in the complex plane. For most aeroelastic problems of practical interest, this condition is fulfilled and *hydlin* converges without difficulty. However an exponential growth of the residual has been encountered for some turbomachinery test-cases caused by a few complex conjugate eigenvalues lying outside the unit disc (*outliers*). In these circumstances the steady flow calculation itself usually failed to converge to a steady-state but instead finished in a small-amplitude limit cycle, related to some physical instability such as flow separations or vortex shedding. The solution procedure of *hyd* is not time-accurate but it nevertheless reflects some physical properties of the flow field due to the pseudo time-marching strategy associated with the Runge-Kutta algorithm. Small-amplitude limit cycles do not prevent the steady solver from converging (their effect is sometimes visible in small oscillations of the residual around a constant low level), but they result in a small number of complex conjugate outliers causing the exponential growth of the residual in the linear calculation. Two different solutions to this problem have been achieved implementing RPM and GMRES in *hydlin*.

3 RPM

RPM is an iterative algorithm for the solution of linear and nonlinear systems [8] and is based on the projection of $M^{-1}A$ onto the orthogonal subspaces $\mathcal{P}$ and $\mathcal{Q}$ of $\mathcal{R}^k$ associated respectively with the subset of m_{out} outliers and that of the remaining $(k-m_{out})$ eigenvalues lying in the unit disc. At each RPM iteration the projection of the linear equations on the low-dimensional subspace $\mathcal{P}$ is solved with Newton's method and that on the subspace $\mathcal{Q}$ with the standard fixed-point iteration (1). Denoting by Z an orthonormal basis of $\mathcal{P}$, the orthogonal projectors P and Q on the subspaces $\mathcal{P}$ and $\mathcal{Q}$ are defined respectively as $P=ZZ^T$ and $Q=I-P$. The basis Z is augmented with the current dominant eigenmode each time the calculation is diverging or converging very slowly. The projections f and g of (1) on $\mathcal{P}$ and $\mathcal{Q}$ are defined respectively as

$$f = P[(I - M^{-1}A)x + M^{-1}b] \qquad g = Q[(I - M^{-1}A)x + M^{-1}b]$$

and the outline of the RPM loop is:

$$p_{init} = Px_{init}, \qquad q_{init} = Qx_{init}$$

Do until convergence:

$$i.\ \ p_{\nu+1} = p_\nu + (I - f_p)^{-1}(f(p_\nu, q_\nu)) - p_\nu)$$

$$ii.\ \ q_{\nu+1} = g(p_\nu, q_\nu))$$

$$x_* = p_* + q_* = p_{\nu_{final}} + q_{\nu_{final}}$$

where $p = Px$, $q = Qx$ and $f_p = P(I - M^{-1}A)P$. It is easily verified that

$$(I - f_p)^{-1} = Z[I - Z^T(I - M^{-1}A)Z]^{-1}Z^T = ZH^{-1}Z^T$$

where H is a small matrix of size m_{out}, whose inversion requires minimum computational effort. The stability analysis of this algorithm shows that its spectral radius is smaller than 1, that is the stabilized RPM iteration is stable. The implementation of RPM in *hydlin* has required only minor changes to the existing code, as q is determined using the core-part of the code performing the standard fixed-point iteration (1) and the remaining computationally cheap operations are performed at the top routine-level.

4 GMRES

GMRES is an iterative method for the solution of linear systems which belongs to the family of Krylov subspace methods [7] and is guaranteed to converge even in the presence of outliers. The Krylov subspace of dimension m generated by $M^{-1}A$ and b is the vectorial space spanned by the powers $((M^{-1}A)^j b, \;\; j = 0, \dots m-1)$, that is

$$\mathcal{K}_m = < b, M^{-1}Ab, \dots, (M^{-1}A)^{m-1} b > .$$

The GMRES algorithm is based on the progressive reduced Arnoldi factorization [7] of $M^{-1}A$:

$$M^{-1}AQ_m = Q_{m+1}\tilde{H}_m \tag{2}$$

where $\tilde{H}_m$ is a Hessemberg matrix of size $((m+1) \times m)$, Q_m is a matrix whose m columns form an orthonormal basis for $\mathcal{K}_m$ and Q_{m+1} is Q_m augmented with a new Krylov vector. The length k of each column of Q_m is equal to that of the complex linear flow field. At the m^{th} GMRES iteration the solution x is approximated by the linear combination of the available m Krylov vectors which minimizes the 2-norm of the residual.

The preconditioned GMRES algorithm implemented in *hydlin* uses its core part as a black-box to determine the Krylov vectors which are preconditioned in the existing way (multigrid+Runge-Kutta+Jacobi preconditioner) and the computationally cheap optimization is carried out at the top routine level. The *restart* option [7] is used in order to limit the required memory. Using between 10 and 30 GMRES iterations per restarted cycle makes the computation affordable even for large problems and a good convergence level is usually achieved within 20 restarted cycles.

5 Results

The considered test-case is a three-dimensional fan rotor whose geometry and surface grid are shown in fig. 1-a. This grid has only 157441 nodes and is quite coarse, but all the phenomena discussed in this section have been also observed with finer computational meshes and for other test-cases. The complete flutter analysis of this rotor is reported in [2] and shows that the rotor is aeroelastically stable for all considered steady working conditions for $IBPA = 180^0$. However all linear calculations using the standard fixed-point iteration (1) do not converge. Figure 1-b provides the residual histories of *hydlin* for the near-stall mean steady conditions and for $IBPA = 180^0$ obtained using the RPM and the GMRES solvers with different numerical parameters. The solid line refers to the RPM solver which adds the current dominant eigenmode to the subspace $\mathcal{P}$ only if the calculation diverges. The iterations at which a new partitioning of $M^{-1}A$ is carried out are labelled from 1 to 4. Before the first dominant mode is added to $\mathcal{P}$ this convergence history is that of the standard preconditioned iteration (1) which therefore does not converge. Conversely the stabilized RPM iteration converges (branch $4_0 - E_0$) once all the unstable modes have been included in $\mathcal{P}$. The subset of the spectrum of $M^{-1}A$ with the first 150 dominant eigenvalues is provided in fig. 2, which reveals the presence of 4 complex conjugate outliers (eigenvalues labelled from 1 to 4). The complex conjugate eigenpair in the unit disc labelled with 5 determines the asymptotic convergence rate of

RPM (slope of the branch $4_0 - E_0$). The dotted line in fig. 1-b refers to the convergence of RPM obtained adding to $\mathcal{P}$ also the eigenpair 5 at the iteration labelled with 5. The slope of the branch $5 - E_1$ is steeper than that of $4_0 - E_0$, since in the former case the asymptotic convergence rate is determined by the eigenpair 6, which is closer than 5 to the centre of the unit disc. The residual of the calculation with restarted GMRES performing 10 iterations per restarted cycle and one multigrid cycle per GMRES iteration stagnates (dashed line in fig. 1) and an acceptable convergence rate can be retrieved only by using 30 GMRES iterations per restarted cycle and three multigrid cycle per GMRES iteration (dashed-dotted line). The analysis of the dominant eigenmodes [2] has shown that these numerical instabilities are due to small physical unsteadiness of the nonlinear flow field such as the hub corner stall.

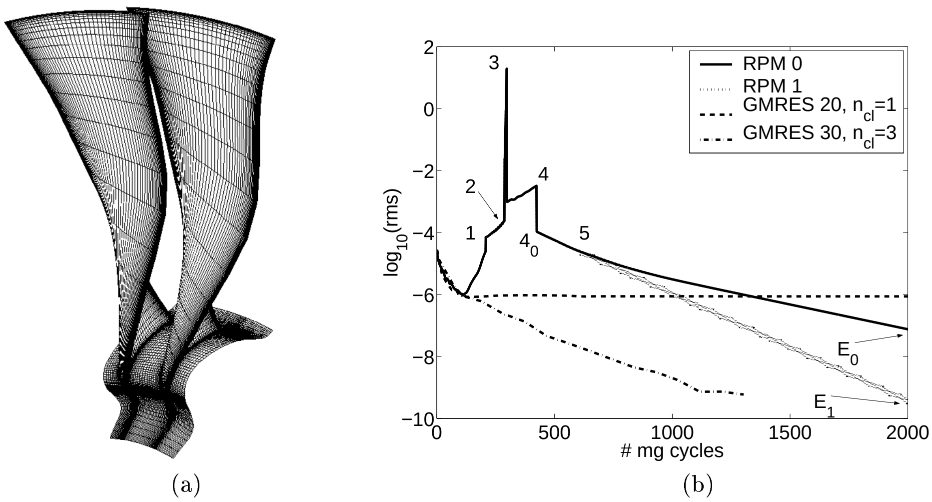

Fig. 1. **(a)** Fan geometry and surface mesh and **(b)** convergence plots of *hydlin*

6 Conclusions

The implementation of the RPM and GMRES algorithms in the existing linear solver based on a preconditioned fixed-point iteration has stabilized the code. This allows one to carry out the aeroelastic analysis even in the presence of small unsteady phenomena in the mean flow, which are believed not to affect significantly the aeroelastic behaviour of the component. The asymptotic convergence rate of the restarted GMRES algorithm depends on the spectrum of the linear operator, on the number of GMRES iterations per restarted cycle and the number of multigrid cycles per GMRES iteration. The extra memory allocation for storing the Krylov vectors depends only on the number of GMRES iterations per restarted cycle and not on the number of outliers. The asymptotic convergence rate of RPM depends on the spectral radius of the projection of the linear operator onto the stable space $\mathcal{Q}$. The required extra memory allocation depends

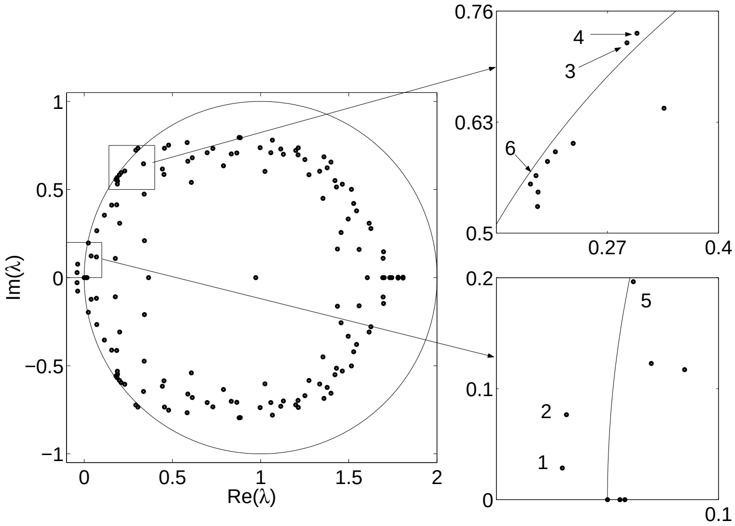

Fig. 2. First 150 eigenvalues of $M^{-1}A$

on the number of outliers and is comparable with that of the restarted GMRES with 10 iterations per restarted cycle if the linear operator has not more than 4 complex conjugate pairs of outliers. Therefore the overall CPU-time and extra memory allocation using either solver is considerably case-dependent.

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